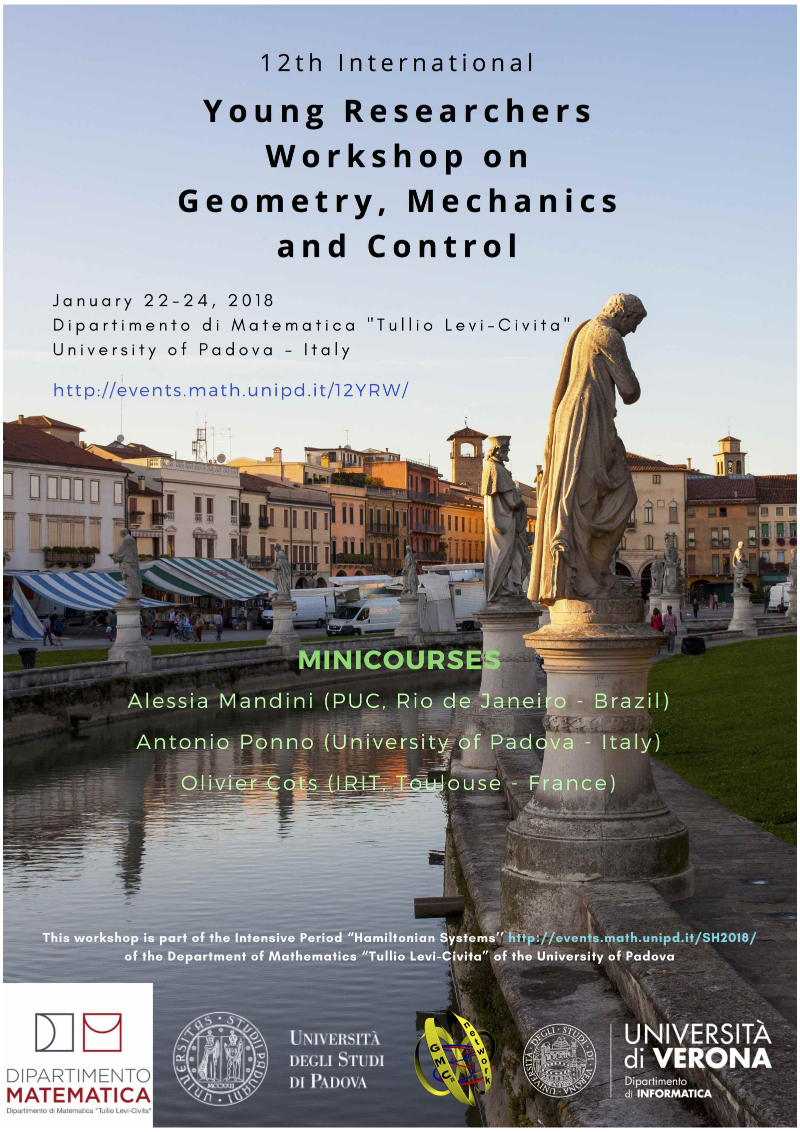
BOOKLET OF THE 12^{TH} YOUNG RESEARCHERS WORKSHOP IN GEOMETRY, MECHANICS AND CONTROL

PADOVA, JANUARY 22–24, 2018,

DEPARTMENT OF MATHEMATICS "TULLIO LEVI–CIVITA",

UNIVERSITÀ DEGLI STUDI DI PADOVA



Program

Schedule. Dipartimento di Matematica "Tullio Levi-Civita", via Trieste 63, Padova, Room 1C150

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DIPARTIMENTO MATEMATICA

Università degli Studi di Padova

Università di Verona

Dipartimento di Informatica

Minicourses.

- *Alessia Mandini* (PUC - Rio de Janeiro, Brazil), “Symmetries and reduction in symplectic geometry”
- *Antonio Ponno* (Università degli Studi di Padova, Italy), “Introduction to Hamiltonian PDEs: an overview of models and methods”
- *Olivier Cots* (Université de Toulouse, INP-ENSEEIHT-IRIT, France), “Introduction to geometric and numerical methods in optimal control with application in medical imaging”

Contributed talks.

