The effective infinity-topos

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Abstract

As an elementary topos, Hyland's effective topos Eff models extensional Martin-Löf type theory with an impredicative universe of propositions [1]. But it also contains another impredicative universe of so-called "modest sets" which is *not* a poset [2]—a fascinating combination not consistent with classical foundations. In joint work with Frey and Speight [3], we proposed a higher-dimensional version of this model with a *univalent* impredicative universe and used it to give new impredicative encodings of some (higher) inductive types. This model was based on cubical assemblies and exploited the constructive character of the recently introduced cubical Quillen model structures [4, 5]. As was subsequently shown, however, the subtopos of 0-types in this model was not equivalent to Eff, but to a larger realizability topos.

In addition to containing a non-degenerate, impredicative, univalent universe, we believe that the elementary ∞-topos Eff_∞ should include Eff as its subtopos of 0-types. This will also provide an example of a non-Grothendieck elementary ∞-topos. As a candidate for Eff_∞ we here propose an ∞-category of *coherent stacks* over the regular category of assemblies. We show that Eff_∞ is locally cartesian closed as an ∞-category, that it is the ∞-exact completion of the 1-category of partitioned assemblies, and that its subcategory of 0-types is indeed the effective 1-topos Eff.

This is joint work with Mathieu Anel and Reid Barton, building on prior joint work with Jacopo Emmenegger and Pino Rosolini [6].

References

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