DYNAMICS AND INTEGRABILITY OF NONHOLONOMIC AND OTHER NON-HAMILTONIAN SYSTEMS

PADOVA, JANUARY 24–27, 2018,

DEPARTMENT OF MATHEMATICS "TULLIO LEVI–CIVITA",

UNIVERSITÀ DEGLI STUDI DI PADOVA

Dynamics and integrability of nonholonomic and other non-Hamiltonian systems

24-27 January, 2018 - Department of Mathematics "Tullio Levi-Civita", University of Padova

List of invited speakers

Paula Balseiro	Ivan Mamaev
Larry Bates	Alessia Mandini
Alexey Bolsinov	David Martin de Diego
Vladimir Dragovic	Eva Miranda
Yuri Fedorov	James Montaldi
Bozidar Jovanovic	Maria Prybylska
Mark Levi	Dmitry Zenkov
Franco Magri	Nguyen Tien Zung

This workshop is part of the Intensive Period "Hamiltonian Systems" http://events.math.unipd.it/SH2018/ of the Department of Mathematics "Tullio Levi-Civita" of the University of Padova

DIPARTIMENTO MATEMATICA

UNIVERSITÀ DEGLI STUDI DI PADOVA

Scientific and organizing commitee

F. Fasso' (Padova), L. Garcia-Naranjo (Mexico City), B. Khesin (Toronto), J.C. Marrero (La Laguna), T. Ratiu (Shanghaj) and N. Sansonetto (Verona)

Booklet Integrability 2018

SCHEDULE

Dipartimento di Matematica "Tullio Levi-Civita", via Trieste 63, Padova, Room 1C150

	Wednesday, January 24	Thursday, January 25	Friday, January 26	Saturday, January 27
9:00		Miranda	Mamaev	Przybylska
9:30				
10:00		Grabowski	Tondo	Rossi
10:30		**Coffee break**	**Coffee break**	**Coffee break**
11:00		Fedorov	Balseiro	Montaldi
11:30				
12:00		Zung	Bates	Jovanovic
12:30				
13:00		**Lunch**	**Lunch**	**Lunch**
13:30				
14:00	***Registration***			
14:30				
14:45	***Openinng***			
15:00	Magri	Bolsinov	Dragovic	
15:30				
16:00	Martin de Diego	Sousa-Dias	Diaz	
16:30		**Coffee break**	**Coffee break**	
17:00	Abenda	Mandini	Levi	
17:30	**Welcome drink**			
18:00		Mestdag	Jimenez Alburquerque	
18:30		Millionshchikov		

DIPARTIMENTO MATEMATICA

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Program

- *Simonetta Abenda*, "On a family of KP multi–line solitons associated to rational degenerations of real hyperelliptic curves and to the finite non–periodic Toda hierarchy"
- *Paula Balseiro*, "The failure of the Jacobi identity of nonholonomic brackets"
- *Larry Bates*, "Reduction without symmetry"
- *Alexey Bolsinov*, "Hamiltonisation of non-holonomic systems"
- *Viviana Alejandra Díaz*, "Reduced equations and integrability for Euler's disk"
- *Vladimir Dragovic*, "Pseudo-integrable billiards"
- *Yuri Fedorov*, "TBA"
- *Janusz Grabowski*, "On Lie's result about integrability of vector fields by quadratures"
- *Fernando Jimenez Alburquerque*, "Fractional variational description of mechanical systems with linear dissipation: Lagrangian and Hamiltonian pictures"
- *Bozidar Jovanovic*, "Symmetries of characteristic line bundles and Noether theorem for non-holonomic systems"
- *Mark Levi*, "Rotation–induced magnetism, Gaussian curvature and the gyroscopes"
- *Franco Magri*, "The Central Equation of Mechanics"
- *Ivan Mamaev*, "Invariant submanifolds of genus 5 and a Cantor staircase in the dynamics of a snakeboard"
- *Mandini Alessia*, "Hyperpolygon spaces and parabolic Higgs bundles"
- *David Martín de Diego*, "On the geometry of mechanical systems subject to affine nonholonomic constraints"
- *Tom Mestdag*, "Reduction and un-reduction of mechanical systems"
- *Dmitry Millionshchikov*, "Growth of Lie algebras and integrability of non-linear hyperbolic PDE"
- *Eva Miranda*, "Constructing and destructing tori in singular symplectic manifolds"
- *James Montaldi*, "The dynamics of an axisymmetric rigid body with Veselova constraint"
- *Maria Przybylska*, "Integrability properties of certain generalisations of Suslov problem"
- *Paolo Rossi*, "Hamiltonian and non-Hamiltonian integrable systems of evolutionary PDEs from the moduli space of curves"
- *M. Esmeralda Sousa–Dias*, "The dynamics of (Poisson) maps from mutation–periodic quivers"
- *Giorgio Tondo*, "Haantjes Algebras and Diagonalization"

