Sparse wavelet approximation of signals and images

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Within the last time, various adaptive methods for signal and image approximation have been suggested. Instead of choosing a priori a basis or a frame to approximate an image, one tries to adapt the approximation scheme to the image geometry.

The proposed methods include e.g. bandelet orthogonal bases by Le Pennec and Mallat, the geometric wavelet transform by Dekel and Leviatan, the tetrolet transform by Krommweh, or image approximation schemes based on lifting schemes by Arandiga et al., Claypoole at al., Baraniuk et al. and others.

In our group, the nonlinear locally adaptive easy path wavelet transform (EPWT) has been explored for sparse image representations. The EPWT is slightly related with the idea of grouplets by Mallat, where one applies a weighted Haar wavelet transform to points that are grouped by a so-called association field. Other generalizations and modifications include the generalized tree-based wavelet transform and redundant wavelet constructions on graphs and high-dimensional data clouds by Ram, Elad and Cohen.

The main idea of the EPWT is as follows. In a first step, we determine a "path vector" through all indices of a given (two-dimensional) index set of the image. Starting with a suitable index resp. the corresponding image value, we apply a "best neighbor strategy" such that on the one hand adjacent pixels in the path are neighbor pixels in the image, and on the other hand the corresponding adjacent image values are strongly correlated. Then, one level of a (one-dimensional) wavelet transform is applied to the image values along the path vector. In the following levels, one needs to find path vectors through index sets of a low-pass image and applies again the wavelet transform.

In the talks, recent methods for adaptive image representation will be summarized with a focus on the EPWT and its generalizations.