- *Philip Arathoon*, “Flag Manifolds and Coadjoint Orbits”
- *Olga Bernardi*, “Lyapunov functions and recurrent sets: from topological dynamics to weak KAM theory”
- *Francesco Boarotto*, “Homotopy properties of horizontal path spaces”
- *Benoit Bonnet*, “Optimal Control Problems in Wasserstein Spaces”
- *Giulia Cavagnari*, “A sparse mean–field control problem”
- *Juan Margalef*, “Understanding the geometry of the Misner spacetime trough topology”
- *Clément Moreau*, “Controllability of a Magneto–Elastic Micro–Swimmer”
- *Michael Orieux*, “Extremal flow of minimum time control affine problems, and applications to space mechanic”
- *Rodrigo T. Sato Martìn de Almagro*, “Higher–Order Geometric Integrators for Nonholonomic Systems ”
- *Marta Zoppello*, “Controllability of the Hydro–Chaplygin sleigh with a moving mass”

Poster Session.

- *Paz Albares*, “Classical Lie symmetries and similarity reductions for a generalized NLS equation in (2 +1)-dimensions”
- *Jone Apraiz*, “Observability Inequalities for Parabolic Equations and Some Applications Related to the Bang–Bang Property for Control Problems”
- *Fabrizio Boriero*, “Trajectory planning for underactuated mechanical system”
- *Hasan Özgür Çildiroglŭ*, “Investigation of Geometric Phases for an Entangled State in Quantum Mechanics ”
- *Benjamin Couéraud*, “Variational discretization of thermodynamical simple systems on Lie groups”
- *Viviana Alejandra Dìaz*, “Equations derived from Hamiltonian orbit reduction”
- *Iván Gutiérrez Sagredo*, “Classical dynamical r–matrices and phase space quantization of (2 + 1)–gravity”
- *Shkelqim Hajrulla*, “Logarithmic wave equation and the initial boundary value problem”
- *Simone Passerella*, “Motion Planning via Reconstruction Theory”
- *Cristina Sardón*, “A Hamilton–Jacobi theory for implicit differential systems”
- *Milo Viviani*, “Efficient geometric integration for Lie–Poisson systems on $\mathfrak{sl}^*(N,\mathbb{C})$ and $\mathfrak{su}^*(N)$”
- *Marcin Zając*, “Gauge theories and Atyiah algebroid”

DIPARTIMENTO MATEMATICA UNIVERSITÀ DEGLI STUDI DI PADOVA UNIVERSITÀ di VERONA Dipartimento di INFORMATICA

Minicourses

Alessia Mandini
PUC - Rio de Janeiro, Brazil

Symmetries and reduction in symplectic geometry

Abstract. In these lectures I will describe how symmetries in a symplectic space can be use to reduce the complexity of the system via symplectic reduction. I will treat in detail with the regular and the singular cases, giving examples to illustrate the phenomena. In particular we will discuss moduli spaces of polygons and other examples.

Antonio Ponno
Università degli Studi di Padova, Italy

Introduction to Hamiltonian PDEs: an overview of models and methods

Abstract. Hamiltonian Partial Differential Equations (PDEs) are introduced starting from a single class of models that includes most of the PDEs of interest to physics, ranging from gas dynamics to quantum mechanics. The main topics of the Hamiltonian formalism, such as (canonical and noncanonical) transformations, symmetries, first integrals, and perturbation theory are presented and discussed through explicit examples. The theory of Lax-integrable systems is also shortly discussed.

Olivier Cots
Université de Toulouse, INP-ENSEEIHT-IRIT, France

Introduction to geometric and numerical methods in optimal control with application in medical imaging

Abstract. An optimal control problem (OCP) is an infinite dimensional optimization problem with algebraic and differential constraints. A simple example of OCP in Lagrange form is the following. Find a command law which steers a dynamical control system from an initial configuration to a target point while minimizing an objective function representing the cost of the trajectory followed by the system during the transfer. The optimal solution can be found as an extremal, solution of the Maximum Principle and analyzed with the techniques of geometric control. As an example, the contrast and saturation problems in Magnetic Resonance Imaging (MRI) are considered. These two control problems are modeled as Mayer problems in optimal control, with a single input affine control system, with respectively a state in dimension two (four) for the saturation (contrast) problem. An analysis with the techniques of geometric control is used first to obtain an optimal synthesis in the case of the saturation problem while for the contrast problem (of higher dimension), this analysis is used first to reduce the set of candidates as minimizers and then to construct the numerical methods. This leads to a numerical investigation combining direct, indirect (multiple shooting) and homotopy methods.

