From dynamics and celestial mechanics to *dark matter*

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Does it even exist?
Intent of this expository lecture is to introduce this dark matter controversy and show how insights can come from celestial mechanics.

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Intent of this expository lecture is to introduce this dark matter controversy and show how insights can come from celestial mechanics.
In doing so, some notions from celestial mechanics, and the N-body problem, are described

It all starts with Kepler’s third law

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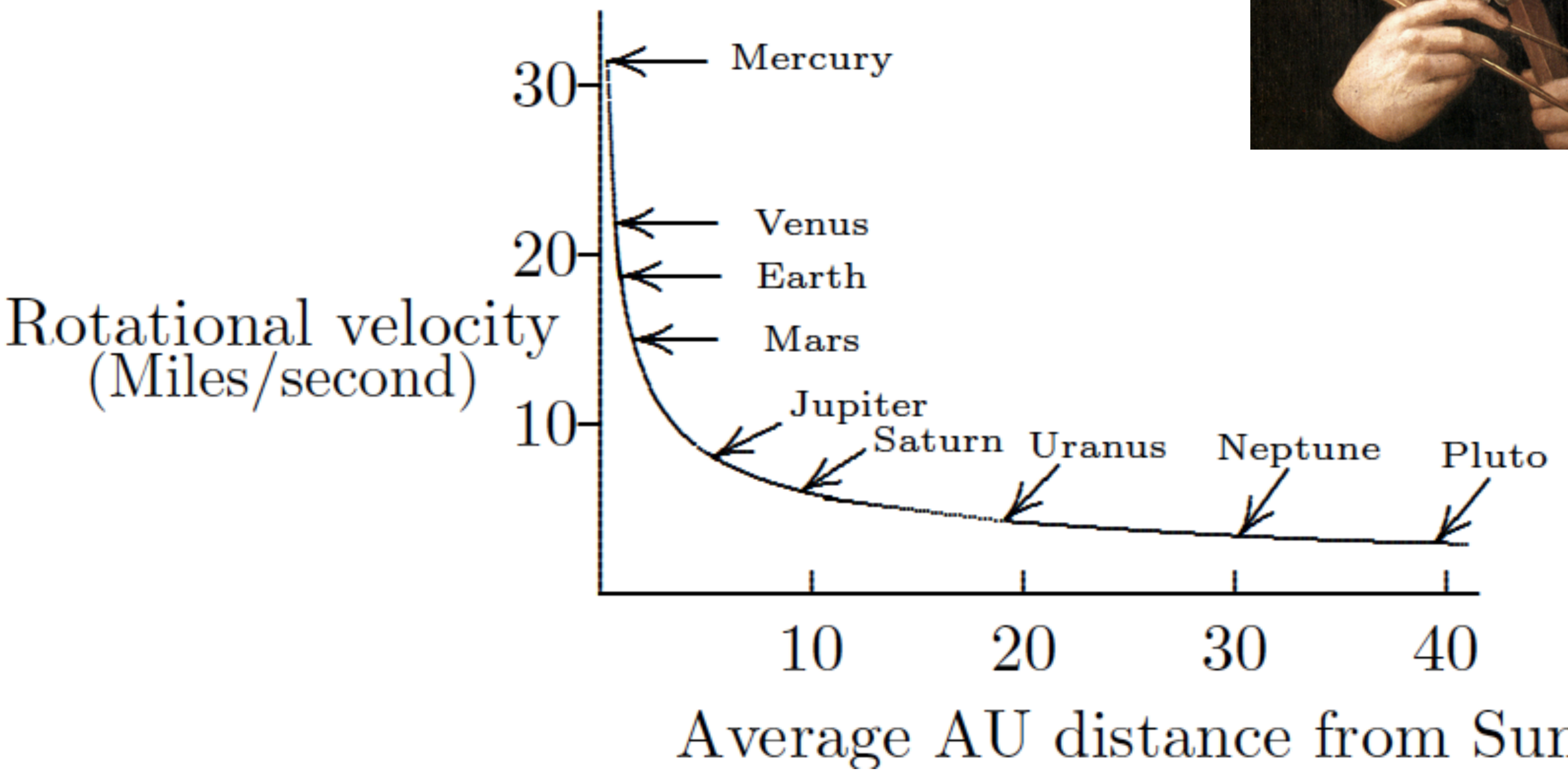

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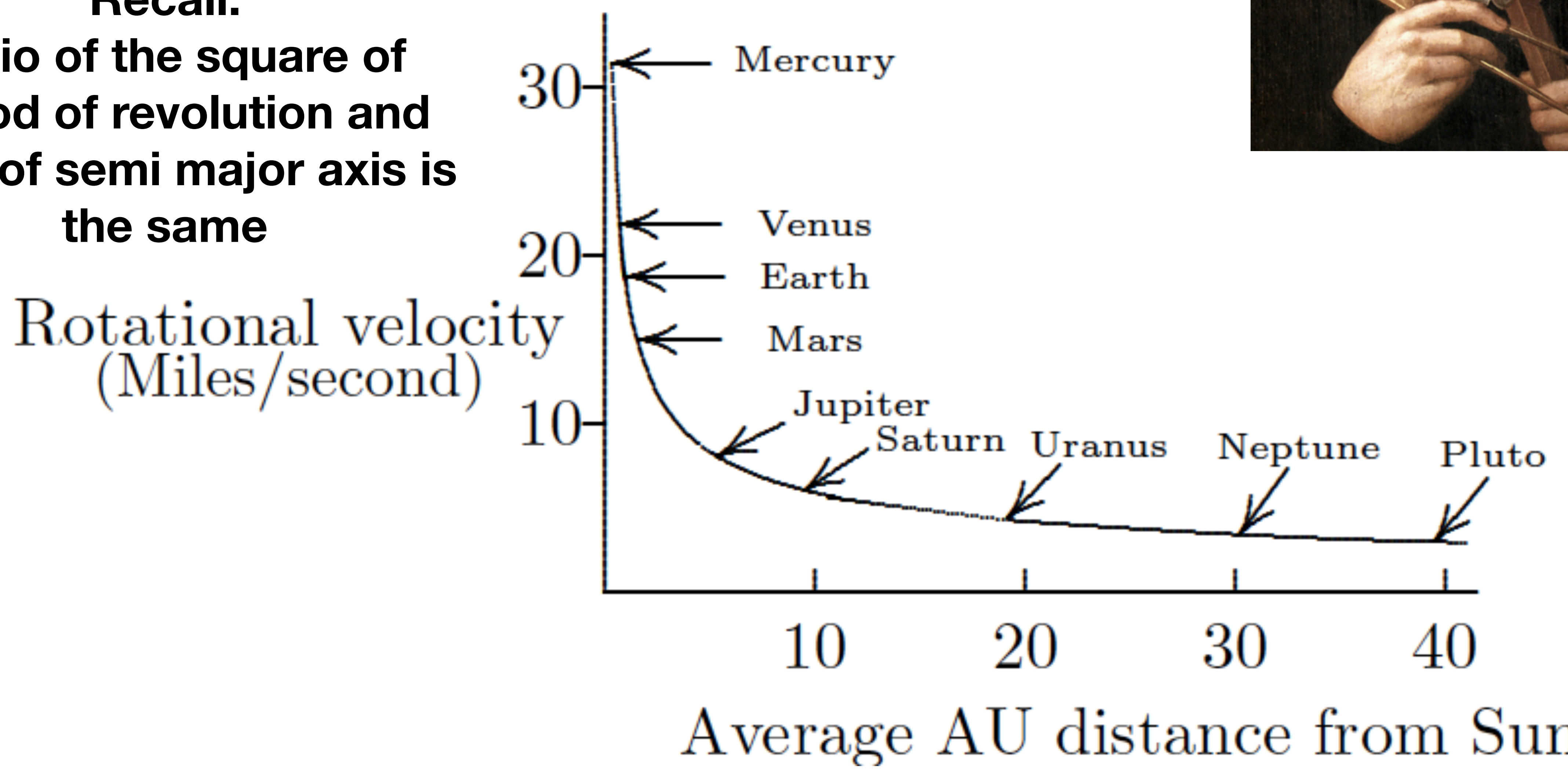
Recall:

Ratio of the square of period of revolution and cube of semi major axis is the same



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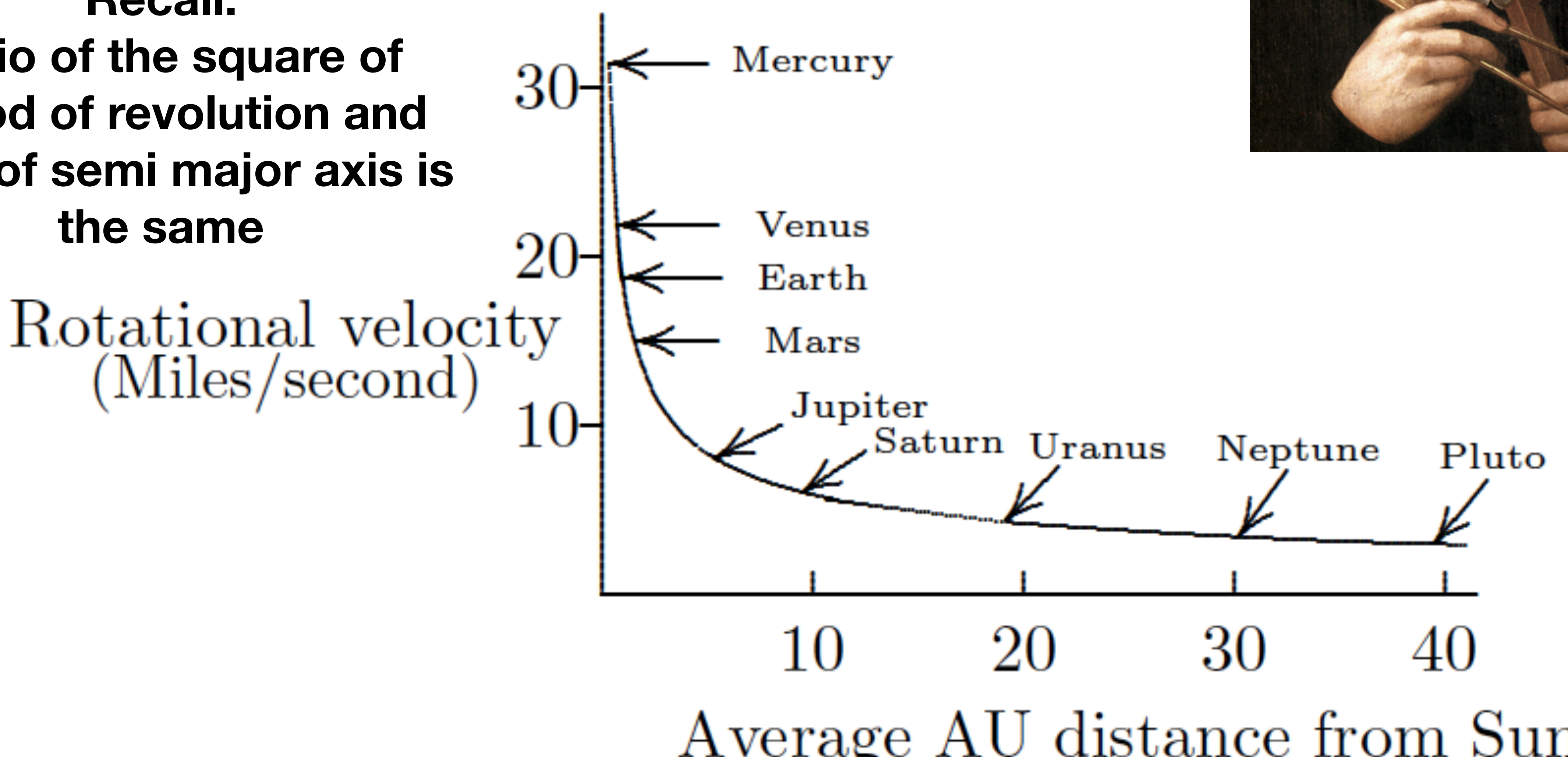
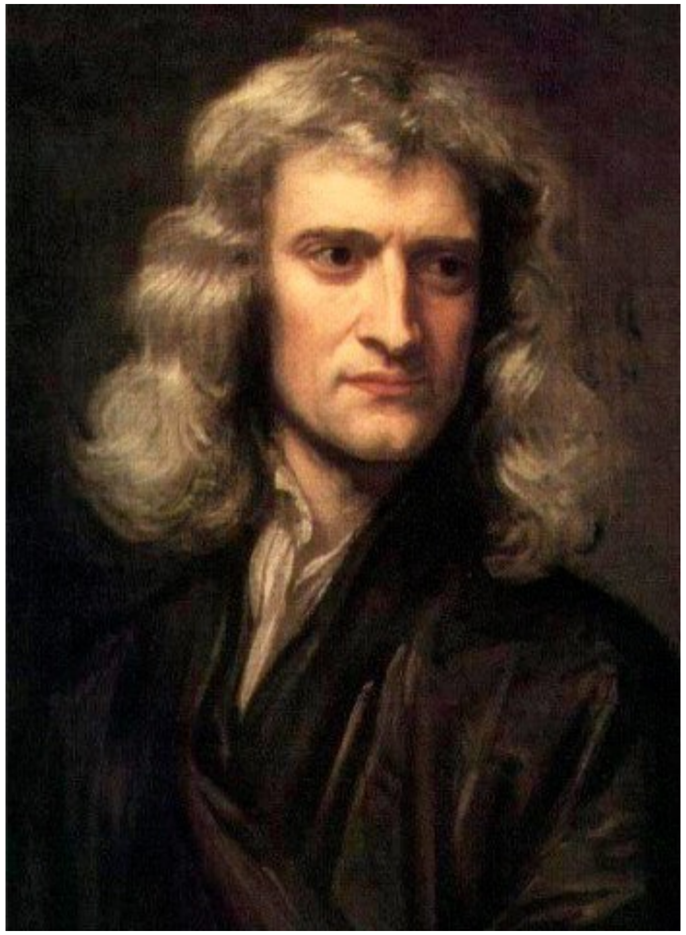


