Control Systems under State Constraints in Wasserstein Spaces

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In this talk I will present some viability and invariance theorems in Wasserstein spaces $\mathcal{P}_p(R^d)$ of Borel probability measures with finite $p-$momentum and $p > 1$. The control system is a nonlocal one, described by the transport equation

$$\partial_t \mu(t) + div\big(f(t, \mu(t), u(t))\mu(t)\big) = 0, \qquad \mu(0) = \mu_0$$

where $f : R_+ \times \mathcal{P}_p(R^d) \times U \to Lip(R^d, R^d)$, U is a compact metric space and $u : R_+ \to U$ is lebesgue measurable.

To state the main results I will discuss the extensions to Wasserstein spaces of the Scorza-Dragoni and mean-value theorems and also of notions of tangents, normals, proximal normals. Applications concern Hamilton-Jacobi-Bellman equation and the Lyapunov second method.

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