Contributed talks

Philip Arathoon
University of Manchester

Flag Manifolds and Coadjoint Orbits

Abstract. An attractive class of symplectic manifolds are those which admit a transitive, symplectic group-action; namely, symplectic homogeneous spaces. For when the group is compact, a well-known result tells us that all such spaces are (up to coverings) given by the coadjoint orbits of the group. In this talk I would like to exhibit these orbits as special kinds of flag manifolds, a generalisation of the projective spaces and Grassmannian manifolds. An aim of the talk will be to show that these spaces may be classified by subsets of Dynkin diagrams. If the group is no longer compact, its symplectic homogeneous spaces become harder to classify. Nevertheless, for the example of the Euclidean group (and indeed other semidirect products) we may once again exhibit these spaces as special flag manifolds; in this case, as affine flag manifolds.

Olga Bernardi
Università degli Studi di Padova

Lyapunov functions and recurrent sets: from topological dynamics to weak KAM theory

Abstract. The aim of this talk is to clarify the intimate relations between Lyapunov functions and chain recurrent sets. The study of this subject comes from a seminal paper by Conley and has had recent important advances by Fathi and Pageault. After an explanation of the state of the art, we present the following improvement of the pre-existent results: every continuous flow on a compact metric space, uniformly Lipschitz continuous on the compact sets of a time, admits a Lipschitz continuous Lyapunov function strict –that is strictly decreasing– outside the strong chain recurrent sets of the flow. We then give two consequences of this theorem. From one hand, we characterize the strong chain recurrent sets in terms of Lipschitz continuous Lyapunov functions. From the other hand, in the case of a flow induced by a vector field, we establish a sufficient condition for the existence of a $C^{1,1}$ strict Lyapunov function and we also discuss various examples.

Francesco Boarotto
UPMC - Paris 6

Homotopy properties of horizontal path spaces

Abstract. We study homotopy properties of endpoint maps for horizontal path spaces, that is spaces of curves on a manifold M, whose velocities are constrained to a subbundle $\Delta \subset TM$ in a nonholonomic way. We prove that in the sub-Riemannian case (that is, when the rank of Δ is smaller or equal than the dimension of M), these maps are Hurewicz fibrations for all

$1 \le p < \infty$, while in presence of a drift field the same property holds for all $1 \le p < p_c$, with $p_c > 1$ depending on the degree of nonholonomy of the structure. The fact that $p_c > 1$ permits to study the number of critical points of geometric costs (the p-energy associated to admissible curves) on these spaces. If there are no abnormal curves and M is compact, then this number is infinite, and a version of Serre theorem, on the existence of infinitely many geodesics between any two points, can be recovered. (Joint work with A. Lerario).

Benoit Bonnet
Aix-Marseille Université / LSIS

Optimal Control Problems in Wasserstein Spaces

Abstract. In this talk, we present a Pontryagin Maximum Principle for optimal control problems in the Wasserstein space of probability measures.
The dynamics is given by a transport equation with non-local velocities and the control is a Lipschitz function of the space.
Taking advantage of the geometrical structure of these equations, we show that it is possible to translate classical results of optimal control theory along with the natural ideas used to prove them to the Wasserstein setting. In particular, to prove the PMP one needs to compute perturbations induced by needle variations and define an Hamiltonian structure in this setting. These results find applications in pedestrian dynamics, traffic control and control on opinion networks.

Giulia Cavagnari
Università degli Studi di Pavia

A sparse mean–field control problem

Abstract. With this talk we present a measure-theoretic approach for a time- optimal control problem. The use of probability measures to describe the state of the system is particularly effective when dealing with the evolution of multi-agent systems. Here the measure describes the statistical distribution of agents/particles. An effective tool which allows us to pass from the microscopic point of view of the agents/particles to the macroscopic point of view of the crowd/mass is the Superposition Principle by Ambrosio-Gigli-Savarè. We provide its generalization to the control case where the microscopic dynamics of characteristics is given by a differential inclusion. Finally, we discuss a time-optimal control problem in this framework where some kind of inter- action is considered. In particular, we put a constraint on the magnitude of the control that we can use on the mass (sparse control). Mainly, we prove in this context the main results valid for the classical case: a Dynamic Programming Principle, existence of optimal trajectories and an Hamilton– Jacobi–Bellman equation solved by the minimum-time function in a viscosity sense. This is a joint work with Antonio Marigonda (University of Verona, Italy) and Benedetto Piccoli (Rutgers University-Camden, USA).

