

# Completeness Theorem, Choice Axioms, and Computation

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The reverse mathematics of the completeness theorem with respect to Tarski, Kripke or Beth semantics show surprising, apparently contradictory, results. In terms of choice principles, depending on the formulation, it is connected either to the ultrafilter theorem, to weak König lemma, or to the fan theorem, itself a weak form of bar induction, itself classically equivalent to dependent choices. This suggests that the ultrafilter theorem and dependent choices can be expressed as particular instances of a common scheme, let us call it generalised dependent choices, relying on coinduction, and whose contrapositive statement generalises bar induction.

Pursuing the comparison between different forms of choice, a connection can be drawn between U. Berger's update induction, a constructive choice principle equivalent to dependent choice, and Teichmüller-Tukey lemma, a set-theoretic principle equivalent to the full axiom of choice. But the former is a well-foundedness principle, so easily realisable by well-founded recursion. A computational content to the full axiom of choice ensues.

The talk will cover joint works with Nuria Brede, Jad Koleilat, Jessica Allegro and Étienne Miquey, with contributions also from Martin Baillon, Yannick Forster and Dimitar Mitov.