Of importance: the downward swoop. This will play an critical role

On to Newton



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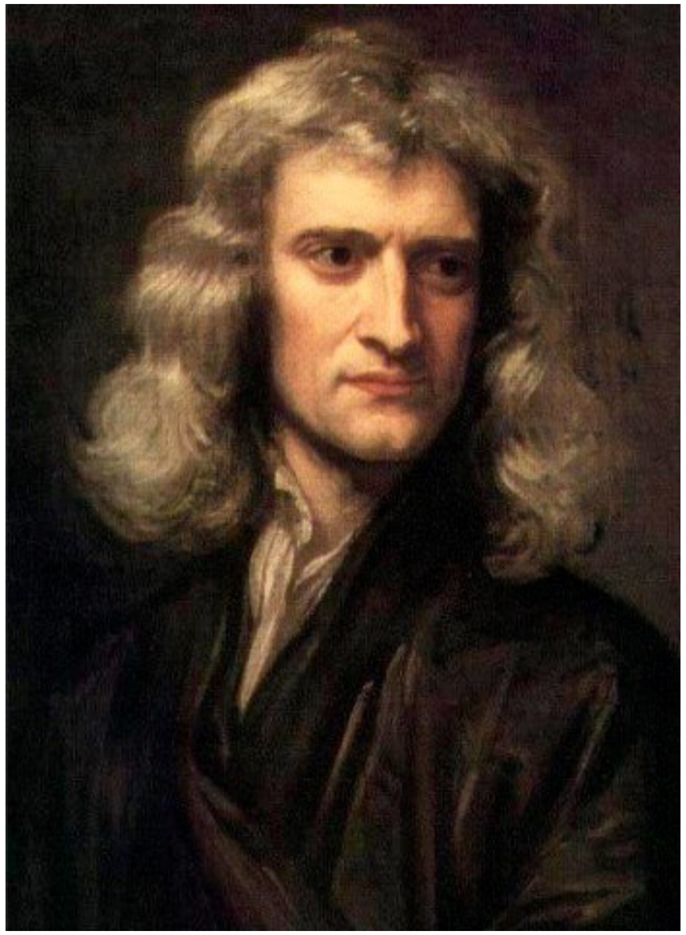
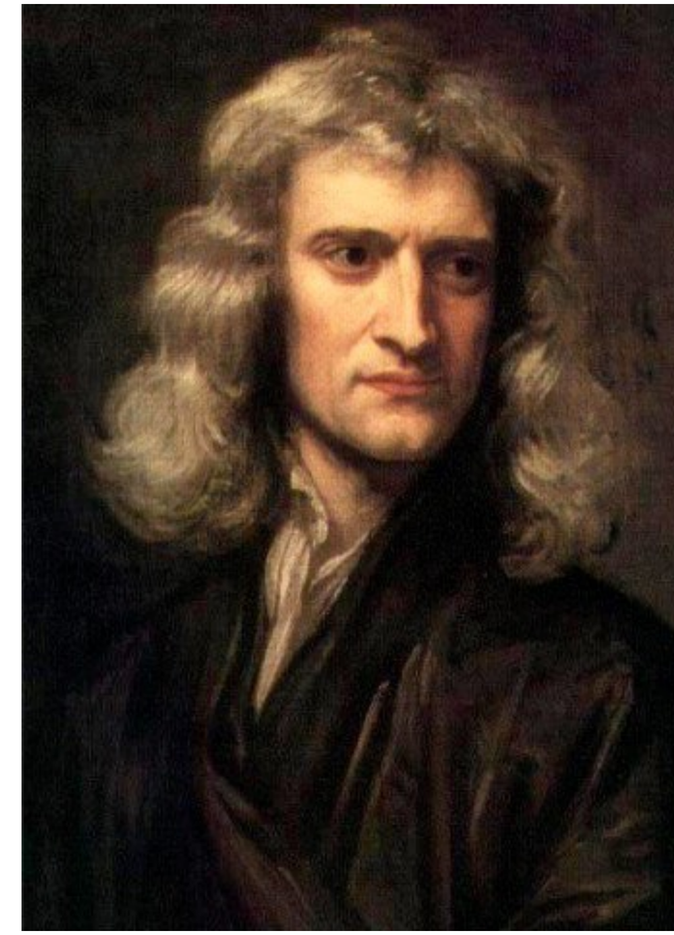


How can we determine the mass of the Sun?

On to Newton

Two-body problem: Sun and planet in circular orbit



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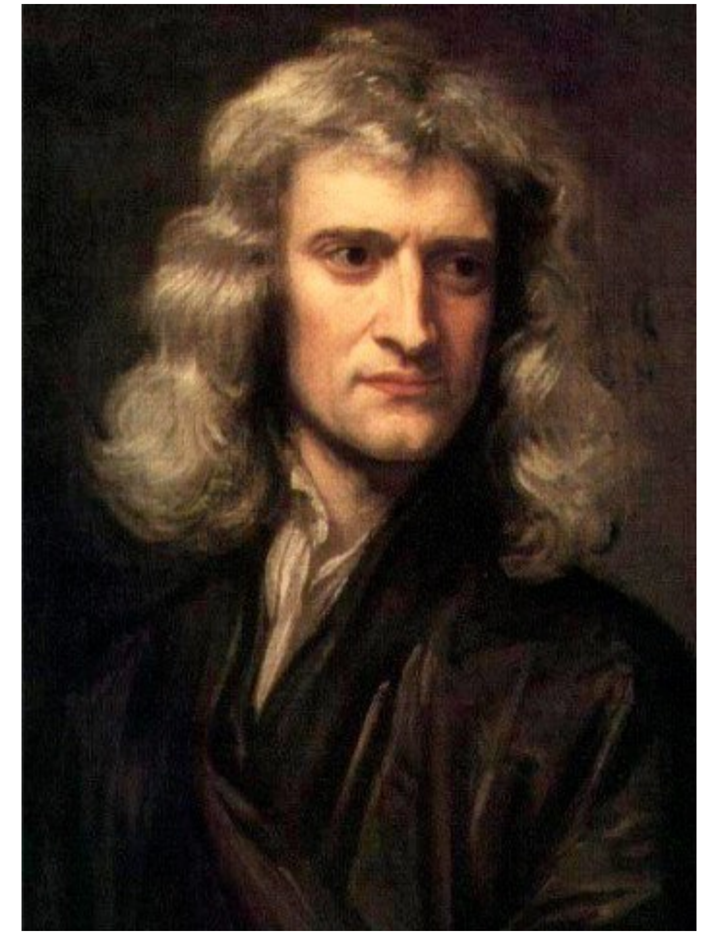

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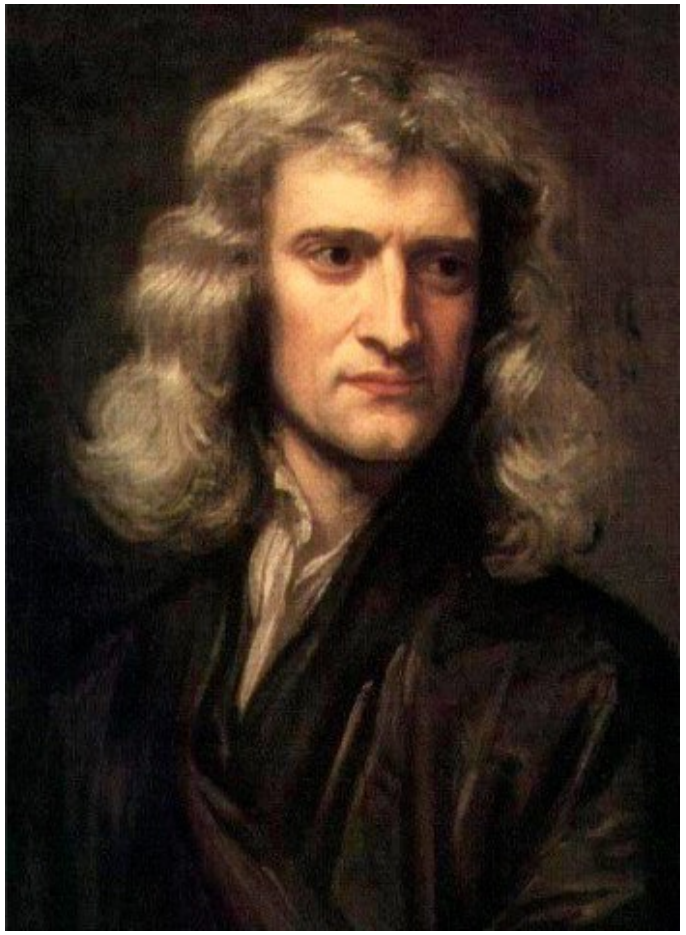

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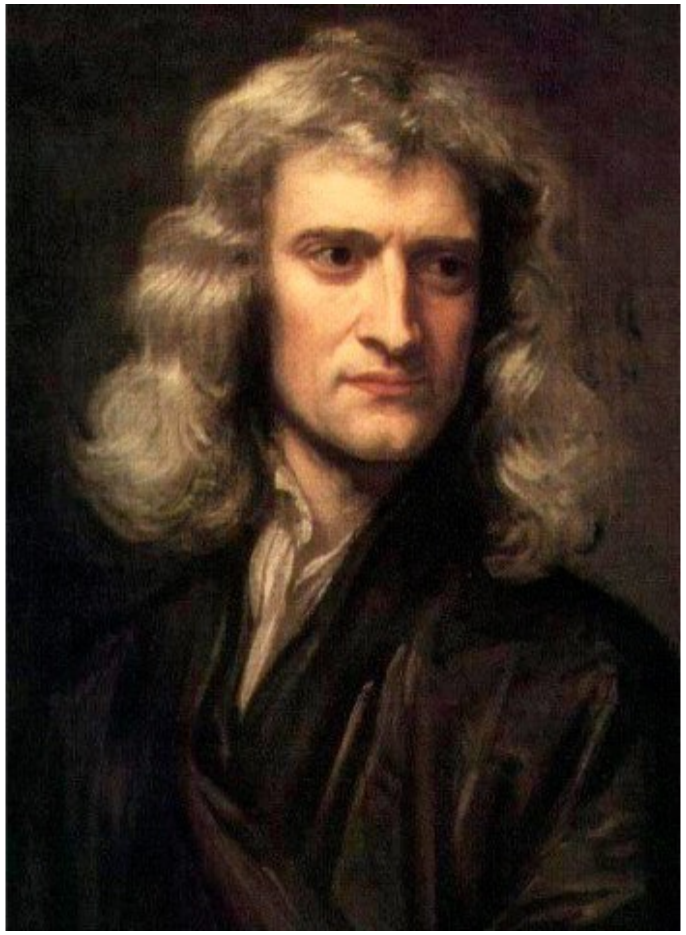

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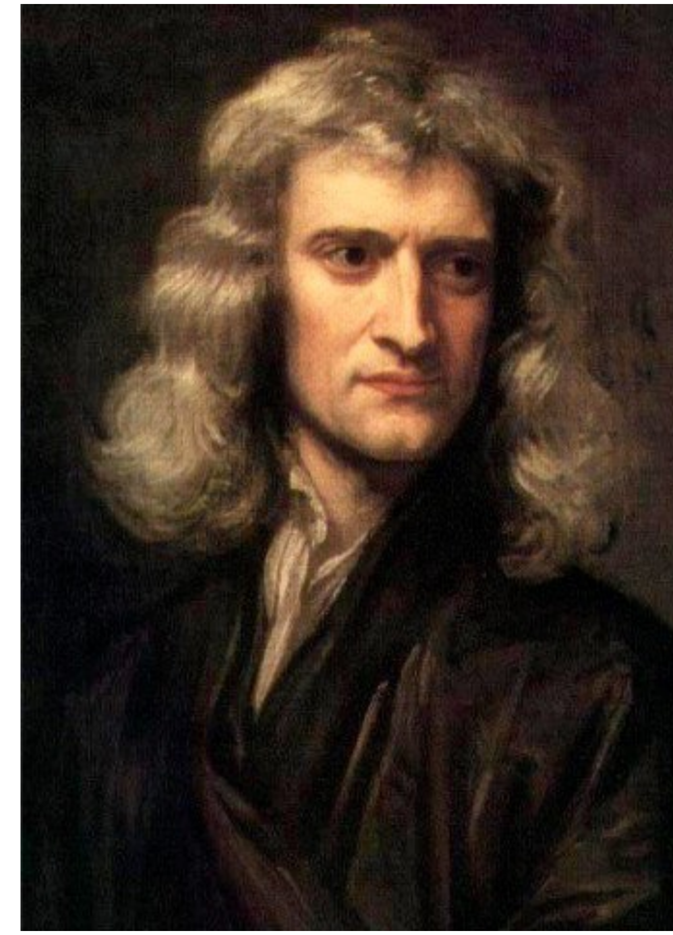