Juan Margalef
UC3M-CSIC

Understanding the geometry of the Misner spacetime trough topology

Abstract. Many apparent space–time pathological behaviours are due to a lack of understanding of the geometry. On the other hand, intrinsic pathologies are the trademark of singular space–times. In this talk I will show how topology might help us understanding both situations in the so called Misner space.

Clément Moreau
Inria Sophia–Antipolis Méditerranée

Controllability of a Magneto–Elastic Micro–Swimmer

Joint work with Laetizia Giraldi, Pierre Lissy and Jean–Baptiste Pomet

Abstract. A recent promising technique to drive robotic micro-swimmers is to use magnetized materials. An external magnetic field can then be applied to bend and propel the swimmer. This leads to the following controllability issue: given an initial state (position and shape) for a micro–swimmer, is it possible to find a magnetic field that brings it to another given state? This talk and related poster focus on a simple planar micro–swimmer model made of two magnetized segments connected by an elastic joint, studied in [1] and [2]. We will recall the definition of small-time local controllability (STLC), focusing on controlling the system in a neighbourhood of an equilibrium point, in small time and with small controls, and show that the system associated to the swimmer cannot be STLC except under a restrictive equality constraint [2]. We will then give perspectives on further research on this matter from the control theory point of view.

References

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Michael Orieux
University Paris Dauphine - Inria Sophia Antipolis

Extremal flow of minimum time control affine problems, and applications to space mechanic

Joint work with J.–B. Caillau and J. Féjoz

Abstract. In this talk, we will address the issue of π–singularities, a phenomenon occouring when one is dealing with minimum time control of a mechanical system. More detailed can be

found in [1, 2]. Minimum time control of planar two and restricted three body problems comes down to tackle the following affine controlled dynamic:

$$\begin{cases} \dot{x} = F_0(x) + u_1 F_1(x) + u_2 F_2(x), \quad x \in M, \ \|u\| \leq 1, \\ x(0) = x_0, \\ x(t_f) = x_f, \\ t_f \to \min \end{cases}$$

Where M is the phase space (4 dimensional), $u = (u_1, u_2)$ is the control, F_0 is the drift coming from the gravitational potential, and F_1, F_2 the two orthogonal vector fields supporting the control. This implies convenient properties on the vector fields and their Lie brackets: (A1) $\forall x \in M$, $\mathrm{rank}(F_1(x), F_2(x), [F_0, F_1](x), [F_0, F_2](x)) = 4$, This dynamic defines a non Hamiltonian system, but necessary conditions coming from the Pontryagin Maximum Principle leads to study the flow of a singular Hamiltonian problem given by $H(x,p) = H_0(x,p) + \sqrt{H_1^2(x,p) + H_2^2(x,p)}$, with $H_i(x,p) = \langle p_i, F_i(x) \rangle$, $i = 0, 1, 2$, being the Hamiltonian lift of the vector fields. It also provides the control feedback $u = \frac{(H_1, H_2)}{\|(H_1, H_2)\|}$ outside of the singular locus $\Sigma = \{H_1 = H_2 = 0\}$.
For extremal crossing Σ, three cases are to be distinguish:

- around $\bar{z} \in \Sigma$ such that $H_{12}(\bar{z})^2 < H_{01}^2(\bar{z}) + H_{02}^2(\bar{z})$, the extremal flow can be stratified, and a co-dimension one sub-manifold is leading to the singular locus. The flow is smooth on each strata and continuous;
- around $\bar{z} \in \Sigma$ such that $H_{12}(\bar{z})^2 > H_{01}^2(\bar{z}) + H_{02}^2(\bar{z})$ no extremal is crossing the singular locus, and the flow is smooth;
- in the rare limit case where $H_{12}(\bar{z})^2 = H_{01}2(\bar{z}) + H_{02}^2(\bar{z})$ no extremal is crossing the singular locus either.

