Contact structures. State of the art and future directions

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Abstract

Contact structures have a long, well-established history dating back to the beginning of the 20th century. The drive for their development stemmed from a philosophically appealing idea: founding geometry and topology on the notion of region (in opposition to the standard theories based on the highly idealized notion of a point) and the relation of nearness between regions.

Early informal proposals by, to name a few, Jean Nicod, Bertrand Russell, Alfred N. Whitehead, and Theodore de Laguna, were later turned into fully-fledged mathematical theories put forward by Alfred Tarski in the 1920s, Andrzej Grzegorczyk, and Hendrik de Vries in the 1960s, Bowman L. Clarke in the 1980s, and Loredana Biacino and Giangiacomo Gerla in the 1990s.

However, the real breakthrough came with the advent of the XXIst century and the emergence of the Contact Algebras in the works of John Stell, Ivo Düntsch, Michael Winter, Ian Pratt-Hartmann, Georgi Dimov, and Dimiter Vakarelov. The main inspiration for these, apart from the aforementioned sources, was the so-called Region Connection Calculus of David A. Randell and Anthony G. Cohn. Roughly speaking, contact algebras are structures obtained by expanding an algebra with the binary relation of contact. Elements of the algebra are usually thought of as regions of space, and contact as a formal embodiment of the nearness or proximity of regions. The seminal papers of the aforementioned authors defined the field, which immediately flourished and gave rise to a vibrant area of research with deep results concerning, among other things, representation and duality. Stone-like connections between classes of algebras and topological spaces have been established; the interactions with modal logic have been analyzed; last but not least, applications in philosophy and computer science have been found.

In my talk, I would like to briefly present the foundations of contact structures, their current state of the art, and point out possible future directions for their development. In particular, I would like to discuss possible interconnections with well-known locale and domain theories.