Book Of Abstracts

Evolution in discrete and continuous mechanics: stability, transients, and asymptotics

Bressanone 1-3 Aprile 2022

W13	Fri 1	Sat 2	Sun 3
all-day			
09:00			
	Opening	09:30 Giuseppe Saccomandi	Giuseppe Mulone
10:00	10:00 Franco Cardin		Andrea Giacobbe
		Marzia Bisi	Paolo Falsaperla
11:00			
	11:30 Tommaso Ruggeri	11:30 Giuseppe Toscani	Carla Perrone
12:00	Vincenzo Sciacca	Maria Groppi	Giulio Giusteri
13:00			
14:00			
	14:30 Florinda Capone	14:30 Andrea Tosin	
15:00	Giuseppe Arnone	Natale Manganaro	
16:00	Valeria Giunta	Giorgio Martalò	
17:00	Nadia Loy	Francesco Gargano	
	Gabriele Grifò	Gaetana Gambino	
18:00	Alessandra Rizzo	Luca Santelli	
19:00			

STABILITY RESULTS FOR PENETRATIVE CONVECTION IN POROUS MEDIA

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Abstract

Penetrative convection occurs in many natural phenomena where an unstable stratified fluid moves into a stable one [1]. This topic is of interest in many research field like, for example, in geophysics and astrophysics [2]. In the preset talk, the onset of penetrative convection in a horizontal porous layer is investigated, on taking into account for quadratic density law. In particular, linear and nonlinear stability analysis of the conduction solution is performed [3].

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A general Boltzmann model for polyatomic gases

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We propose a kinetic Boltzmann model for polyatomic gases, where the internal structure of a molecule is described by a single internal state parameter, belonging to a suitable space. Proper options for such a space of internal states and for the measure defined on it allow to recover some models commonly used in kinetic theory for polyatomic particles: the description based on a set of discrete internal energy levels [1,2], and the one involving a continuous internal energy variable [3]. Moreover, within this general framework it is possible to build up new models desirable in physical applications, as a description able to separate the internal energy into two different components, the rotational and the vibrational ones, with the former approximated by means of a continuous variable, keeping the latter discrete. We prove the H-Theorem for the proposed kinetic equation of Boltzmann type in this general framework, and we characterize the equilibrium Maxwellian distribution and the thermodynamic number of degrees of freedom. We also show how is possible to reduce some models fitting this general setting to a one-real-variable description with a suitable measure, similar to the classical continuous model with integration weight. Possible generalizations of our formulation to gas mixtures of polyatomic and monoatomic constituents, even in presence of chemical reactions, are also discussed.

References

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[3] L. Desvillettes, R. Monaco, F. Salvarani, A kinetic model allowing to obtain the energy law of polytropic gases in the presence of chemical reactions, *Eur. J. Mech. B Fluids* **24** (2005), 219-236.

ONSET OF THERMOSOLUTAL CONVECTION IN HORIZONTAL NANOFLUID LAYERS

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ABSTRACT

Onset of thermal convection in nanofluids is having great attention in the recent literature.

Nanofluids are engineered colloids composed of nanoparticles (oxides and metals) dispersed in a base fluid [1]. Three different behaviours with respect to the base fluid have been observed: abnormal thermal conductivity increase; abnormal viscosity increase; abnormal single-phase convective heat-transfer coefficient increase. The fundamental mathematical theory of nanofluids was introduced by Buongiorno in [2]. He introduced a model for convective transport in nanofluids incorporating Brownian diffusion and thermophoresis effects. Tzou in [3], [4] found that the instability threshold was lower by one or two orders of magnitude than that one of regular fluids.

In this talk the onset of thermosolutal convection in a uniformly rotating nanofluid layer with physically meaningful boundary conditions (requiring the vanishing of the nanoparticle flux) is investigated.

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Quantum asymptotic ground states for multiple-well problems on $\mathbb{T}^n$

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Alcune teorie 'riduzionistiche', correlanti descrizioni macroscopiche della meccanica dei continui con descrizioni puntuali cristallografiche finito-dimensionale, hanno prodotto importanti risultati di comprensione della materia. Tali meccanismi conoscitivi, quando riconosciuti ben funzionanti, hanno sempre avuto delle valenze di tipo locale o asintotico. La comunicazione proposta si colloca in questo naturale ordine d'idee, coerente con l'indirizzo scientifico del meeting.

