A Type Theory for Comprehension Categories

Benedikt Ahrens
Delft University of Technology

Recent models of intensional type theory have been constructed in algebraic weak factorization systems (AWFSs). AWFSs give rise to comprehension categories that feature non-trivial morphisms between types; these morphisms are not used in the standard interpretation of Martin-Löf type theory in comprehension categories.

We develop a type theory that internalizes morphisms between types, reflecting this semantic feature back into syntax. Our type theory comes with Π-, Σ-, and identity types. We discuss how it can be viewed as an extension of Martin-Löf type theory with coercive subtyping, as sketched by Coraglia and Emmenegger. We furthermore define semantic structure that interprets our type theory and prove a soundness result. Finally, we exhibit many examples of the semantic structure, yielding a plethora of interpretations.

This talk is based on joint work with Niyousha Najmaei, Niels van der Weide, and Paige Randall North [1].

References

[1] Niyousha Najmaei, Niels van der Weide, Benedikt Ahrens, and Paige Randall North. From semantics to syntax: A type theory for comprehension categories. *Proc. ACM Program. Lang.*, 10(POPL):2409–2438, 2026.