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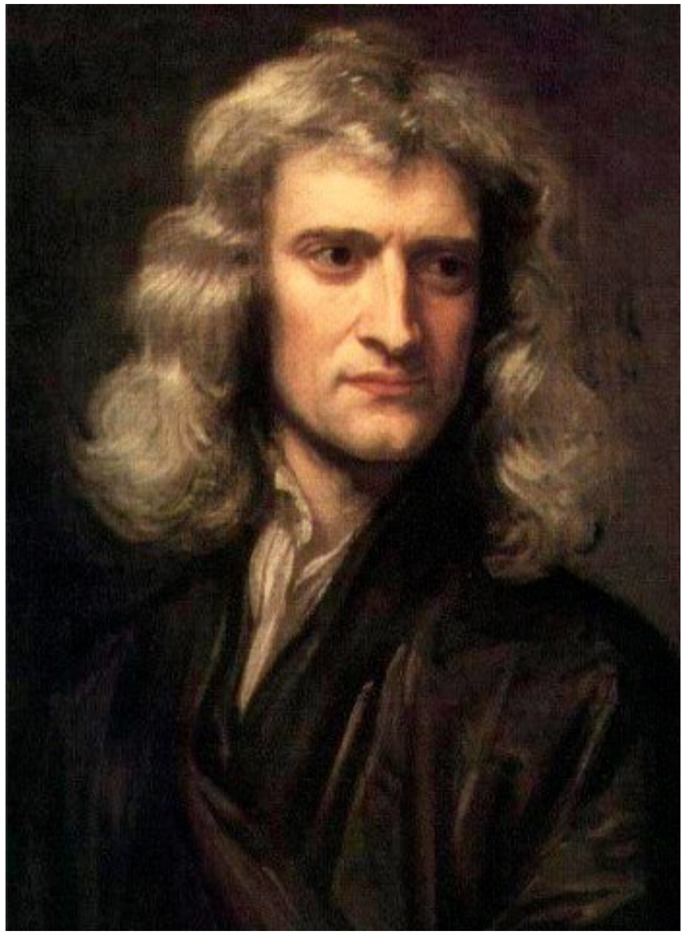

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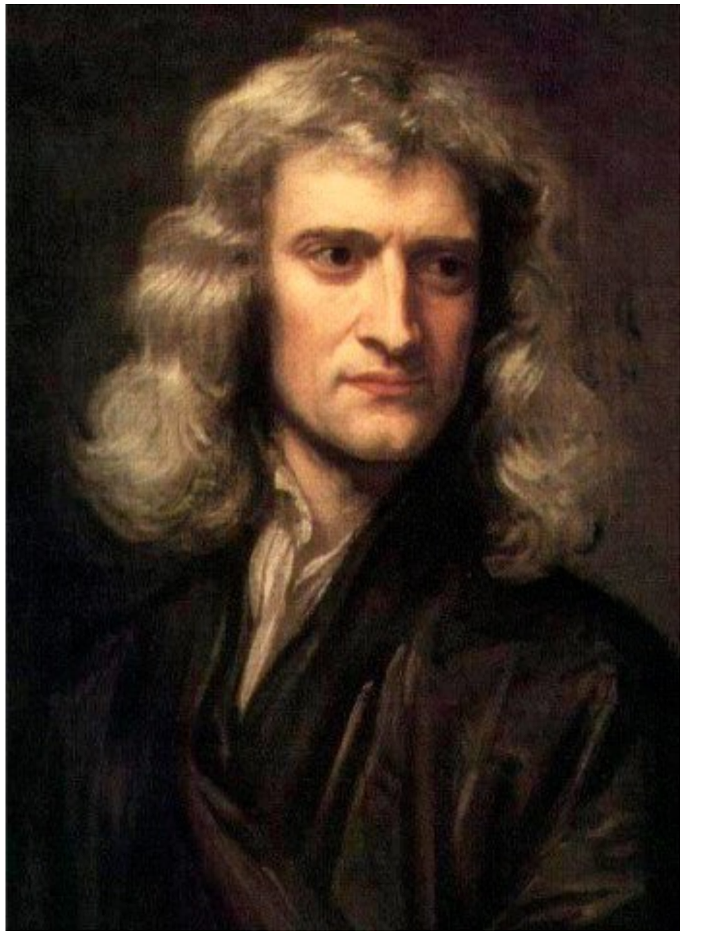

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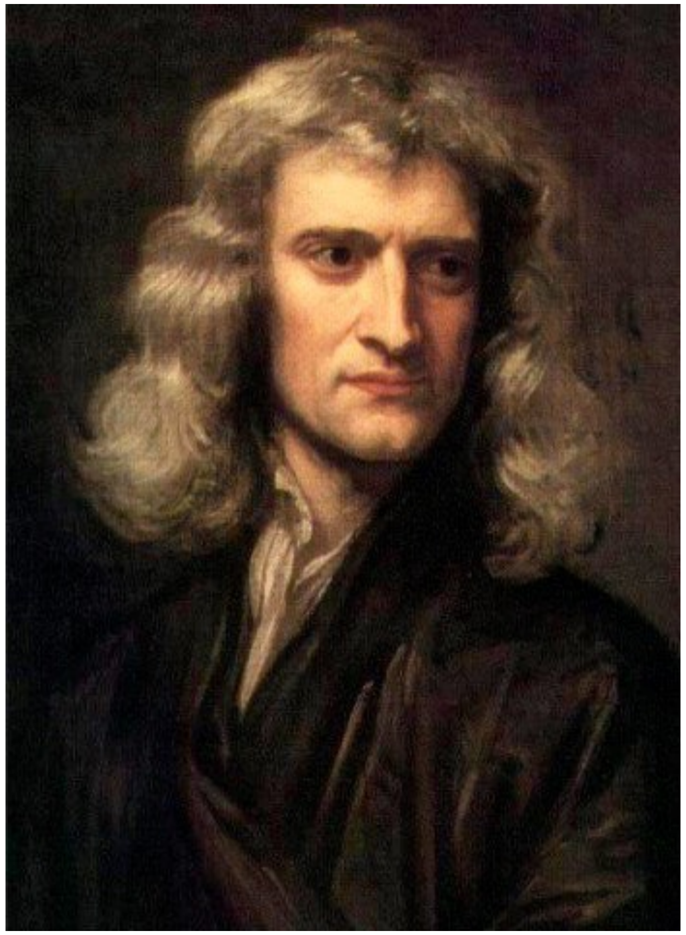
Different planets, different mass values for the Sun?



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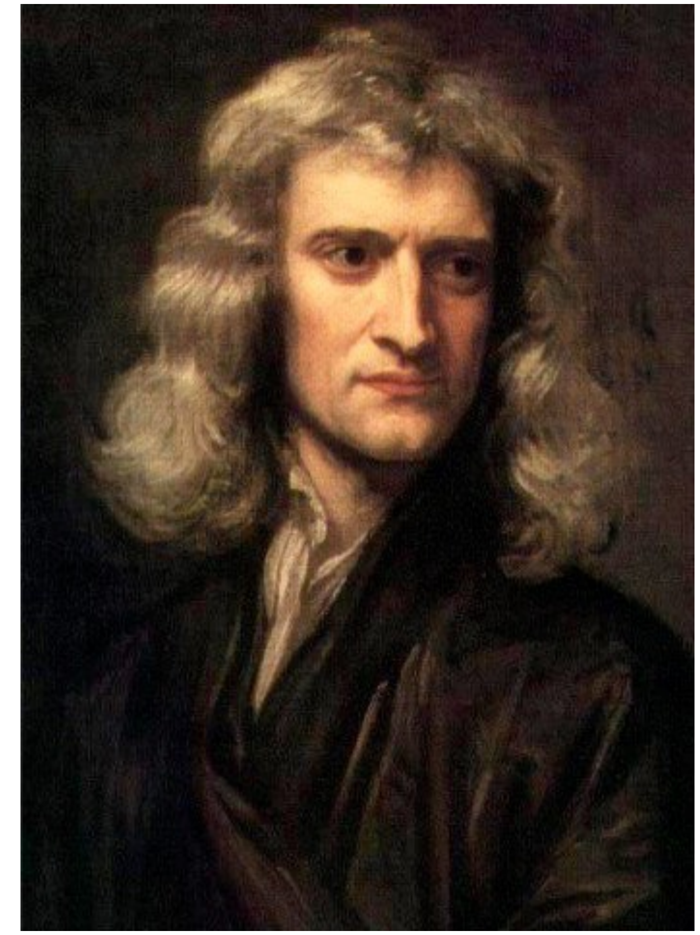

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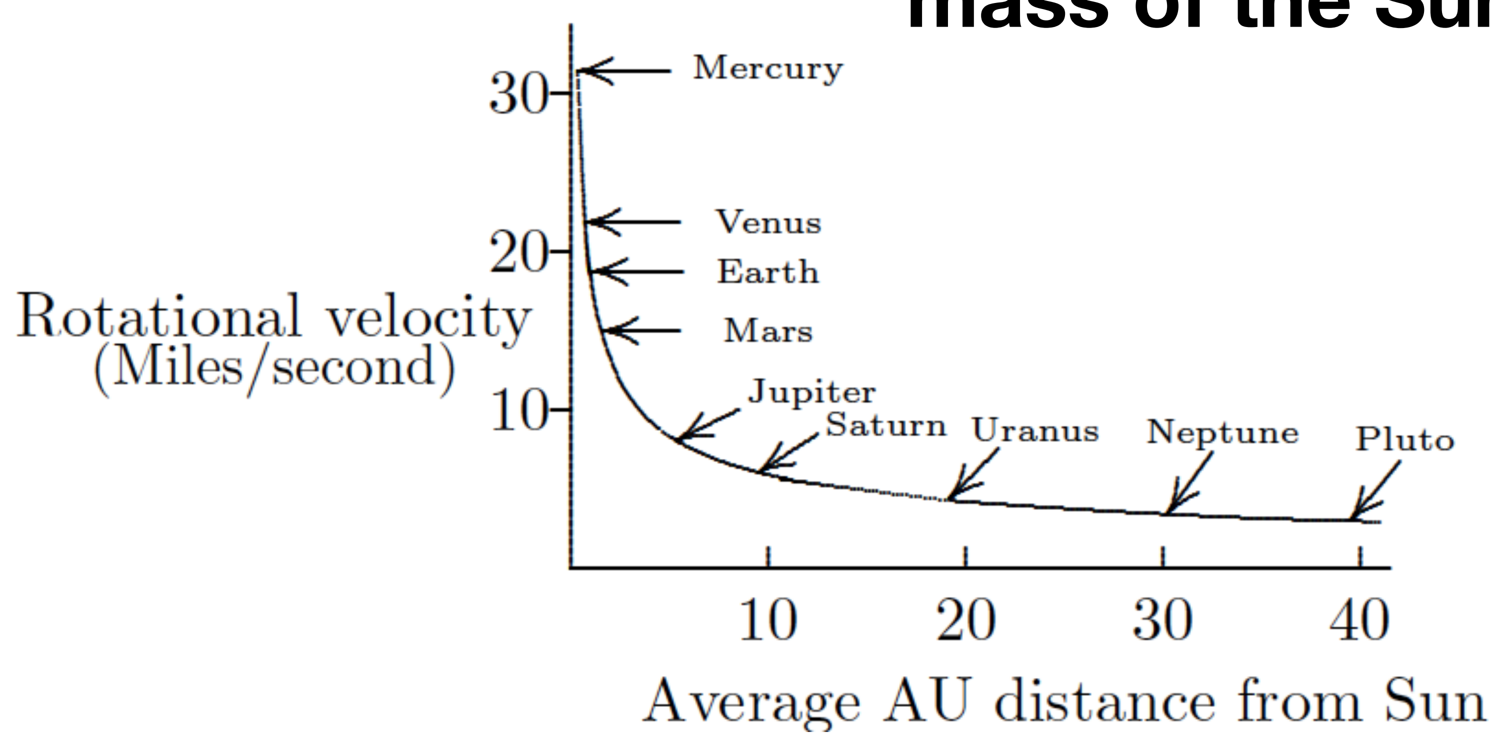