Si considera un sistema particellare a struttura periodica, definito in tori $\mathbb{T}^n$ a dimensione grande a piacere e dotato di energia potenziale con un numero finito di buche (*multiple-well*). Se ne studiano aspetti quantistici asintotici, in particolare si calcola lo stato fondamentale, in un preciso senso asintotico. Le tecniche qui usate riguardano le soluzioni di viscosità delle equazioni di H-J, ed elementi della teoria weak KAM. Si giunge alla generalizzazione di un risultato di Barry Simon del 1983 coinvolgente la doppia buca (*double well*). Quel risultato fu ottenuto mediante considerazioni legate alla teoria delle Grandi Deviazioni.

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Stability of Bingham–Poiseuille flows in an inclined channel

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We study the stability of Bingham-Poiseuille flows in an inclined layer. This problem has geophysical applications to the evolution of landslides [1]. We generalise recent results of Falsaperla et al. [2, 3] on laminar Couette and Poiseuille flows for Newtonian fluids.

The stability of the basic motion of the generalised Navier-Stokes system for a Bingham fluid in a horizontal channel against linear perturbations has been studied in [4, 5, 6] We show here the stabilizing effect of the Bingham parameter B. We also study the stability of the linear system with the energy method (Lyapunov method) and prove that the streamwise perturbations are always stable, while the spanwise perturbations are energy-stable if the Reynolds number Re is less than a critical value Rc, obtained by solving a generalised Orr equation of a maximum variational problem.

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Cross-diffusion-induced effects on stationary pattern formation

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ABSTRACT

In recent years, the theory initially introduced by Turing for the mathematical description of pattern formation has raised increasing interest: the need to go beyond the classical reaction-diffusion framework has triggered the formulation of generalizations of the original theory along several directions. One of them is the inclusion of nonlinear/cross-diffusion effects that appears crucial for a realistic description of phenomena occurring in biological tissues, on surfaces, in gel media and in population dynamics, where density-dependent migration effects are ubiquitous.

This talk shall discuss the role of cross-diffusion terms on the onset of stationary non-homogeneous structures in reaction-diffusion systems. We shall show that the presence of cross-diffusion terms can relax, or even remove, the differential diffusivities constraint imposed by the Turing mechanism; namely, that with an activator-inhibitor-type reaction kinetics, the inhibitor must diffuse much faster than the activator. Therefore, including cross-diffusion in the model system enlarges the parameter region where the pattern can develop [2, 7]; this makes the mechanism more plausible than Turing's original one in explaining the variety of spatial scales and environments where one observes biological patterns [8].

In particular, we shall see that cross-diffusion boosts pattern formation and intricate sequences of bifurcations in reaction-diffusion systems where the mere coupling of the kinetics with the classical diffusion would prevent any interesting spatio-temporal dynamics [3, 4, 5, 6, 9].

We shall also present a recently discovered phenomenon induced by cross-diffusion: it is the existence of a double bifurcation threshold of the control parameter for the onset of stationary structures. The pattern can emerge for values of the diffusion ratio above a given threshold, as in the classical Turing systems, and when the diffusion ratio is below a new lower threshold. In the latter case, the pattern formation mechanism is opposite to what is prescribed by the classical Turing theory [1]. We shall see that the patterns originated by this new mechanism are out-of-phase and subcritical in large part of the instability region. We shall finally discuss some implications of the above results on selecting the desired patterning features and on the emergence of localized solutions.

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Transition phenomena in high Re number vortex layers flows

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Abstract

We study the evolution of a 2D vortex layer at high Reynolds number governed by the incompressible Navier-Stokes equation. Vortex layer flows are characterized by intense vorticity distributed around a planar curve with the vorticity rapidly decaying to 0 away from the curve. We consider vortex layers whose initial thickness is proportional to the square-root of the inverse of the Reynolds number. We investigate the typical roll-up process, showing that the crucial phases in the initial flow evolution are related to the growth of the palinstrophy and the competition between the vorticity gradients stretching and the dissipation. We show the presence of two Reynolds-regimes, low and moderate-high. For moderate-high Reynolds the flow evolution shows transition phenomena in which small scale structures develop, leading to the formation of self-similar structures in the flows. Moreover, we compare the obtained solution with the vortex-sheet solution obtained through the governing equation of the vortex sheet flows, the Birkhoff-Rott equation. Through the analysis of the complex singularities we show that, in the limit $Re \to \infty$, the Navier-Stokes solution behaves differently from the Birkhoff-Rott solution.

