Stone types in neutral mathematics

Martín Escardó
University of Birmingham

Types are inherently topological, and I will explore this in neutral mathematics.

By "neutral" I mean that classical axioms (such as excluded middle or choice), or Brouwerian intuitionistic axioms (such as "all functions are continuous") are not asserted or denied.

I will focus on "Stone types": types that are compact and totally separated in a suitable sense expressed in type theory, without endowing types with topologies of any kind.

I will give plenty of examples of Stone types, as well as a number of theorems that match the classical development of the topology of Stone spaces.

One point of view is that this is "neutral synthetic topology". The results we describe hold in *all* toposes, not just toposes for topology or toposes for computability, although I will discuss the implications for particular toposes, such as these two kinds.

All of this is done in a Spartan Martin-Löf type theory, with extensionality assumptions, and has been implemented in the Agda proof assistant.