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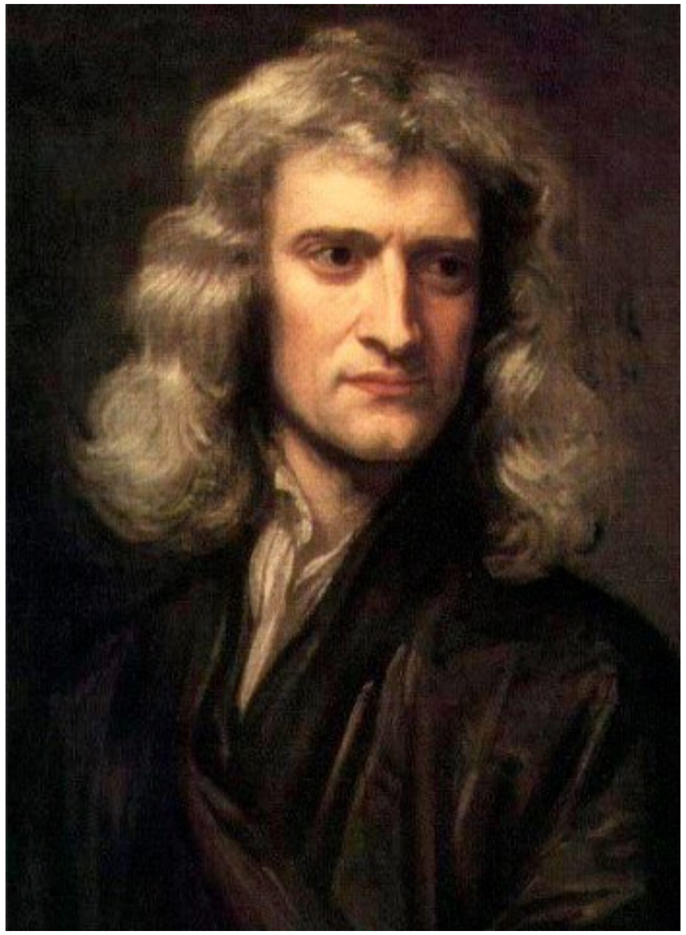

For circular orbits, Kepler's third law

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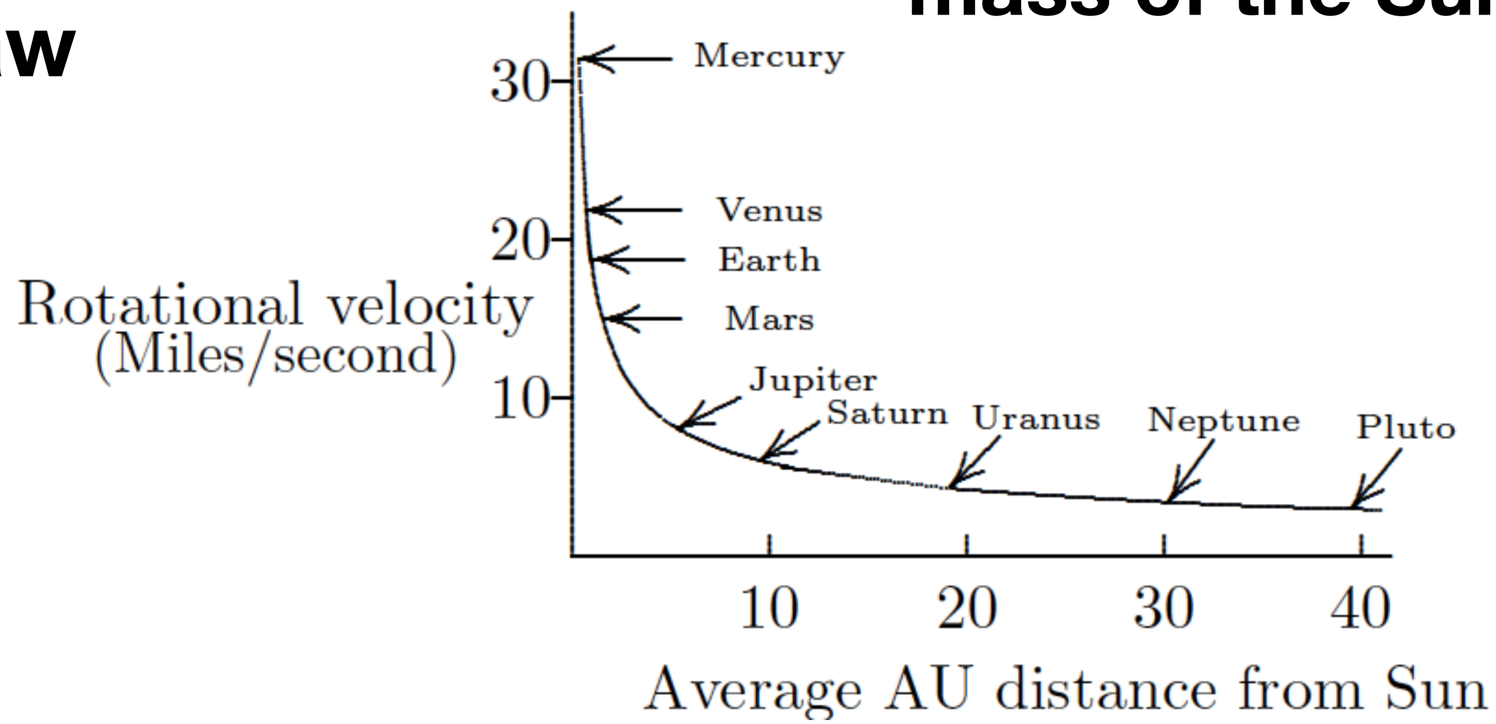

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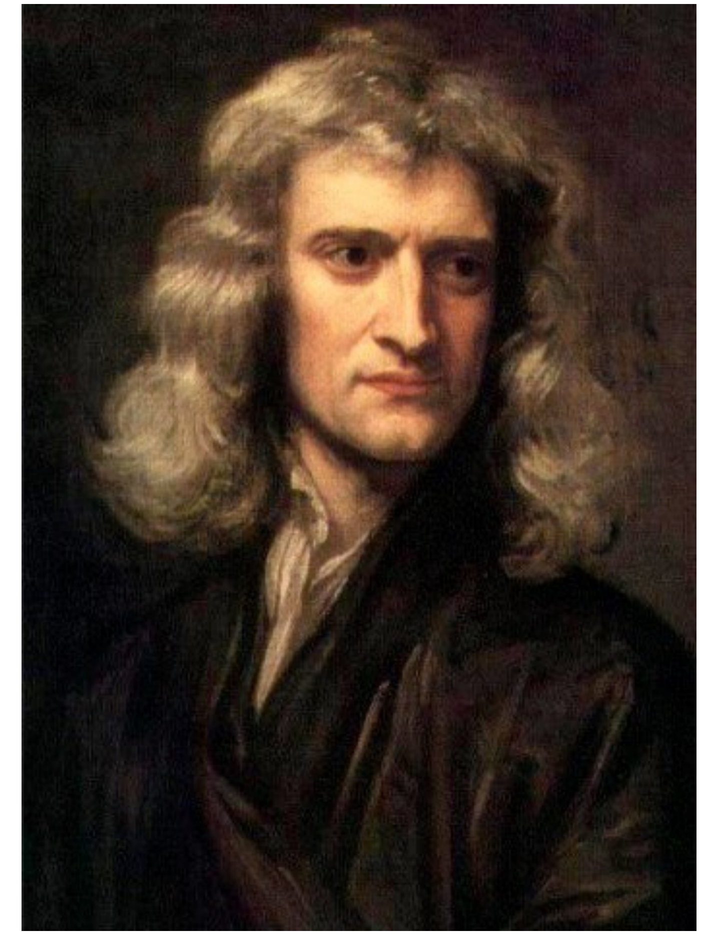

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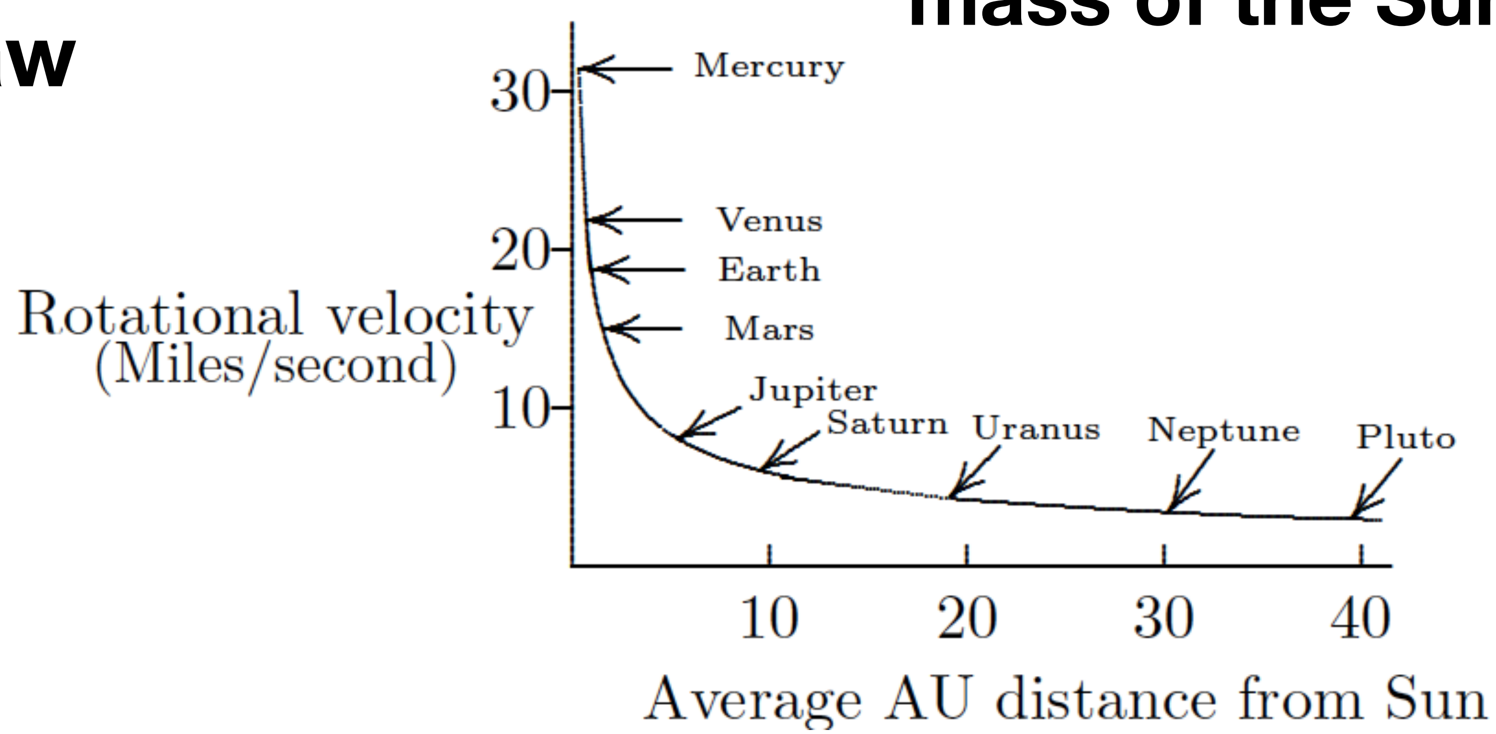

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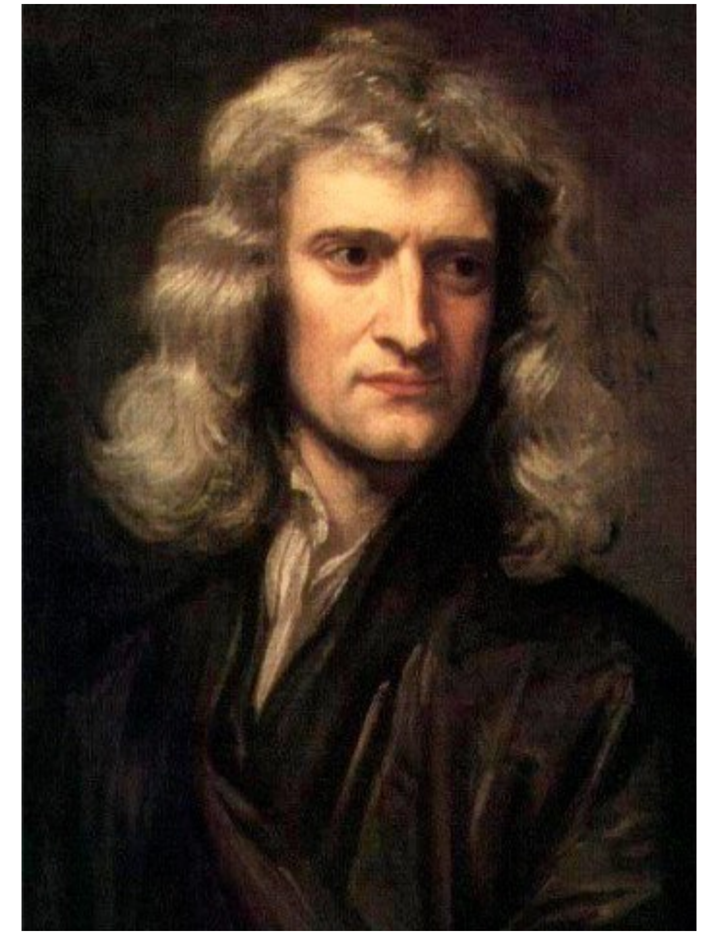