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[1]Courant Institute, NY (USA).
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Monotonic stability for Couette and Poiseuille flows

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Theorical, numerical, and experimental investigation of shear flows, their stability, and the structure of turbulent flows above threshold is actual and still debated. The first investigations on this argument date back to works of Orr and Joseph, and still the results have some controversial incongruences among them and with the experiments.

Even in a linear stability regime, the evolution of the energy from a given initial field of velocity, its monotonicity, its initial increase, its maximal value, are all subject of investigation. One example is the attention given to a regime called $t$ransient (the interval of time in which the energy increases before eventually decreasing to zero).

We present a theoretical and a numerical investigation of the time evolution of the energy, which reproduces and confirms the numbers that both, Joseph and Orr obtained.

This presentation couples with that of Giuseppe Mulone, that will give his point of view on the higher reasonability of Orr's deductions with respect to Joseph's ones.

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Title: *Detecting stability and multi-stability in multispecies non-local advection-diffusion models*

Speaker: *Valeria Giunta*

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Abstract:

In many biological systems, it is essential for individuals to gain information from their local environment before taking decision. In particular, through sight, hearing or smell, animals detect the presence of other individuals and adjust their behavior accordingly. Interestingly, this feature is not only restricted to higher level species, such as animals, but is also found in cells. For example, some human immune cells are able to interact non-locally by extending long thin protrusions to detect the presence of chemicals or signaling molecules. Indeed, the process of gaining information about the surrounding environment is intrinsically non-local and mathematically this leads to non-local advection terms in continuum models.

In this talk, I will focus on a class of nonlocal advection-diffusion equations modeling population movements generated by inter and intra-species interactions. I will show that the model supports a great variety of complex spatio-temporal patterns, including stationary aggregations, segregations, oscillatory patterns, and irregular spatio-temporal solutions. However, if populations respond to each other in a symmetric fashion, the system admits an energy functional that is decreasing and bounded below, suggesting that patterns will be asymptotically stable. I will describe novel techniques for using this functional to gain insight into the analytic structure of the stable steady state solutions. This process reveals a range of possible stationary patterns, including regions of multi-stability. These will be validated via comparison with numerical simulations.

Modelling Shear Jamming and Fragility of Concentrated Suspensions

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Under an applied traction, highly concentrated suspensions of solid particles in fluids can turn from a state in which they flow to a state in which they counteract the traction as an elastic solid: a shear-jammed state. Remarkably, the suspension can turn back to the flowing state simply by inverting the traction. A tensorial constitutive model is presented and its effectiveness discussed in paradigmatic cases [1]. We show that, to reproduce the phenomenology of shear jamming in generic geometries, it is necessary to link this effect to the elastic response supported by the suspension microstructure rather than to a divergence of the viscosity.

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Pattern formation in two-compartments hyperbolic models

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Self-organizing patterns are quite ubiquitous in nature and the study of the associated nonlinear spatial processes has become nowadays a sub-area of complexity science. Despite pattern formation is observed in different fields, such as biology, ecology, chemistry and medicine, the underlying phenomena share some common features [1]. To better understand those processes that characterize pattern formation and stability, different analyses are here conducted in two-compartments hyperbolic models. The use of an hyperbolic framework is justified by the need of overcoming the paradox of infinite propagation speed of disturbances and of taking into account inertial effects and long transient regimes [2, 3]. The main aim of this talk is to elucidate the role of inertial times in the formation and stability of stationary and non-stationary patterns.

In the first part of the talk, we will tackle the study of Eckhaus instability [1, 4] of stationary periodic patterns on large finite domains, in both supercritical and subcritical regimes, in the framework of reaction-diffusion hyperbolic models. In particular, we deduce the equation ruling the evolution of pattern amplitude close to criticality as well as the explicit expressions of the most relevant dynamical features characterizing the quantized periodic branches, i.e. stationary amplitude, existence and stability thresholds and linear growth rate. Moreover, this analysis has also allowed to inspect the functional dependence of phase slip on inertial times.