Titles and abstracts

Simonetta Abenda
Università di Bologna, Italy

On a family of KP multi–line solitons associated to rational degenerations of real hyperelliptic curves and to the finite non–periodic Toda hierarchy

Abstract. In our research project with P.G. Grinevich, to any point of the real totally positive Grassmannian $Gr^{\mathrm{TP}}(k,n)$ we associateÂ a reducible curveÂ which is a rational degeneration of an M–curve of minimal genus $g = k(n-k)$, and we reconstruct the realÂ algebraic-geometric data á la Krichever for the underlying real bounded multiline KP soliton solutions.Â In particular, if $k = 1$, Γ is a certain rational degeneration of a hyperelliptic M–curve of genus $n-1$. The characterization of the soliton data in $Gr^{\mathrm{TP}}(k,n)$ which produce real and regular KP divisors on Γ singles outÂ a special family of KP multi–line solitons (T–hyperelliptic solitons) which are naturally connected to the finite non-periodic Toda hierarchy.Â I shall present the relations between the algebraic-geometric description of the open Toda system by Krichever and Vaninsky and that of KP T–hyperelliptic solitons following from our construction. Finally, I also explain the effect of the space–time transformation which conjugates soliton data in $Gr^{\mathrm{TP}}(k,n)$ to soliton data in $Gr^{\mathrm{TP}}(n-k,n)$ on the Krichever divisor for such KP solitons.

Paula Balseiro
UFF

The failure of the Jacobi identity of nonholonomic brackets

Abstract. In this talk we will discuss geometric features of nonholonomic systems and their behaviour after a reduction by a group of symmetries. In particular, we will show how the failure of the Jacobi identity is modified after a reduction by symmetries and also by considering 'gauge related brackets'. We will present some concrete examples where Poisson and twisted Poisson brackets appear in the description of the reduced dynamics.

Larry Bates
University of Calgary

Reduction without symmetry

Abstract. We discuss the reduction of some examples that lack symmetry, but nevertheless reduce as if they were symmetric.

Alexey Bolsinov
University of Loughborough

Hamiltonisation of non-holonomic systems

Abstract. The problem of Hamiltonization of non-holonomic systems, both integrable and non-integrable, is discussed. This question is important in the qualitative analysis of such systems and it enables one to determine possible dynamical effects.

Viviana Alejandra Díaz
Universidad Nacional del Sur, Argentina

Reduced equations and integrability for Euler's disk

Abstract. The presence of symmetry in the case of nonholonomic systems leads, upon the choice of an arbitrary principal connection, to a reduced d'Alembert's principle and to the Lagrange–d'Alembert–Poincaré equations. In the talk I shall describe how we can find reduced equations for the classical example of a thick disk rolling on a rough surface, sometimes called Euler's disk, using a 3–dimensional abelian group of symmetry. I shall also show that the reduced system can be transformed into a single second order equation, which is an hypergeometric equation. This is a joint work with Hernán Cendra.

References
[1] Cendra, H., J.E. Marsden and T.S. Ratiu [2001a] Lagrangian reduction by stages, *Memoirs of the AMS*, Volume 152, 2001.
[2] Cendra, H., J.E. Marsden and T.S. Ratiu [2001b] Geometric mechanics, Lagrangian reduction, and nonholonomic systems, *Mathematics Unlimited 2001 and Beyond*, Springer-Verlag, pages 221–273, 2001.

Vladimir Dragovic
University of Texas at Dallas

Pseudo-integrable billiards

Abstract. We will present dynamical, topological, arithmetic and geometric aspects of the pseudo-integrable billiards. We will emphasize a relationship with the rectangular billiards.

Yuri Fedorov
Universitat Politecnica de Catalonia

TBA

Abstract.

