Control Systems with Path-wise State Constraints

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Control systems whose state trajectories must satisfy path-wise constraint are ubiquitous in control engineering. The constraint typically arises in connection with some safety protocol where, for example, the pressure in a chemical reactor must remain below dangerous levels, or the heat shield of a space re-entry vehicle must not exceed a critical level, or a driverless car must stay on the road and obey speed limits. This course provides the analytic tools for investigating state constrained control systems. The focus is on optimal control problems for such systems. We derive first order necessary conditions, in the form of a maximum principle. We identify some cases of interest when these conditions do not supply useful information about minimizers (the 'degeneracy' phenomenon) and show how these shortcomings can be overcome. We also introduce dynamic programming techniques for state constrained optimal control, which provides global optimality conditions expressed in terms of the Hamilton Jacobi equation.