Then, the focus is moved on reaction-advection-diffusion hyperbolic models, where oscillatory periodic pattern can be observed. In particular, linear stability analysis techniques is used to deduce the conditions under which wave instability takes place and multiple-scale weakly nonlinear analysis is applied to determine the equation which rules the spatio-temporal evolution of pattern amplitude close to criticality. In this framework, some intriguing consequences due to hyperbolicity are emphasized.

To show in detail the richness of these scenarios through an illustrative example, we will show the pattern dynamics occurring in the hyperbolic generalization of the Klausmeier model [5, 6], where vegetation patterns arise in the context of dryland ecology.

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BGK Models for inert mixtures: comparison, hydrodynamic limits and applications

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BGK Relaxation-time approximations constitute the most used simplified kinetic models of the true integrodifferential Boltzmann equations of Rarefied Gas Dynamics, since they retain the most significant mathematical and physical features of the Boltzmann description. Their extension to mixtures is not trivial since some inconsistencies can arise, like breakdown of positivity of density and temperature fields and of the indifferentiability principle.

In this talk, consistent BGK models [1,2,3] for inert mixtures of gases are compared, first in their kinetic behavior and then versus the hydrodynamic limits that can be derived in different collision-dominated regimes. In particular, the structure of the BGK model presented in [3] allows to deduce two different hydrodynamic limits (at Euler and Navier-Stokes level), characterized by global velocity and temperature or multi-velocity and multi-temperature, respectively [4]. The comparison is carried out both analytically and numerically, for the latter using an asymptotic preserving, conservative semi-Lagrangian scheme for the BGK models. Application to realistic binary mixtures of noble gases is also presented [5].

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A viral load-based model for epidemic spreading by kinetic equations on a graph

Nadia Loy, Università degli Studi Di Parma

Abstract

In this talk, I will discuss a Boltzmann-type kinetic model of the spread of an infectious disease on a network, which describes the connections among countries, cities or districts depending on the spatial scale of interest. The disease transmission is represented in terms of viral load of the individuals and is mediated by social contacts among them and their displacements across the nodes of the network. The state of each individual with respect to the disease is thus characterized by the viral load and no classic division in epidemic compartments is considered. In particular, I will discuss various migration modes on the network as well as the impact of confinement measures, such as quarantines or localised lockdowns, on the diffusion of the disease on the network. Both analytical and numerical results will be presented.

References

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Double waves for hyperbolic systems

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Among the class of the multiple wave exact solutions, double waves and simple waves play a prominent role for studying problems of interest in nonlinear wave propagation. Within such a theoretical framework, we propose a suitable reduction procedure for determing particular double wave solutions for first order hyperbolic systems. The idea is to reduce the problem of integrating a full system of equations to that of solving a suitable 2×2 reduced system along with a differential constraint. An example of interest on gas dynamics is given.

Regular singularities and sub-shocks
in moment equations for gasdynamics

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Abstract. Since Navier-Stokes equations do not satisfactorily describe the shock thickness, the classical problem of shockwave solutions is analyzed and discussed by means of moment equations. Unfortunately, since Grad's work, these models have immediately shown some limitations due to the hyperbolic structure of equations; in particular, 13-moment description provides a continuous shock structure up to a certain Mach number (M=1.65), and from this value on the solution exhibits a non-physical discontinuity (sub-shock). To improve such result, other hyperbolic models involving a higher number of moments have been proposed in extended thermodynamics. This approach seems not to allow significant extensions of the Mach number range that guarantees a continuous solution; moreover, the solutions can exhibit additional singularities due to other characteristic speeds. In this talk, by means of a geometrical approach, we want to discuss such singularities for different moment closures.

