

# Towards a syntax-semantics description of non-commutative Stone duality via étendues

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## Abstract

Many dualities can be understood as ‘syntax-semantics’ dualities for certain logics. Currently, the literature is missing such a description for non-commutative Stone duality – a dual equivalence between classes of inverse semigroups and topological groupoids. The purpose of this talk is to describe what a topos-theoretic answer to this question might look like, using the fact that the topoi naturally associated with inverse semigroups and étale groupoids are \*étendues\* – the ‘locally localic’ topoi.

We will characterise the geometric theories classified by étendues by drawing a parallel with Gaifman’s co-ordination theorem in model theory. Using properties of these ‘uniformly co-ordinated’ theories, we sketch how the inverse semigroups and étale groupoids of non-commutative Stone duality yield, respectively algebraic syntax and topological semantics.