The controlled Kepler and restricted three body problem has a simpler structure and we have the additional hypothesis: (A2) $[F_1, F_2] = 0$ identically, as such, we find ourselves in the first case. From the feedback given above and the involution condition (A2), it appears that when Σ is crossed, we get an instant rotation of angle π on the control (the so-called π-singularities), and the flow is stratified as above. This stratification will also answer the question of which trajectories leads to a singularity. One could now investigate integrability properties of such systems. The minimum time controlled Kepler problem is studied in that matter, namely, we will present our

Theorem 1. *The extremal flow of the minimum time Kepler problem is non integrable in the class of meromorphic functions.*

This result was obtained as a consequence of the Morales–Ramis theorem. The talk will end with on going work on sufficient conditions for local optimality of minimum time extremals.

References
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Rodrigo T. Sato Martín de Almagro
ICMAT

Higher–Order Geometric Integrators for Nonholonomic Systems

Abstract. In this talk I present a newly obtained family of geometric, arbitrarily high-order partitioned Runge–Kutta integrators for nonholonomic systems, both on vector spaces and Lie groups. These methods differ from those of J. Cortés and S. Martínez [1] in that we do not require the discretisation of the constraint, and contrary to L. Jay's SPARK integrators [2] we do not require extraneous combinations of constraint evaluations. Our methods preserve the continuous constraint exactly and can be seen to extend those of M. de León, D. Martín de Diego and A. Santamaría [3]. Work in collaboration with D. Martín de Diego.

References

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Marta Zoppello
Università degli Studi di Padova

Controllability of the Hydro–Chaplygin sleigh with a moving mass

Abstract. In the framework of nonholonomic mechanics we analyze a steering control for the Chaplygin sleigh with a moving mass as in [4], but immersed in an ideal irrotational fluid (see [3] for the case without the moving mass). We prove that the system is asymptotic controllable using as controls the mass velocity, then, by assuming that it remains controllable also by applying external control forces on the mass, we investigate an optimal control problem with a cost quadratic in the controls. We provide both an Hamel version of the approach used by Bloch et all in [1, 2] and numerical results. (Based on a joint work with Nicola Sansonetto).

References

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POSTERS

Paz Albares, University of Salamanca

Classical Lie symmetries and similarity reductions for a generalized NLS equation in $(2+1)$-dimensions

Abstract. This work is dedicated to the study of an integrable generalization of the nonlinear Schrödinger equation in 2+1 dimensions, which includes dispersive terms of third and fourth order, and its associated non-isospectral problem, [6]. The classical Lie symmetries of the Lax pair and the similarity reductions are widely studied in several cases, providing both the reductions for the original equation and for the spectral problem. In addition, some interesting results that lead to non-isospectral problems in $(1+1)$–dimensions are presented. This is part of a joint work with J. M. Conde and P. G. Estévez.

References

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Jone Apraiz, University of the Basque Country

Observability Inequalities for Parabolic Equations and Some Applications Related to the Bang–Bang Property for Control Problems

Abstract. In this talk we will see two observability inequalities for the heat equation from measurable sets over $\Omega \times (0, T)$. In the first one, the observation will be from a subset of positive measure in $\Omega \times (0, T)$, while in the second, the observation will be from a subset of positive surface measure on $\partial\Omega \times (0, T)$. We will provide some applications for the above-mentioned observability inequalities, the bang–bang property for the minimal time, time optimal and minimal norm control problems.

Fabrizio Boriero
Università degli Studi di Verona

Trajectory planning for underactuated mechanical system

Joint work with Nicola Sansonetto and Paolo Fiorini

Abstract. Underactuated mechanical systems (UMS) are present in our every-day lives. Autonomous cars, airplanes, boats, and quadrotors are concrete examples of systems that will affect our lifestyles in the near future. Trajectory planning for UMS is a research field born in the first days of robotics, but there are still challenging research questions to overcome. In this presentation, we will show how to take advantage of differential geometry tools to deal with UMS, in particular how to plan a trajectory when the actuated degrees of freedom are less than the observable ones.