Monotonic energy stability of planar shear flows: Orr vs Joseph
G. Mulone (A. Giacobbe e C. Perrone)

The critical Reynolds numbers for the monotonic exponential energy stability of the Couette and Poiseuille plane flows were obtained by Orr (1907) in famous paper, and by Joseph (1966), Joseph and Carmi (1969) and Busse (1972). They used variational methods applied to a maximum functional ratio which comes from the Reynolds-Orr energy identity. They got different results, for instance in the Couette case: Orr obtained the value 44.3 (on the spanwise perturbations) and Joseph 20.65 (on streamwise perturbations).
Recently in a paper by Falsaperla et al (2022), the authors made a conjecture by restricting the search for the maximum on a subspace of the space of kinematically admissible perturbations; with this conjecture, the critical nonlinear energy Reynolds number was found among the spanwise perturbations.
Here we try to prove that this conjecture is true. For this, *we split the dissipation terms* in the Reynolds-Orr identity into two terms and prove that if the Reynolds numbers are less than those obtained from Orr then the basic motion is monotonically energy stable. This result implies *the validity of a Squire theorem in the nonlinear energy norm*: the least stabilizing perturbations are the two-dimensional spanwise perturbations (this communication is connected with that of A. Giacobbe).

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A time delayed deterministic model for the spread of COVID-19 with calibration on a real dataset

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During the evolution of the COVID-19 pandemic, each country has adopted different control measures to contrast the epidemic's diffusion. Restrictions to mobility, public transport, and social life in general have been actuated to contain the spread of the pandemic.

In this talk, we start by considering the deterministic SIRD model with delays proposed by Calleri et al. which is improved by adding the vaccinated compartment V (SIRDV model) and modeling a time-dependent contact frequency. The three delays take into account the incubation time of the disease, the healing time, and the death time.

The aim of this study is to understand the effect of the vaccination campaigns in Great Britain (GBR) and Israel (ISR) during the pandemic period. The different restriction periods are included by fitting the contact frequency on real datasets as a piecewise constant function. As expected, the vaccination campaign reduces the amount of deaths and infected people. Furthermore, for the different levels of restriction policy, specific values of the contact frequency that can be used to predict the trend of the pandemic are found.

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Differential Constraints for Solving Nonlinear Wave Problems

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Simple wave solutions are of great interest for nonlinear wave problems. Such a class of solutions is admitted by first order quasilinear hyperbolic homogeneous systems and it is useful for solving different problems of interest in the applications as, for instance, Riemann problems. Unfortunately simple waves are not usually admitted by hyperbolic systems when dissipative effects are taken into account (non-homogeneous case).

Within such a theoretical framework, in this talk, we would like to show how the Method of Differential Constraints [1]-[3] can be used for determing a class of exact solutions which generalize simple waves so that different nonlinear wave propagation problems can be studied.

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Recent Results on Rational Extended Thermodynamics

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In this talk, some recent results of Rational Extended Thermodynamics both in the classical and relativistic framework are presented. In particular, the problem in which the relaxation times for internal states are different and the relativistic case of polyatomic gases, including its classical limit, are discussed. Finally, some thermodynamic aspects of similarity between gas mixtures and flocking and synchronization are studied.

Mathematics of Materials with Fibers

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Questo breve intervento considera una rassegna dei modelli che vengono comunemente usati per descrivere materiali solidi e fluidi con fibre e alcune delle loro peculiarità matematiche. Questi modelli trovano non solo applicazioni per descrivere materiali dove una matrice solida viene rinforzata per mezzo dell'aggiunta di fibre ma anche nelle sospensioni colloidali e nei manufatti "orditi" dalle fibre stesse. In particolare, si dimostra come il carattere anisotropo di questi modelli abbia delle conseguenze molto particolari nello studio di alcuni questioni di stabilità. Questo fatto è dovuto a come la presenza delle fibre riesce ad accoppiare i vari stati di deformazione e/o di flusso.

Anche nel caso lineare, il classico principio di sovrapposizione degli effetti deve essere completamente rivisto e semplici stati tensionali possono generare la presenza di deformazione secondarie che cambiano in modo pesante la struttura matematica e meccanica della descrizione di alcuni fenomeni anche comuni.

Rayleigh–Bénard Convection in Spherical Shells

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A numerical scheme for simulations of three-dimensional Navier-Stokes equations for incompressible viscous flows in spherical coordinates has been developed with the purpose of analysing and characterising Rayleigh–Bénard convection between spherical shells [1].