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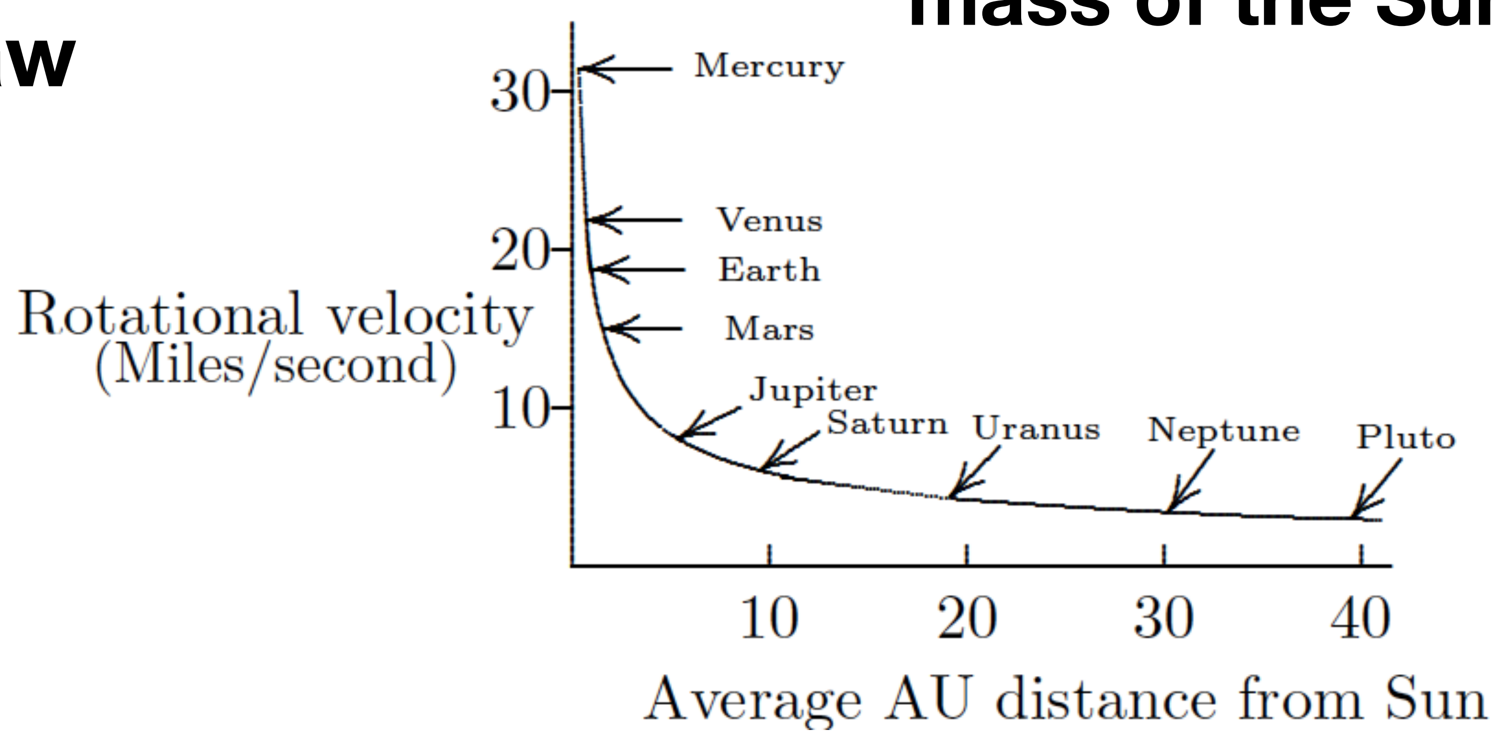

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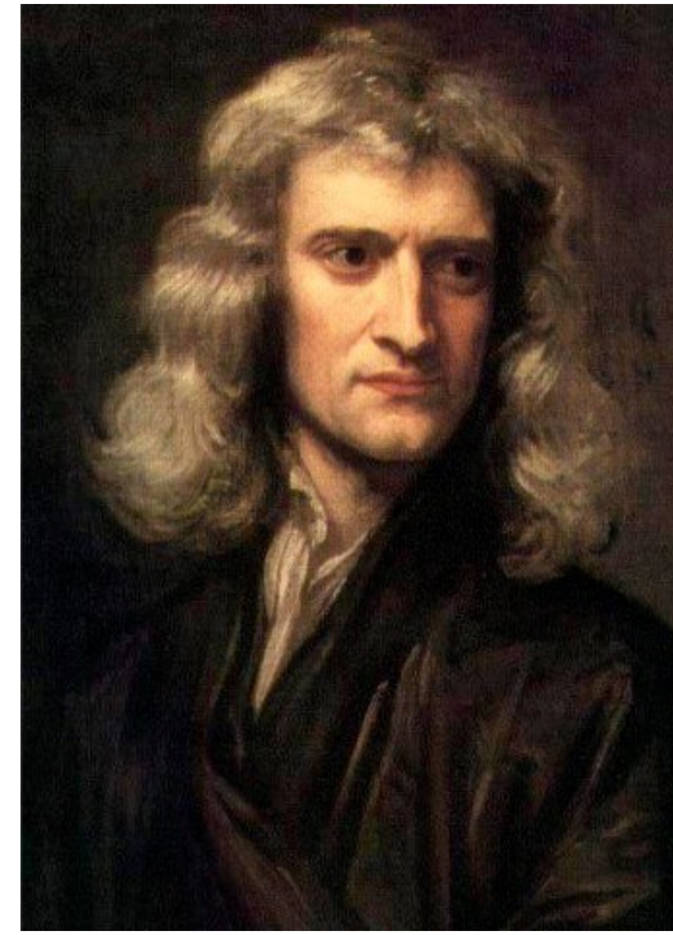
Smaller M_s dissipate!



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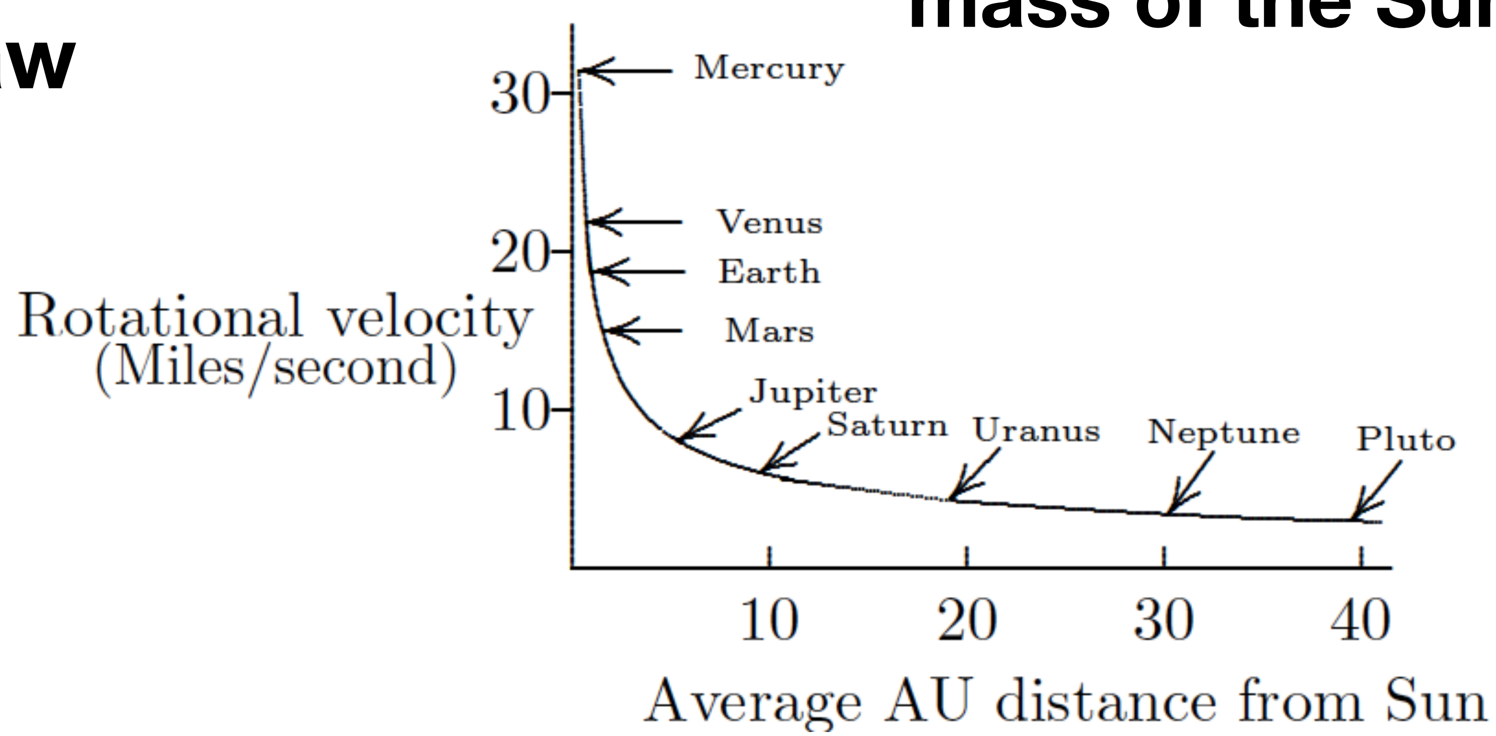

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Replace Newton’s laws with continuum approximation

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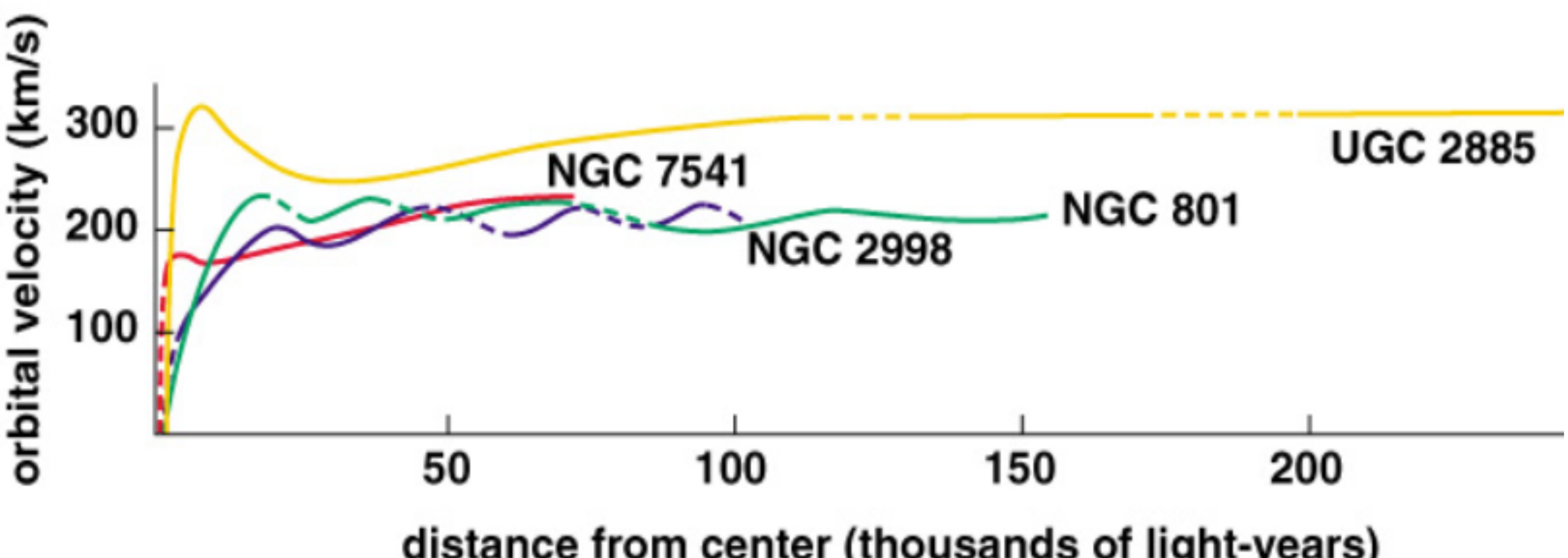

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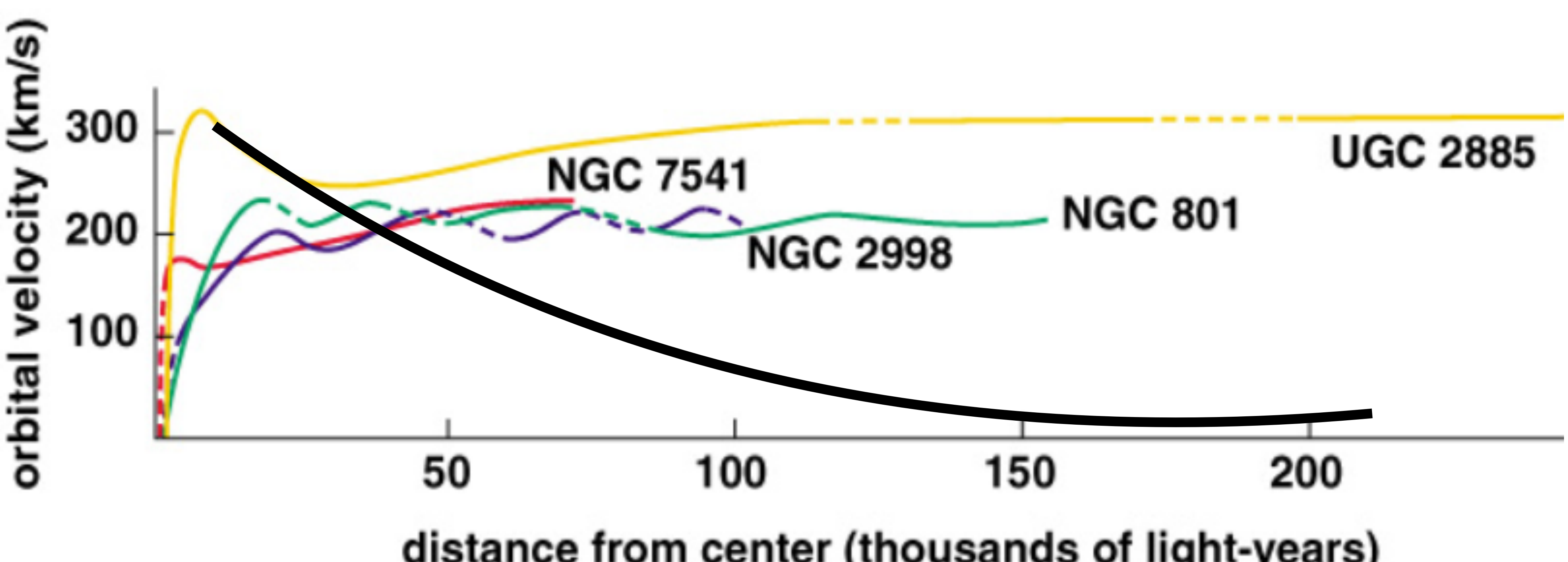

