Sparse Interpolation

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Throughout computational science and engineering, several attempts have been made to represent data in a parsimonious way. Mathematical models are proposed in which the major features of the data are represented using only a few terms, in other words, models that use a sparse combination of generating elements instead of a linear combination of all basis elements. Besides the accuracy of a representation, its sparsity has gradually become a priority. This is because a sparser model means higher compression, less data collection, storage or transmission, and a reduced model complexity.

A representation is called t-sparse if it is a combination of only t elements. In sparse interpolation, the aim is to determine both the support of the sparse linear combination and the scalar coefficients in the representation, from a small or minimal amount of data samples. Sparse techniques solve the problem statement from a number of samples proportional to the number t of terms in the representation rather than the number of available data points or available generating elements.

We indicate the connections between sparse interpolation, coding theory, generalized eigenvalue computation, exponential analysis and rational approximation. In the past few years, insight gained from the computer algebra community combined with methods developed by the numerical analysis community, has lead to significant progress in several very practical and real-life signal processing applications. We make use of tools such as the singular value decomposition and various convergence results for Padé approximants to regularize an otherwise inverse problem. Classical resolution limitations in signal processing with respect to frequency and decay rates, are overcome.

In the illustrations we particularly focus on multi-exponential models

$$\phi(t) = \sum_{i=1}^{t} \alpha_i \exp(\phi_i t), \qquad \alpha_i = \beta_i + \mathtt{i}\gamma_i, \quad \phi_i = \psi_i + \mathtt{i}\omega_i, \tag{1}$$

representing signals which fall exponentially with time. These models appear, for instance, in transient detection, motor fault diagnosis, electrophysiology, magnetic resonance and infrared spectroscopy, vibration analysis, seismic data analysis, music signal processing, dynamic spectrum management such as in cognitive radio, nuclear science, and so on.