Janusz Grabowski
Polish Academy of Sciences, Poland

On Lie's result about integrability of vector fields by quadratures

Abstract. We present a substantial generalisation of a classical result by Lie on integrability by quadratures. Namely, we prove that all vector fields in a finite–dimensional transitive and solvable Lie algebra of vector fields on a manifold can be integrated by quadratures.

References

[1] Cariñena, J. F. ; Falceto, F. ; Grabowski, J. *Solvability of a Lie algebra of vector fields implies their integrability by quadratures.* J. Phys. A **49** (2016), no. 42, 425202, 13 pp.

Fernando Jimenez Alburquerque
Department of Engineering Science, University of Oxford, UK

Fractional variational description of mechanical systems with linear dissipation: Lagrangian and Hamiltonian pictures

Abstract. Dissipative systems are essentially non–Hamiltonian. Along the years, there have been several attempts to provide a variational description for them, task that we address in order to construct both Lagrangian and Hamiltonian pictures of their dynamics.
Employing a phase space which includes the (Riemann–Liouville) fractional derivative of curves evolving on real space, we develop a variational principle for Lagrangian systems yielding the so-called restricted fractional Euler–Lagrange equations, which, as we show, are invariant under linear change of variables. This variational principle relies on a particular restriction upon the admissible variation of the curves. In the case of the half-derivative and mechanical Lagrangians, i.e. kinetic minus potential energy, the restricted fractional Euler–Lagrange equations model a dissipative system in both directions of time, summing up to a set of equations that is invariant under time reversal. This process accounts for the Lagrangian picture of the linear dissipative dynamics.
After performing the usual Legendre transformation in the new phase space, we obtain the so–called restricted fractional Hamilton equations, which are based on the same restriction over the varied curves. Needless to say, these Hamilton equations are fully consistent with the Euler–Lagrange ones. This accounts for the Hamiltonian picture of the linear dissipative dynamics, and consequently for a particular Hamiltonisation of this kind of systems

Bozidar Jovanovic
Mathematical Institute SANU, Belgrade

Symmetries of characteristic line bundles and Noether theorem for non-holonomic systems

Abstract. We consider Noether symmetries of the equations defined by the sections of characteristic line bundles of nondegenerate 1-forms and of the associated perturbed systems. It appears that this framework can be used for time-dependent systems with constraints and

nonconservative forces, allowing a quite simple and transparent formulation of the momentum equation and the Noether theorem in their general forms.

Mark Levi
Penn State

Rotation–induced magnetism, Gaussian curvature and the gyroscopes

Abstract. I will describe the appearance of a magnetic–like effective force acting on particles in rapidly spinning potentials. Such a force acts, for instance, on a spinning tethered satellite, or on a spinning binary orbiting another star. On a closely related subject, I will also describe an intimate connection between the Gaussian curvature and the gyroscopic effect, and the role of Jacobi fields in this connection. The talk is based on joint work with Oleg Kirillov and with Graham Cox.

Franco Magri
Dipartimento di Matematica - Università di Milano Bicocca

The Central Equation of Mechanics

Abstract. The talk is a survey of the work of Boltzmann and Hamel on the theory of nonholonomic systems, centered around the concept of "Central Equation of Mechanics", introduced by Heun and Hamel. The purpose is to present this equation from a new perspective, and to show its role inside Classical Mechanics as well as inside Geometric Mechanics .

Ivan Mamaev
Udmurt State University

Invariant submanifolds of genus 5 and a Cantor staircase in the dynamics of a snakeboard

Abstract. In this paper we address the free (uncontrolled) dynamics of a snakeboard consisting of two wheel pairs fastened to a platform. The snakeboard is one of the well-known sports vehicles in which the sportsman executes necessary body movements. From the theoretical point of view, this system is a direct generalization of the classical nonholonomic system of the Chaplygin sleigh. We carry out a topological and qualitative analysis of trajectories of this dynamical system. An important feature of the problem is that the common level set of first integrals is a compact two-dimensional surface of genus 5. We specify conditions under which the reaction forces infinitely increase during motion and the so-called phenomenon of nonholonomic jamming is observed. In this case, the nonholonomic model ceases to work and it is necessary to use more complex mechanical models incorporating sliding, elasticity, etc.