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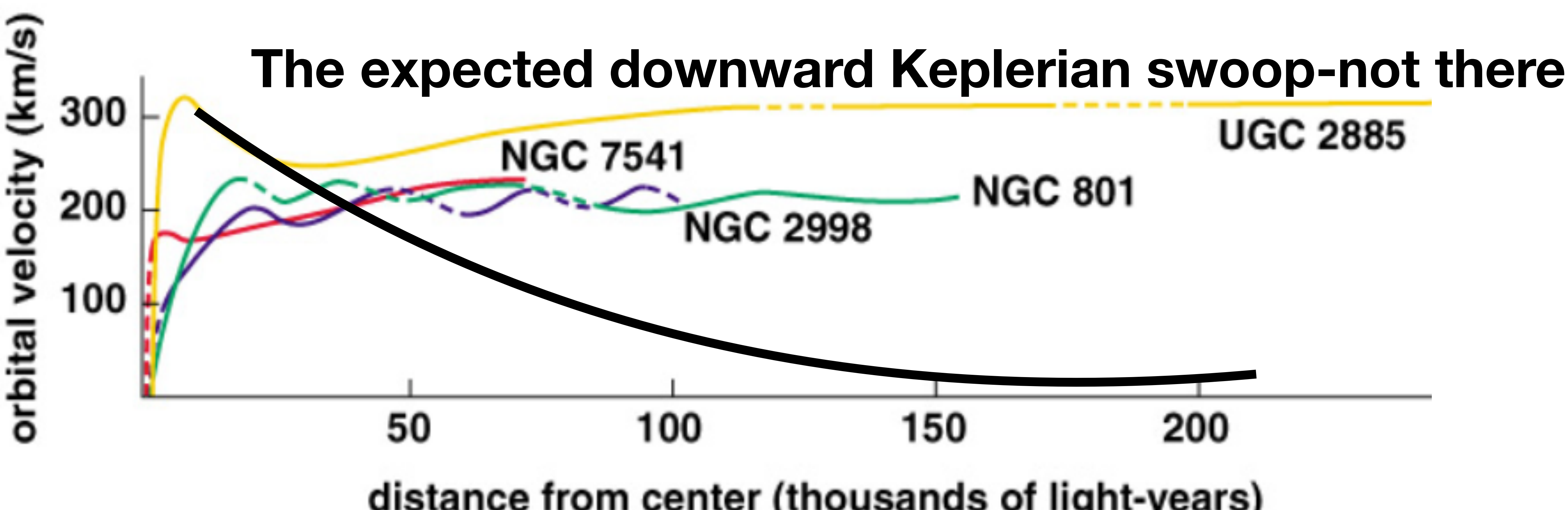

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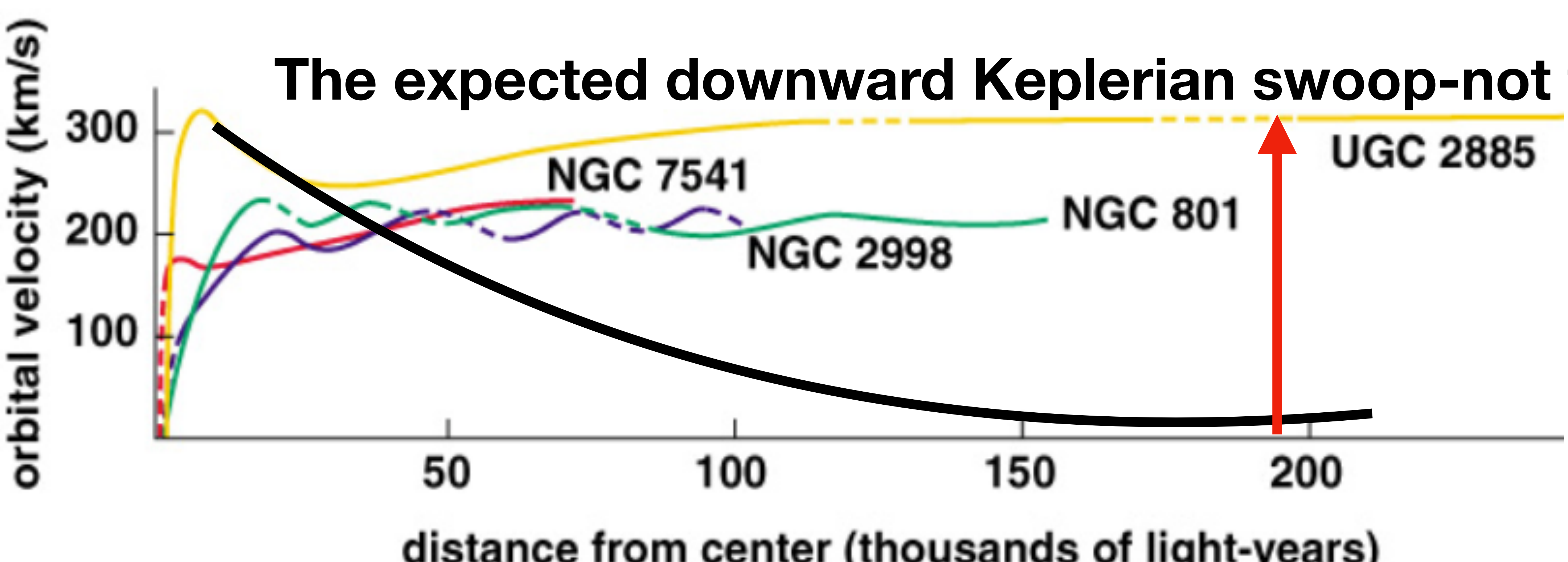

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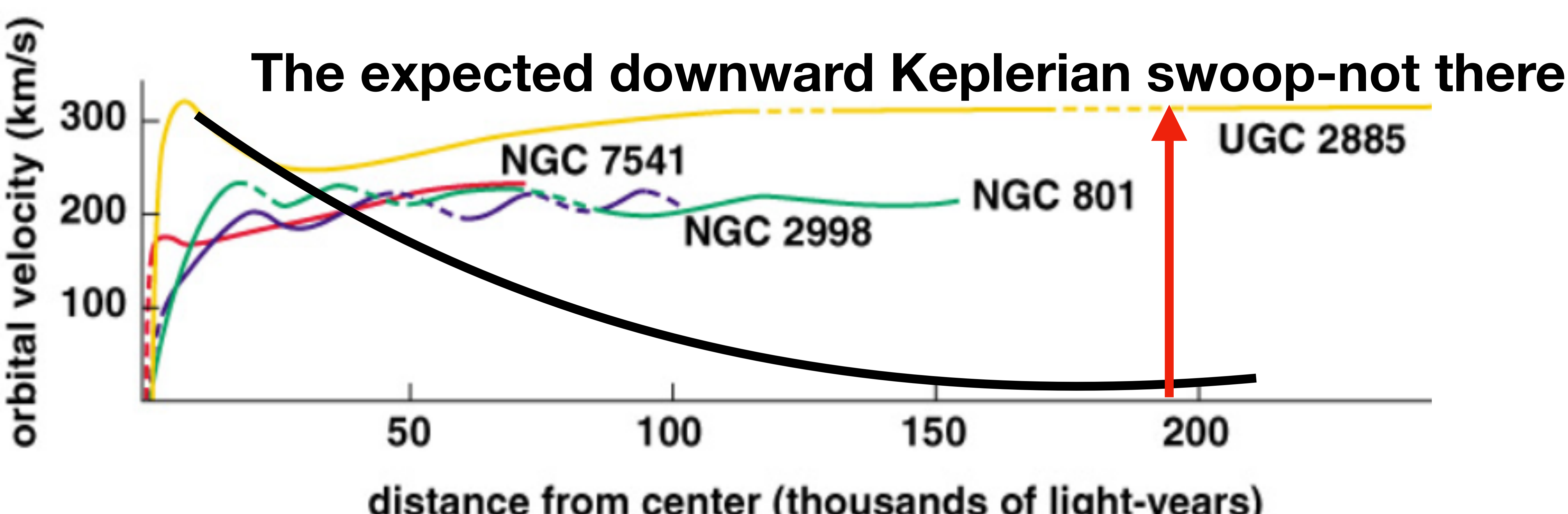

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Problem:

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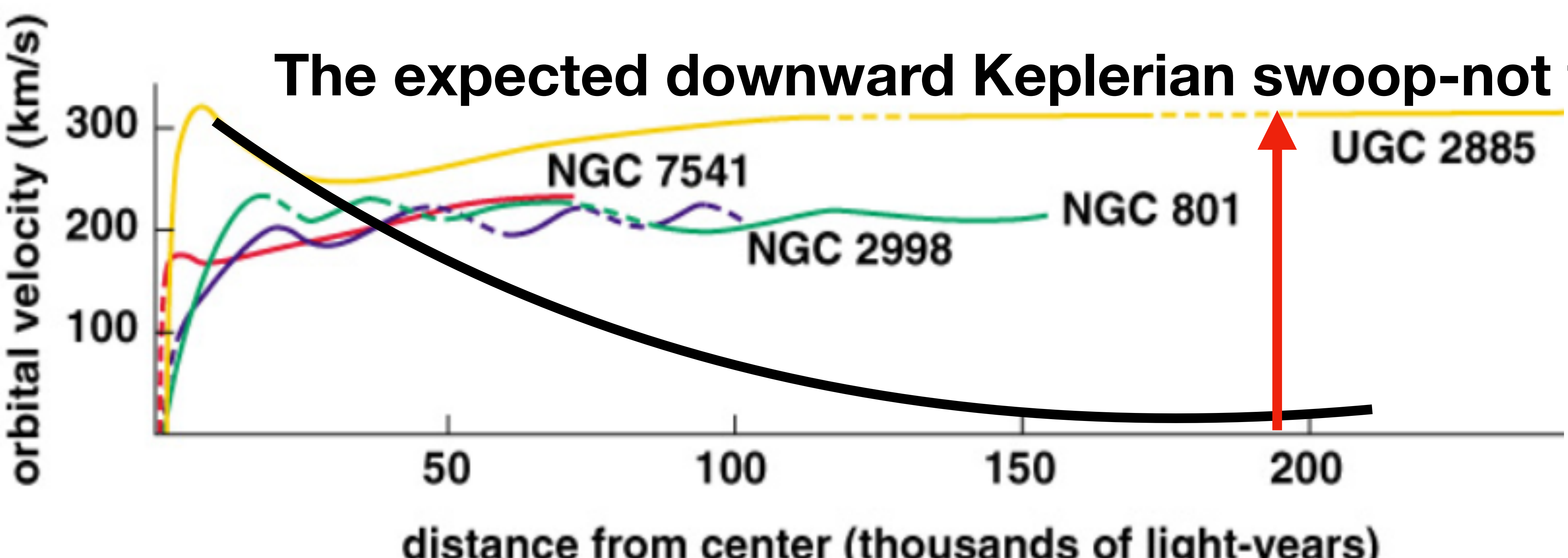
The expected downward Keplerian swoop-not there!



