

## Abstracting point-free topology and localic sheafification via doctrines

Maria Emilia Maietti Davide Trotta

University of Padova

The notion of Formal Topology, introduced by Per Martin-Löf and Giovanni Sambin in the 1980s, is a constructive, predicative version of a locale, a point-free notion of space.

In a topos, this idea generalizes to Lawvere-Tierney topologies. More generally, in the framework of Lawvere doctrines, a topology can be described as a monad induced by a geometric morphism between doctrines. This approach applied to triposes was first introduced by Martin Hyland, Peter Johnstone, and Andrew Pitts, though using different foundational setting in [1].

Using this generalization, we show that every topos obtained from a tripos whose base category has weak dependent products and a generic proof can be described as a topos of  $j$ -sheaves for an appropriate Lawvere–Tierney topology over an abstract presheaf topos. Ordinary localic sheafification appears as a particular instance of this framework.

This result allows us, on the one hand, to recover the geometric meaning underlying the tripos-to-topos construction, encompassing not only localic toposes but also realizability toposes. On the other hand, it represents a first step toward an abstract predicative and constructive generalization of localic sheafification within the setting of predicative toposes.

This talk is based on a joint work with Maria Emilia Maietti [1].

### References

[1] Maietti, M.E., Trotta, D., 2025. An Algebraic Abstraction of the Localic Sheafification via the Tripos-to-Topos Construction. Preprint