Alessia Mandini
PUC-Rio

Hyperpolygon spaces and parabolic Higgs bundles

Abstract. Hyperpolygons spaces are a family of (finite dimensional, non-compact) hyperkähler spaces, that can be obtained from coadjoint orbits by hyperkaehler reduction. In joint work with L. Godinho, we show that these space are diffeomorphic (in fact, symplectomorphic) to certain families of parabolic Higgs bundles. In this talk I will describe this relation and use it to analyse the fixed points locus of a natural involution on the moduli space of parabolic Higgs bundles. The fixed point locus of this involution is identified with the moduli spaces of polygons in Minkowski 3-space and the identification yields information on the connected components of the fixed point locus.

This is based on joint works with Leonor Godinho and with Indranil Biswas, Carlos Florentino and Leonor Godinho

David Martín de Diego
ICMAT

On the geometry of mechanical systems subject to affine nonholonomic constraints

Abstract. We analyze the geometry of nonholonomic systems with affine nonholonomic constraints. We construct an almost-Poisson affine bracket to describe the dynamics and we study the existence of moving energies and the geometrical interpretation.

Tom Mestdag
University of Antwerp, Belgium

Reduction and un-reduction of mechanical systems

Abstract. A Lagrangian or Hamiltonian system with a symmetry Lie group can be reduced to a dynamical system on a quotient manifold. Un-reduction is the inverse process, where one associates to a Lagrangian system on a manifold a dynamical system on a principal bundle over that manifold, in such a way that solutions project [1]. We show (in [2]) that, when written in terms of second-order ordinary differential equations (SODEs), one may associate to the given system a "primary un-reduced SODE", and we explain how all other un-reduced SODEs relate to it.

References
[1] M. Bruveris, D. Ellis, D.D. Holm, F. Gay-Balmaz, *Un-reduction.* J. Geom. Mech. **3** (2011), 363–387.

[2] E. García-Toraño Andrés and T. Mestdag, *Un-reduction of systems of second–order ordinary differential equations.* Symmetry, Integrability and Geometry: Methods and Applications (2016), **115**, 20 pages.

Dmitry Millionshchikov
Lomonosov Moscow State University, Russia

Growth of Lie algebras and integrability of non-linear hyperbolic PDE

Abstract. The notion of characteristic Lie algebra of a hyperbolic system of non-linear PDE was introduced by Leznov, Shabat and Smirnov in 1982 and last years charactreristic Lie algebras of different hyperbolic systems were actively studied by Zhiber's and Habibullin's schools. We show that characteristic Lie algebras of Sine-Gordon and Tzitzeica equations are isomorphic to non-negative parts of two Kac-Moody affine algebras $A_1^{(1)}$ and $A_2^{(2)}$. Both of them are slowly growing infinite-dimensional Lie algebras. We will discuss Habibullin's conjecture which states that the integrability of a hyperbolic PDE system (continuous or discrete) implies the "minimal growth of its corresponding Lie ring".

Eva Miranda
UPC-Fondation Sciences Mathématiques de Paris

Constructing and destructing tori in singular symplectic manifolds

Abstract. Toric actions and integrable systems have always been hand in hand in the symplectic realm. Liouville-Mineur-Arnold's theorem for integrable systems on a symplectic manifold guarantees that in a neighbourhood of their regular compact fibers a toric action exists. This toric action was already used by Duistermaat to extend local action-angle coordinates (Darboux-Carathéodory) to a neighbourhood of the fiber. Duistermaat's trick works pretty well for regular Poisson manifolds (Laurent-Miranda-Vanhaecke). In this talk I will find an extension of Duistermaat's trick to the singular setting (one of the action functions is no longer smooth). Our motivation to work with singular symplectic manifolds (mainly b^m-symplectic manifolds and folded-type symplectic manifolds) comes from several examples native to celestial mechanics where regularization transformations (McGehee, Kustanheimo, etc..) yield such singularities and it is convenient to have the power of symplectic-type techniques close to the collision set/line at infinity.
In this talk, I will give a general overview of this theory and I will briefly present a desingularization procedure for the singular symplectic structure which can be extended to desingularize also integrable systems and action-angle coordinates. This game can be played further to obtain KAM theorems in the singular setting and new results concerning periodic orbits and singular Hamiltonian Dynamics.