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Not enough known mass

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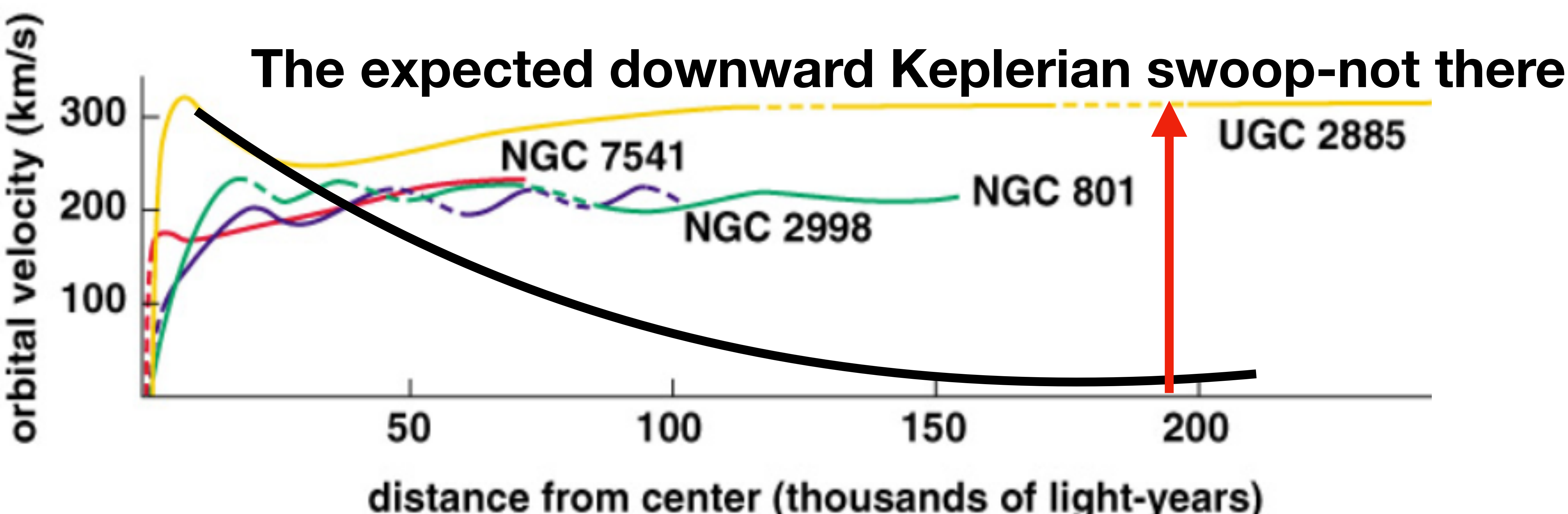

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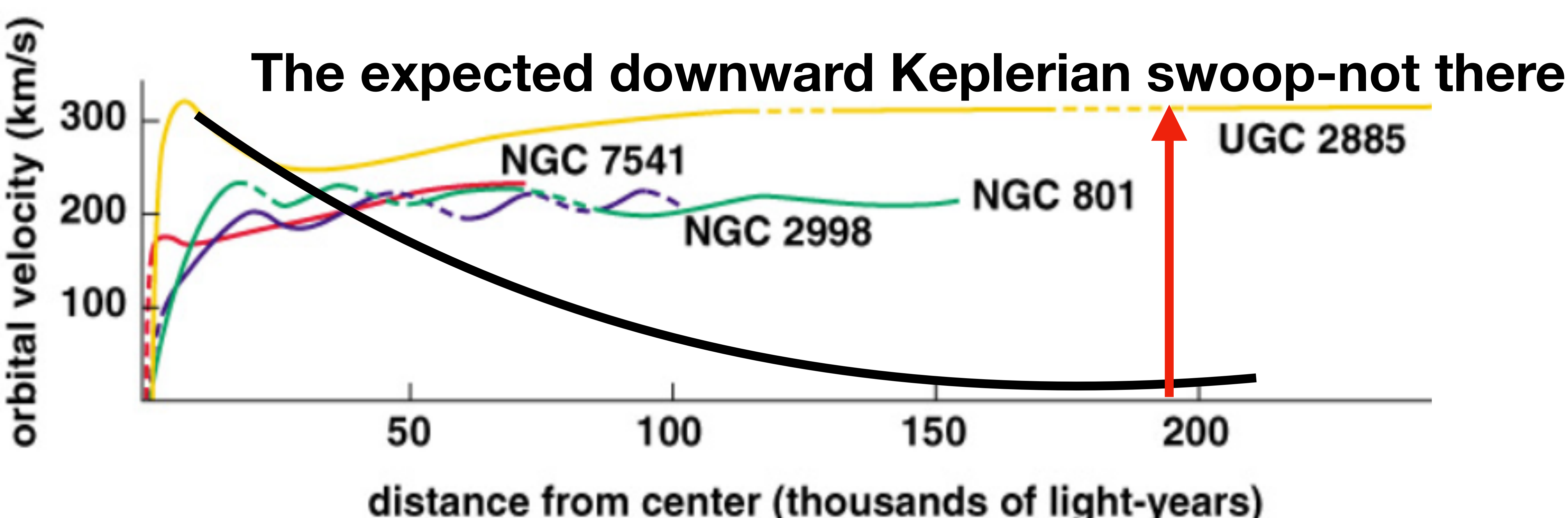

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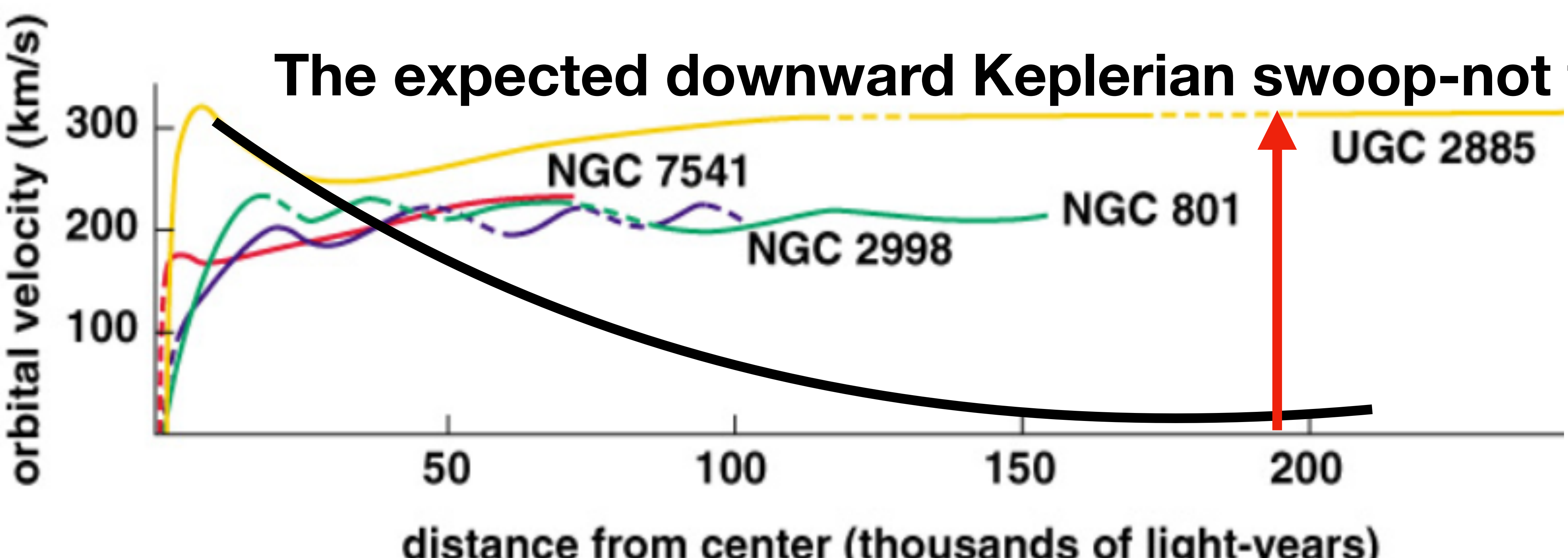

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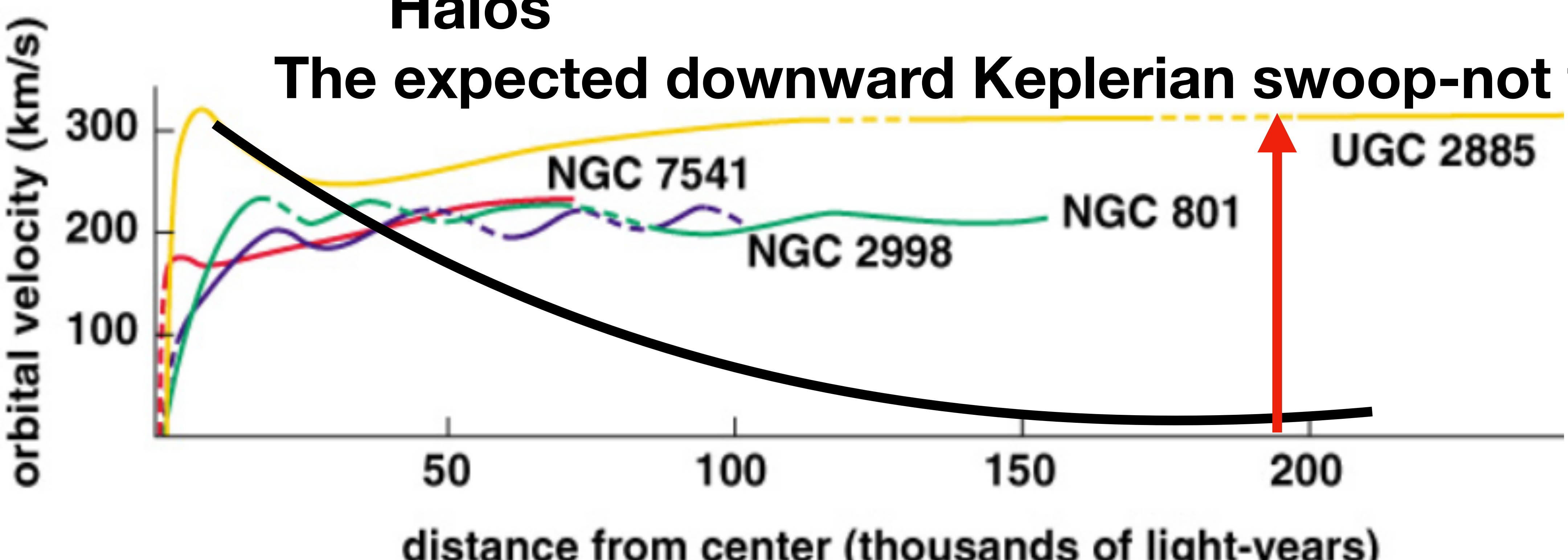

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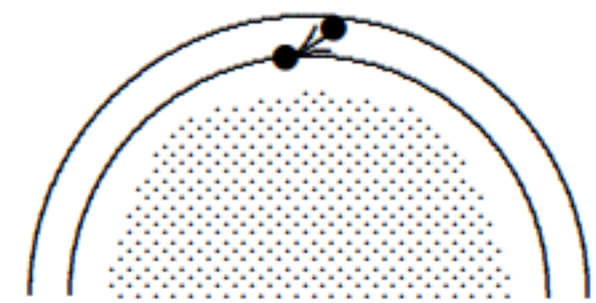

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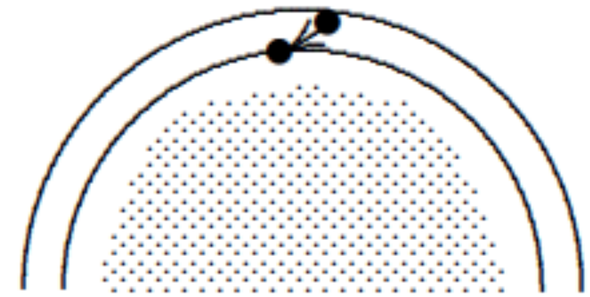

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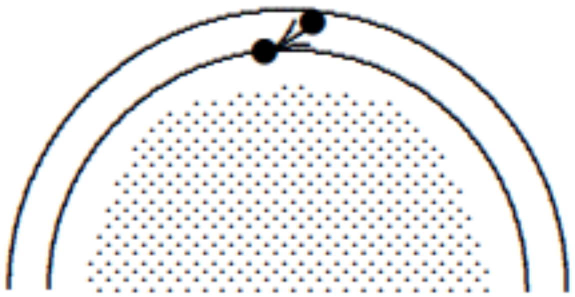

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Rather than a star being just in two-body interaction, both pictures indicate there also is a pulling effect!

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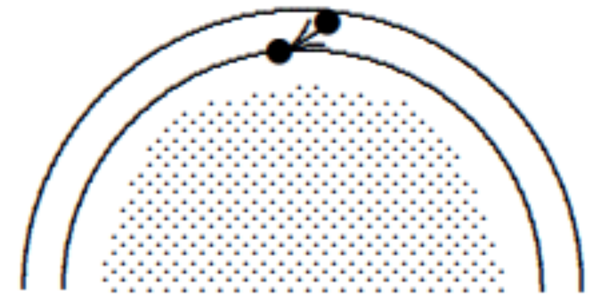

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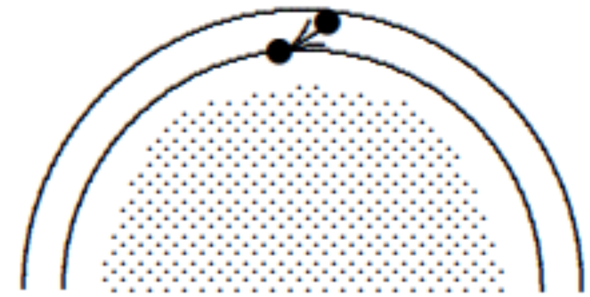

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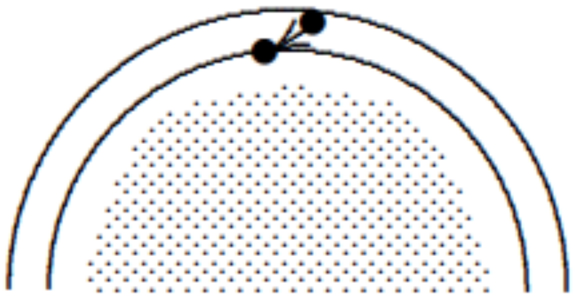

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Angular momentum
Finding M(r) must involve c

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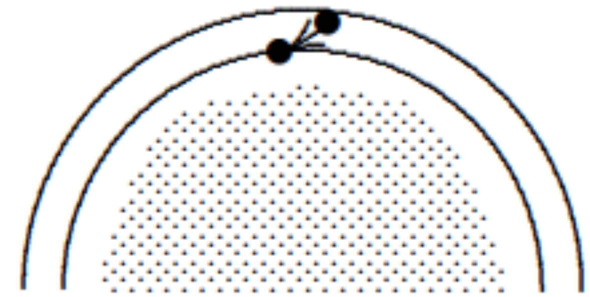

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A first step: create the analytic solution for a billion body problem and show that the M(r) expression does not hold.

