Hölder and Markov capacities in $\mathbb{C}^N$

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Abstract. Let $V_E$ be the pluricomplex Green function associated to a compact subset $E$ of $\mathbb{C}^N$. The well known Hölder Continuity Property of $E$ means that there exist constants $A > 0, \gamma \in (0,1]$ such that

$$|V_E(w) - V_E(z)| \leq A |w - z|^\gamma.$$  

It turns out that this condition is equivalent to a Vladimir Markov type inequality, i.e.

$$\|D^\alpha P\|_E \leq M^{|\alpha|} (\deg P)^m |\alpha|! \gamma^{1-m} \|P\|_E,$$

where $m, M > 0$ are independent of the polynomial $P$ of $N$ variables and $\| \cdot \|_E$ is the supremum norm on $E$. In this context we give a definition of Markov and Hölder capacities that are strictly related to the best constants in Markov and Hölder inequalities. We will present some links between these capacities and the L-capacity of $E$ given by

$$C(E) = \liminf_{\|z\|_2 \to \infty} \frac{\|z\|_2}{\exp V_E^*(z)}.$$