Hasan Özgür Çildiroglŭ
University of Ankara

Investigation of Geometric Phases for an Entangled State in Quantum Mechanics

Abstract. Entanglement which takes place in systems that has two or more parts is one of the most profound and significant aspect of quantum mechanics. Emergence of new applications and the theoretical studies carried out in full swing is an indication of its importance for

physics. In connection with entanglement, there are many researches in progress to reveal the relationship between different quantum mechanical processes whose roots common and thence to carry out the discussion from a more systematic and a complementary frame work. In the first part of this study, effective Aharonov Casher Hamiltonian in two dimensional space in framework of relativistic quantum mechanics is depicted entirely without any approximation. Then, with the discussion of Bell inequalities in two and three dimensional spaces, a convenient basis is prepared to investigate AC effect on quantum entangled systems. In the last section, for completeness, Bell inequalities which could be violated with AC effect will be derived as the main goal of the study.

Benjamin Couéraud,
Laboratoire de Météorologie Dynamique, Ecole Normale Supérieure

Variational discretization of thermodynamical simple systems on Lie groups

Abstract. The subject of this work is the variational discretization of geometric thermodynamical systems whose dynamics are obtained from the constrained variational principle established by Gay–Balmaz and Yoshimura. More particularly, we are concerned with the case of thermodynamical simple systems whose configuration space is a finite dimensional Lie group. Starting from this variational formalism on the Lie group, we perform an Euler–Poincaré reduction in order to obtain the reduced evolution equations of the system on the Lie algebra of the configuration space. We obtain as corollaries the energy balance and a Kelvin-Noether theorem. Then based on the formalism introduced in Marsden, Pekarsky, and Shkoller, a compatible discretization is developed resulting in discrete evolution equations that take place on the Lie group. Then, these discrete equations are transported onto the Lie algebra of the configuration space with the help of a group difference map, in the spirit of the work of Bou–Rabee and Marsden. Finally we illustrate our framework with an heavy top immersed in a viscous fluid modeled by a Stokes flow and proceed with a numerical simulation.

Viviana Alejandra Díaz
Universidad Nacional del Sur

Equations derived from Hamiltonian orbit reduction

Abstract. It is known that a process of orbit reduction in a Hamiltonian system with symmetry can be performed. Following this theory of symplectic reduction and taking coordinates in the reduced space, we write a coordinate expression for the two-form and the consequent system of equations. We can develop this procedure in one stage considering the whole symmetry group, and in two stages in the case in which the group has a normal subgroup. This is a joint work with Marcela Zuccalli.

References

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Iván Gutiérrez Sagredo
Universidad de Burgos

Classical dynamical r–matrices and phase space quantization of $(2+1)$–gravity

Abstract. Let G be a Lie group and $\mathfrak{g} = Lie(G)$ its Lie algebra. Classical dynamical r-matrices [1] are a natural generalization of classical r-matrices, where the classical Yang-Baxter equation (CYBE) is replaced by the dynamical classical Yang-Baxter equation (DCYBE). They are related with a given Lie subalgebra $\mathfrak{h}$ of the Lie algebra $\mathfrak{g}$, being $\mathfrak{h} = 0$ the corresponding to the 'non-dynamical' case. As the 'non-dynamical' ones, classical dynamical r-matrices are related with some Poisson structures on the group G, although now we must consider in general some enlarged space. In this short talk, I will give precise definitions of dynamical classical r-matrices, emphasizing their relation with 'non-dynamical' ones and I will present two, quite different, related Poisson structures, namely the dynamical Sklyanin and dual brackets. Finally, I will briefly sketch how these dynamical classical r-matrices are related with the phase space of $(2+1)$–gravity (see [2] for the Poincaré case) and I will comment on some work in progress trying to generalize previous results and introducing an observer into the theory.

References

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Shkelqim Hajrulla
University of Vlora

Logarithmic wave equation and the initial boundary value problem

Joint work in collaboration with L. Bezati and F. Hoxha

Abstract. We can consider the problem of solutions for logarithmic wave equation. Considering the initial boundary value problem logarithmic wave equations, we create the premises to stand it for a class of logarithmic wave equations. In particular we consider the logarithmic wave equations with linear damping and initial boundary value problem of the solutions.

