Sharp Elements and the Points of the Patch Locale

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Abstract

Locatedness is a central notion in formal topology. Spitters previously studied located subsets in the context of compact regular formal topologies, relating them to the fundamental notion of positivity. More recently, de Jong introduced the notion of sharp element in a domain-theoretic context, capturing a located filter of Scott open neighbourhoods in terms of the elements of a domain. In this talk, we present our work on characterizing the sharp elements of a domain via its patch locale: for a Scott domain D, we establish a bijection between the set #D of sharp elements and the set 1 $\to$ Patch(Scott(D)) of points of the patch of the Scott topology viewed as a locale. Our development is constructive and predicative, taking place in the foundational setting of Univalent Type Theory. We build on two previous constructive and predicative developments in this foundational setting: (1) de Jong and Escardó's development of domain theory and (2) the authors' development of locale theory. In future work, we plan to use this characterization to construct the Lawson topology of a Scott domain in the same foundational setting.