The low Rayleigh number regime for non–rotating Rayleigh-Bénard convection is explored with various radial gravity profiles for both air-like and water-like fluids [2]. We observe how, in most cases, the effect of the different gravity can be reabsorbed by the introduction of an effective Rayleigh number, yielding a critical $Ra_c \approx 1750$ for the onset of convection regardless of the specific gravity profile. The exploration of higher values of Rayleigh number shows that the system is subjected to hysteresis, i.e. the dynamic has a very strong dependence on initial conditions and flow parameters.

We then investigate the effect of an offset between the sphere center and the gravity center [4], which might be used to simulate the effect of a dishomogeneity in the Earth core. Even a small displacement between the two points gives rise to a distorted temperature profile, with a hot jet emerging from the inner sphere in the direction opposite to the shift. Nevertheless, while the local heat flux and temperature profile are greatly modified, the global heat flux seems to be mostly unaffected by these changes.

Lastly, we analysed the diffusion–free scaling regime for slowly rotating Rayleigh-Bénard convection between spherical shells [3]. This regime is characterized by a bulk–dominated flow and its emergence, for the parameters used, is due to the peculiar properties of the spherical geometry compared to the more commonly studied planar geometry.

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Discontinuous-Galerkin Methods for a Kinetic Cucker-Smale Model

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Abstract

We consider a kinetic Cucker-Smale type flocking models introduced in [4]. The jargon "flocking" represents collective phenomena in which self-propelled particles (or agents) is organized into an ordered motion from a disordered state using only limited environmental information and simples rules. Such an organized motion is ubiquitous in our nature: aggregation of bacteria, flocking of birds, swarming of fish, herding of sheep, etc... These models have been extensively studied recently because of their possible applications to sensor networks, controls of robots, unmanned aerial vehicles, and opinion formation of social networks [2,5]. We apply a discontinuous Galerkin method [1,3] for the numerical resolution of the kinetic Cucker-Smale type flocking models, and we analyse consistency and stability of the semi-discrete scheme.

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On the body size distributions of species. A kinetic approach

Giuseppe Toscani *

Abstract

In this talk, by means of classical methods of statistical mechanics, we present a qualitative and quantitative description of the possible mechanisms leading to the observed statistical weight distribution of species. This evolutionary problem can be perfectly treated within the framework of kinetic theory, that provides a powerful approach to the mathematical modeling of systems composed of a huge number of agents interacting with each other and/or the environment, and has as its primary product the understanding of the relationship between parameters in microscopic rules and the resulting macroscopic statistical outcomes. Because of this broad spectrum of application, in the last two decades mathematical modeling based on kinetic theory entered in interdisciplinary fields ranging from the biological context [4] to new aspects of socio-economic phenomena [1, 3]. Following the arguments in [2], the kinetic description of the time variations of the weight distribution will be based on elementary interactions that describe successive evolutionary updates, and for large times will determine explicit equilibrium distributions. Numerical fittings on mammalian eutherians of the order Chiroptera population illustrate the effectiveness of the approach [2].

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Title: A statistical mechanics approach to macroscopic limits of car-following traffic dynamics

Speaker: Tosin, Andrea

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Abstract:
We present the derivation of macroscopic traffic models from car-following vehicle dynamics by means of hydrodynamic limits of an Enskog-type kinetic description. We consider the superposition of Follow-the-Leader (FTL) interactions and relaxation towards a traffic-dependent Optimal Velocity (OV) and we show that the resulting macroscopic models depend on the relative frequency between these two microscopic processes. If FTL interactions dominate then one gets an inhomogeneous Aw-Rascle-Zhang model, whose (pseudo) pressure and stability of the uniform flow are precisely defined by some features of the microscopic FTL and OV dynamics. Conversely, if the rate of OV relaxation is comparable to that of FTL interactions then one gets a Lighthill-Whitham-Richards model ruled only by the OV function. Unlike other formally analogous results, our approach builds the macroscopic models as physical limits of particle dynamics rather than simply assessing the convergence of microscopic to macroscopic solutions under suitable numerical discretisations.

The talk is based on a joint work with F. A. Chiarello (Politecnico di Torino) and B. Piccoli (Rutgers University - Camden, USA):

F. A. Chiarello, B. Piccoli, A. Tosin. A statistical mechanics approach to macroscopic limits of car-following traffic dynamics, *Internat. J. Non-Linear Mech.* 137 (2021) 103806.