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Motion Planning via Reconstruction Theory

Joint work with: Francesco Fassò, Marta Zoppello.

Abstract. In Geometric Control Theory problems of motion planning consist in finding a control that steers from a starting configuration to a prescribed final one. This problems are common in several fields of research: biology, chemistry, medicine, robotics. In this work we

present an approach to study these problems based on the techniques of reconstruction of dynamical systems with symmetry. In particular we focus on a class of control problems, that we called robotic locomotion systems, which are driftless affine control systems with configuration space the total space of a trivial principal fiber bundle $\pi : G \times S \longrightarrow S$, with G a Lie group and S the shape space which is a manifold [4]. The problem is, for every assigned loop in the base space, to determine the motion of the system on the fiber over the considered loop. This problem is linked to the one of reconstruction in dynamical systems. The main difference is that now the curve on the base manifold is not the integral curve of a (reduced) differential equation, but it is assigned by the controller. The theory of reconstruction under the action of a connected Lie group is well studied from different points of view [5]. Our purpose is to the importance in motion planning of reconstruction results. In particular there are two cases: if the group is compact, the motion on the fiber over a loop is quasi-periodic [3]; if the group is not compact, the motion on the fiber over a loop is either quasi-periodic or there is a drift [1]. The generic case is determined by the group itself. As a consequence some relevant information on the possible motion can be a priori infered from the structure of the group alone. We present some examples involving some classical Lie groups: $SE(2)$, $SO(2) \times \mathbb{R}^2$, $SE(3)$, $SO(3) \times \mathbb{R}^3$.

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Cristina Sardón
ICMAT

A Hamilton–Jacobi theory for implicit differential systems

Abstract. In this paper, we propose a geometric Hamilton-Jacobi theory for systems of implicit differential equations. In particular, we are in- terested in implicit Hamiltonian systems, described in terms of Lagrangian submanifolds of TT^*Q generated by Morse families. The implicit character implies the nonexistence of a Hamiltonian function describing the dynamics. This fact is here amended by a generating family of Morse functions which plays the role of a Hamiltonian. A Hamilton–Jacobi equation is obtained with the aid of this generating family of functions. To conclude, we apply our results to singular Lagrangians by employing the construction of special symplectic structures.

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Efficient geometric integration for Lie–Poisson systems on $\mathfrak{sl}^*(N,\mathbb{C})$ and $\mathfrak{su}^*(N)$

Abstract. Lie–Poisson dynamics encompasses a large class of Hamiltonian systems both in classical and quantum mechanics, for example in the description of 2D inviscid hydrodynamics and spin systems dynamics. A Lie–Poisson Hamiltonian system encodes the symmetries of a physical problem in the structure constants of the Lie algebra underlying the Lie–Poisson brackets of the system. Generally, for simulations, we want to create a discrete flow which is a Lie–Poisson map, such that we have near-conservation of the Hamiltonian and conservation of the Casimir functions. The main difficulty here is that, in general, these first integrals are high order polynomials, for which classical methods show very poor behaviour. Here we show a class of efficient numerical methods for Lie–Poisson Hamiltonian systems on $\mathfrak{sl}^*(N,\mathbb{C})$ and $\mathfrak{su}^*(N)$, for $N > 0$, which preserve, up to machine precision, the Casimir functions and nearly conserve the Hamiltonian. As examples, we will show the results for the Euler equations on a compact surface and the dynamics of a spin system.