James Montaldi
University of Manchester

The dynamics of an axisymmetric rigid body with Veselova constraint

Abstract. We show that the n-dimensional axisymmetric rigid body with Veselova constraint is an integrable system and discuss its dynamics. (Joint work with Francesco Fassò and Luis García-Naranjo.)

Maria Przybylska
University of Zielona Gora

Integrability properties of certain generalisations of Suslov problem

Joint work with Andrzej J. Maciejewski.

Abstract. Two models are considered. The first one is the classical heavy gyrostat. Its equations of motion are restricted by the non-holonomic Suslov constrain: the projection of the agular velocity of the body onto a vector constant in the body frame vanishes. Integrability of obtained system is analysed. It appears that certain integrable cases of the Suslov problem have their integrable generalisation. Additionally it is proved that for wide range of parameters of the problem system is not integrable in the Jacobi sense.
The second model is a Lie-Poisson system on six dimensional class A co-algebras generate by a quadratic Hamiltonian and restricted by a non-holonomic constrain which is a generalisation of the Suslov constrain. It appears that for all class A co-algebras there exists a generalised version the Kozlov case when the system is described by a natural Hamiltonian with two degrees of freedom. It is shown that this system is not integrable except one case. Joint work with Andrzej J. Maciejewski.

Paolo Rossi
Institut de Mathématiques de Bourgogne, France

Hamiltonian and non-Hamiltonian integrable systems of evolutionary PDEs from the moduli space of curves

Abstract. In a collaboration with A. Buryak, J. Guéré and B. Dubrovin, we have introduced a new construction of integrable Hamiltonian systems on spaces of fields in one space dimension (like the KdV equation), called the double ramification hierarchy. This construction produces an integrable system for each partial cohomological field theory, a class of geometric objects in the moduli space of stable algebraic curves. The range of integrable systems produced this way is vast, including basically all the known example of integrable tau-symmetric Hamiltonian PDEs. It also provides a new recursion for their Hamiltonians and a general quantization procedure. Tweaking the definition of a cohomological field theory it is possible to produce non–Hamiltonian integrable systems too, retaining most of the properties of their Hamiltonian counterparts, including the mentioned recursion. This last part is a joint work in progress with P. Lorenzoni.

M. Esmeralda Sousa–Dias
Instituto Superior Tecnico, Portugal

The dynamics of (Poisson) maps from mutation–periodic quivers

Joint work with: Inês Cruz and Helena Mena–Matos

Abstract. The maps arising from mutation-periodic quivers are birational maps which always preserve a presymplectic form (defined by the quiver) and often they are also Poisson maps with respect to distinct Poisson structures of quadratic type. Several aspects of the interplay between these geometric structures will be addressed as well as the respective consequences to the dynamics of such maps.

Giorgio Tondo
Università of Trieste, Italy

Haantjes Algebras and Diagonalization

Abstract. I will present the notion of Haantjes algebra, which consists in an assignment of a family of operators over a differentiable manifold, with vanishing Haantjes torsion and fulfilling suitable compatibility conditions among each others. Haantjes algebras generalize several interesting geometric structures arising in Riemannian geometry and in the theory of integrable systems. At the same time, they play a crucial role in the theory of diagonalization of operators over differentiable manifolds. Whenever the generators of a Haantjes algebra are semisimple and commute, I shall prove that there exists a set of local coordinates where all operators can be diagonalized simultaneously. Moreover, in the non-semisimple case, they acquire simultaneously a block-diagonal form.

Dmitry Zenkov
North Carolina State University

TBA

Abstract.

