Orbits of Functions and their Topologies

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Abstract. We introduce a topology on a set X induced by an endofunction on X. We show that the iterates (or orbits) of the function form a basis for this topology.

Several properties of functions can be characterized in terms of the associated topology. In particular, a function is the identity if and only if its topology is Hausdorff (T_2), and it is bijective if and only if every open set is clopen. We also give a characterization of T_0 spaces.

Finally, we study continuity with respect to this topology and characterize the corresponding compact spaces.