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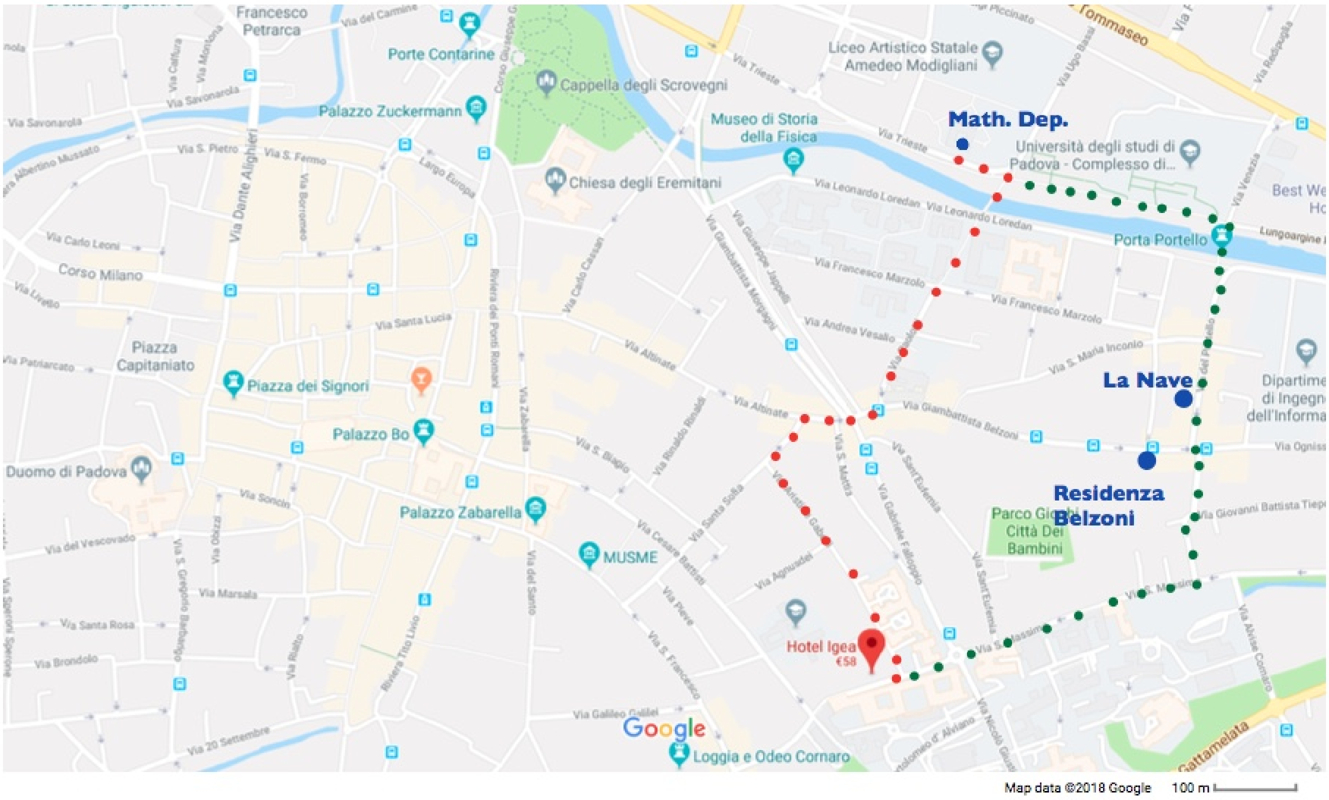
Gauge theories and Atyiah algebroid

Abstract. Gauge field theories are usually described in terms of principal bundles P over M where fields are connections on that bundles. Then, the Lagrangian is defined on a bundle of first jets J^1E, where E is a bundle of connections on P. Since gauge connections are equivariant with respect to the group action, we can divide E by this action and obtain a bundle which has a structure of an Atiyah algebroid over M. In my talk I will show the pass from the principal bundle description to algebroidal one for gauge theories. In particular, I will show the basic objects (connection, curvature etc.) of gauge theories in a language of Atiyah algebroid and the process of generating the fields dynamics. Despite the fact that it is a very natural idea, this approach is not very common in a literature yet.

Some practical information

Here you can find a map of the area of the Department of Mathematics, with in evidence Foresteria La Nave, Residenza Belzoni and Hotel Igea.



Related events

This Workshop is part of the Intensive Period "**Hamiltonian System**" (www.events.math.unipd.it/SH2018) supported by the Department of Mathematics "Tullio Levi-Civita" of the Università degli Studi di Padova. Other related events are:

- Workshop on Dynamics and integrability of nonhoonomic and other non-Hamiltonian systems 24-27 January, 2018. www.events.math.unipd.it/Integrability2018
- Recent advances in Hamiltonian dynamics and symplectic topology 12-16 February, 2018. www.events.math.unipd.it/hamschool2018
- Workshop on Fermi-Pasta-Ulam problem: open questions and perspectives, 12-14 April, 2018. www.events.math.unipd.it/fpu2018

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