A j-translation with Kripke forcing relation

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Abstract

It is well known that the concept of a *nucleus* (i.e., a meet-preserving monotone function) arises naturally in both intuitionistic proof theory and formal topology theory. From a proof-theoretic perspective, a nucleus j induces the associated *j-translation*. This translation generalizes important examples such as the double negation translation and Friedman's A-translation, and plays a role in establishing not only relative consistency but also partial conservation results.

In this talk, we propose a new translation that combines the j-translation with the Kripke forcing relation in the internal logic of an elementary topos. First, we show that our translation is sound for intuitionistic first-order logic and Heyting arithmetic. Next, by interpreting this translation in the effective topos, we see that it extends the sheaf model of realizability introduced by de Jongh and Goodman [1] (for more details, see [3]).

Finally, to clarify the connection between this translation and formal topology, we investigate a corresponding covering relation. More specifically, we discuss a candidate for the corresponding internal site and the associated internal sheaf topos.

References

[1] Nicolas D. Goodman. Relativized realizability in intuitionistic arithmetic of all finite types. *Journal of Symbolic Logic*, 43(1):23–44, 1978. doi:10.2307/2271946.

[2] Satoshi Nakata. A j-translation with Kripke forcing relation, 2026. URL: https://arxiv.org/abs/2602.23218, arXiv:2602.23218.

[3] Jaap van Oosten. *Realizability: an introduction to its categorical side*, volume 152 of *Studies in Logic and the Foundations of Mathematics*. Elsevier, 2008.