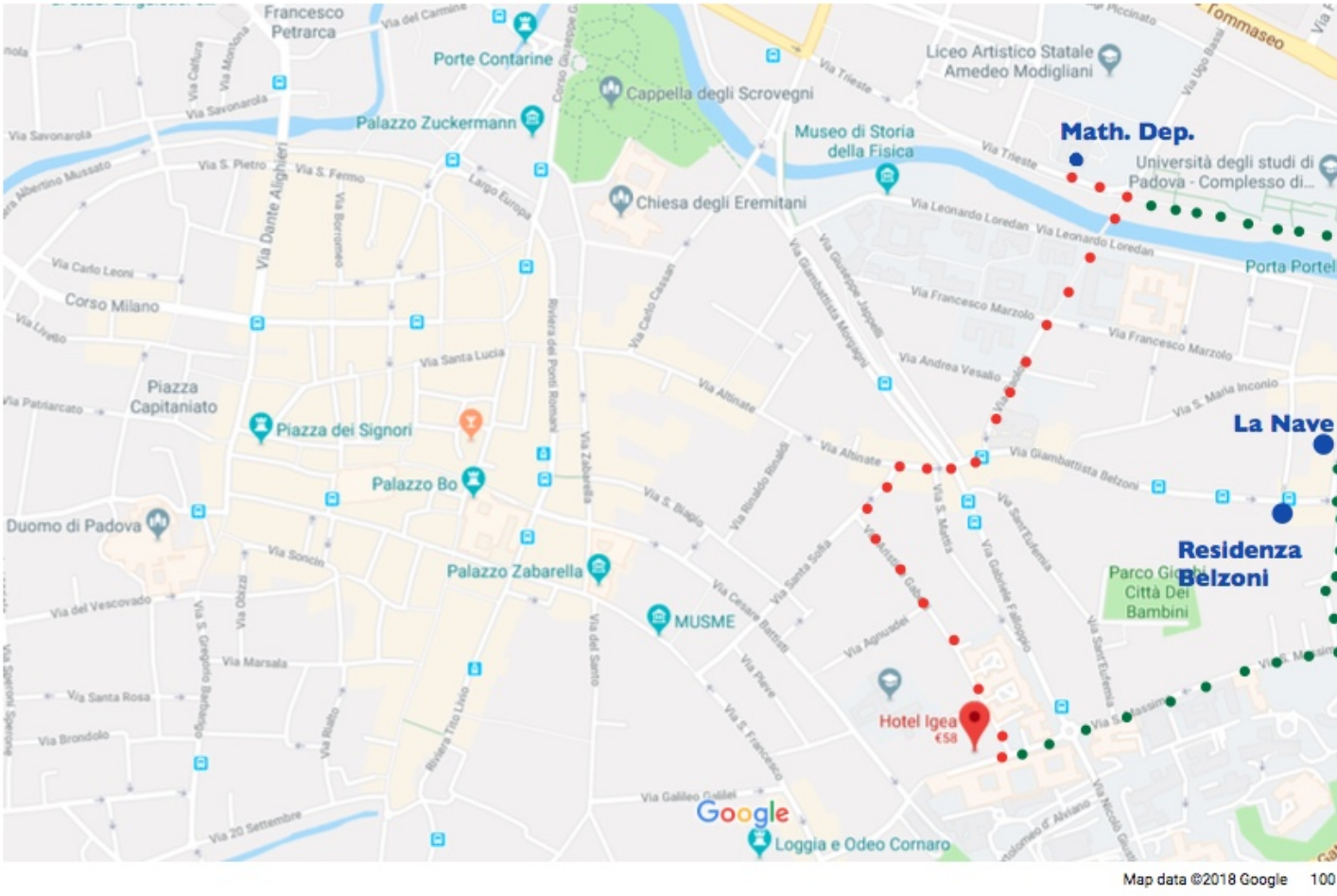
Zung Tien Nguyen
Institut de Mathématiques de Toulouse

Hamiltonianization of integrable non-Hamiltonian systems

Abstract. I want to discuss about relations between Hamiltonianity and integrability, and both positive and negative results about Hamiltonianization of integrable systems.

Some practical information

Here you can find a map of the area of the Department of Mathematics, with in evidence Foresteria La Nave, Residenza Belzoni and Hotel Igea.



Related events

This Workshop is part of the Intensive Period "**Hamiltonian System**" (www.events.math.unipd.it/SH2018) supported by the Department of Mathematics "Tullio Levi-Civita" of the Università degli Studi di Padova. Other related events are:

- 12th Young Researchers Workshop on Geometry, Mechanics and Control 22-24 January, 2018. www.events.math.unipd.it/12YRW
- Recent advances in Hamiltonian dynamics and symplectic topology 12-16 February, 2018. www.events.math.unipd.it/hamschool2018
- Workshop on Fermi-Pasta-Ulam problem: open questions and perspectives, 12-14 April, 2018. www.events.math.unipd.it/fpu2018

DIPARTIMENTO MATEMATICA

Università degli Studi di Padova