

Cubical assemblies do not have a subobject classifier

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Results due to Menni [Men03] show that the ex/lex completion of a locally cartesian closed category is a topos iff the original category has a generic proof. For cubical sets [Coh+18], the picture is more complicated. It is possible to show that if the model of HoTT in cubical sets has a subobject classifier, then there is a generic proof in the metatheory. However, unlike with ex/lex completions, subobject classifiers can fail to exist in cubical set models of HoTT even when there is a generic proof in the metatheory. I'll show exactly this happens when constructing cubical sets internal to the category of assemblies, i.e. for cubical assemblies [Uem19].

References

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