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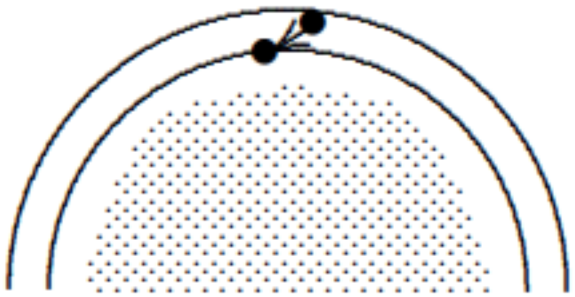

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A first step: create the analytic solution for a billion body problem and show that the M(r) expression does not hold.
Where is the tugging (and associated angular momentum) the strongest?

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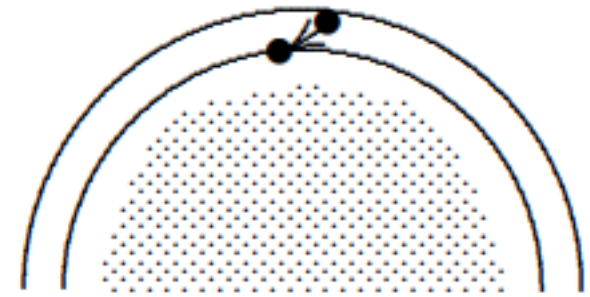
First, notice that the two-body and n-body representation agree

$$M_s = \frac{r\,v_{rot}^2}{G} \qquad M(r) = \frac{r v_{rot}^2}{G}$$



JPL

Rather than a star being just in two-body interaction, both pictures indicate there also is a pulling effect!
How do we measure pulling effect in celestial mechanics?
Angular momentum
Finding M(r) must involve c
A first step: create the analytic solution for a billion body problem and show that the M(r) expression does not hold.
Where is the tugging (and associated angular momentum) the strongest?
Central configurations!

Central configurations

Central configurations

$$\mathbf{r}_j'' = \lambda_j \mathbf{r}_j$$

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All λ_j are equal

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Euler, 1767

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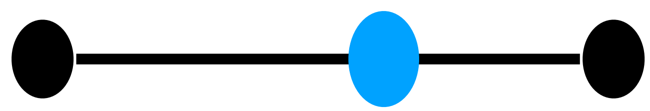

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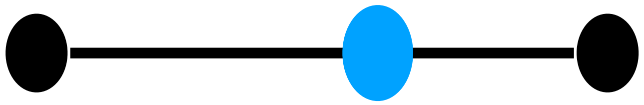

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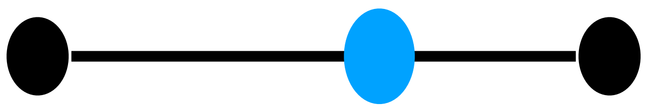

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With proper choices of velocities, two-body type motion occurs keeping fixed the configuration

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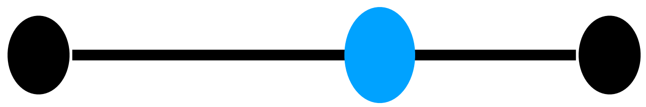

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Earth, moon, space prob

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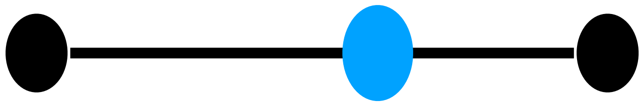

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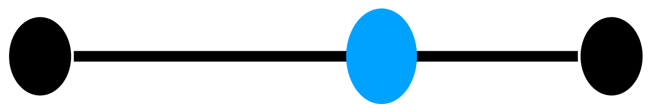

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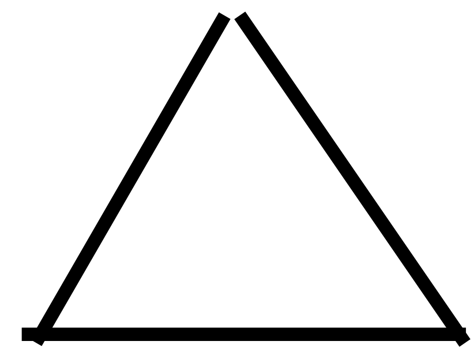

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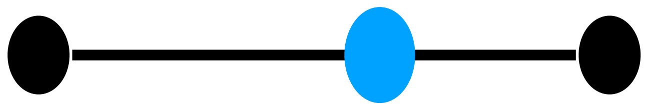

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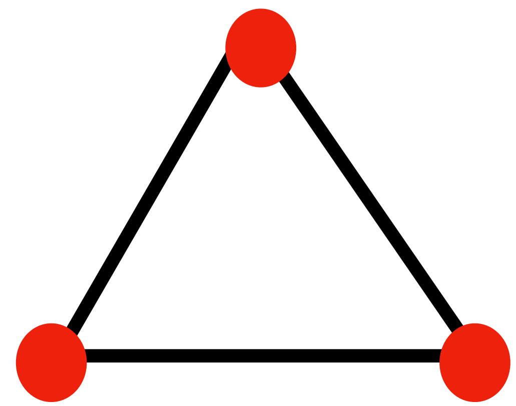

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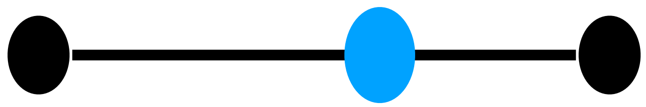

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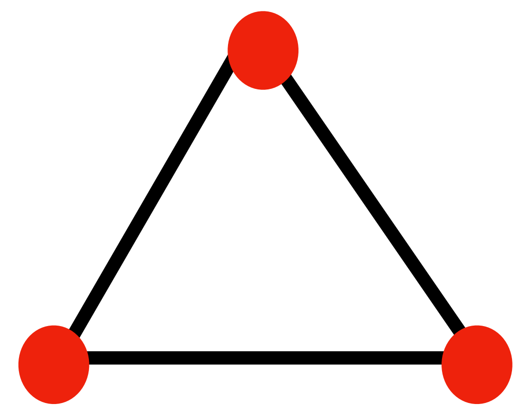

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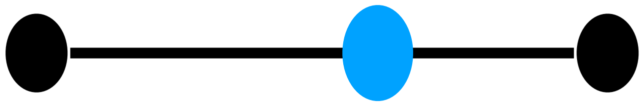

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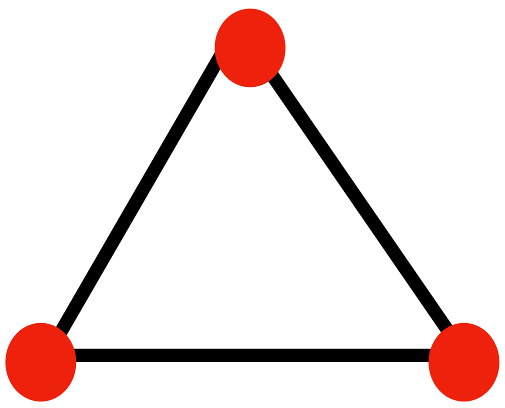
Sun, Jupiter, Trojan asteroids



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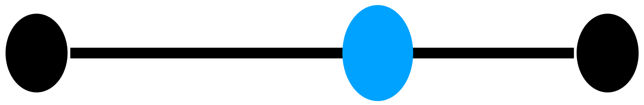

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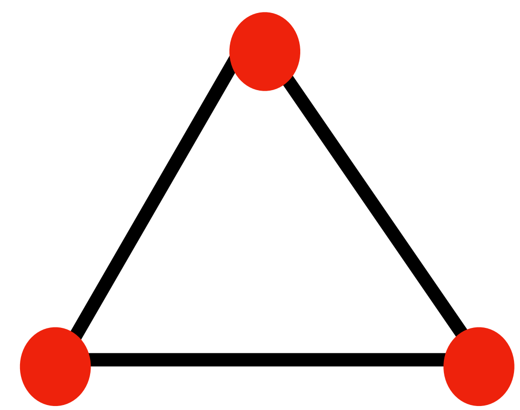

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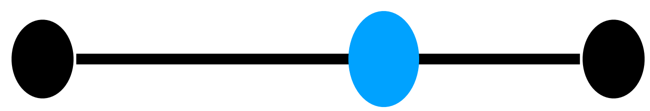

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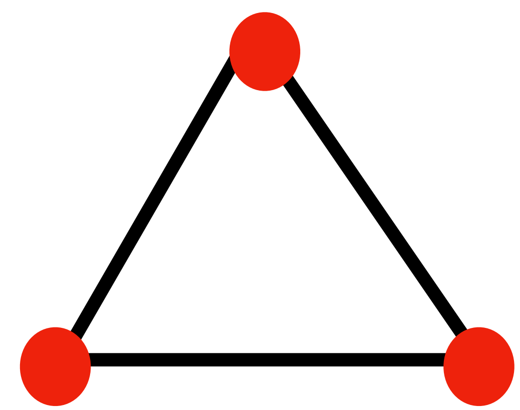
Earth, moon, space prob

Problem: For any n and mass choices, are there a finite number of central configurations?



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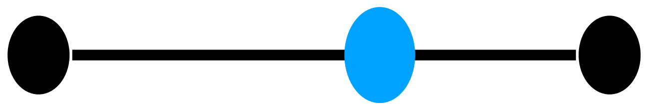

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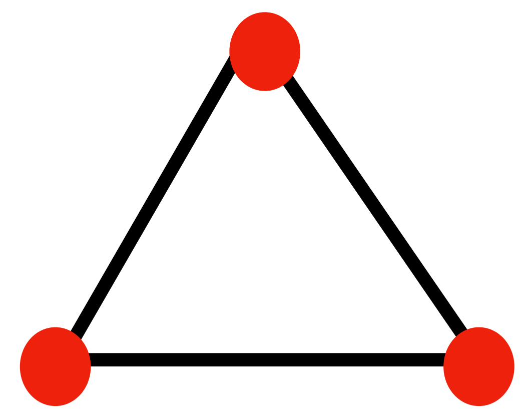

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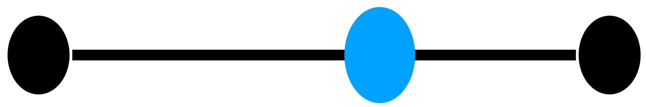

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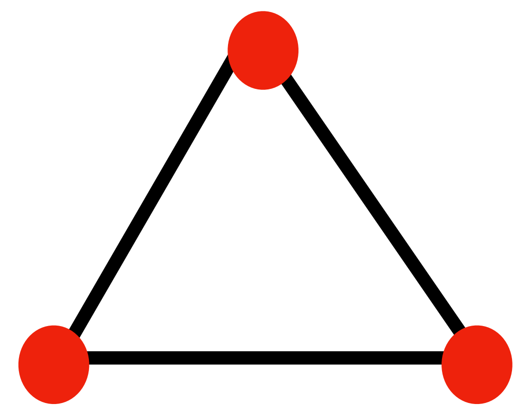

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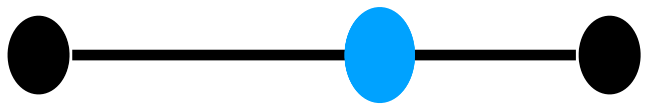

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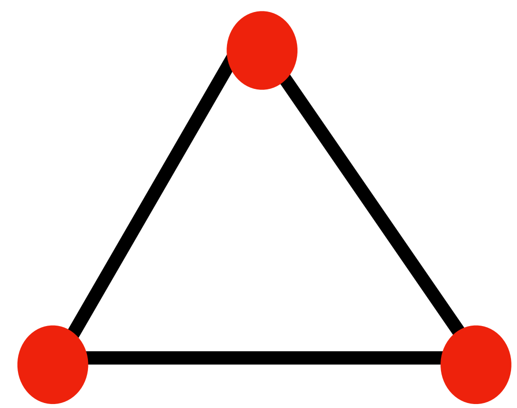

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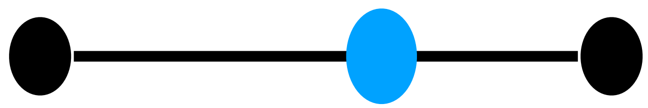

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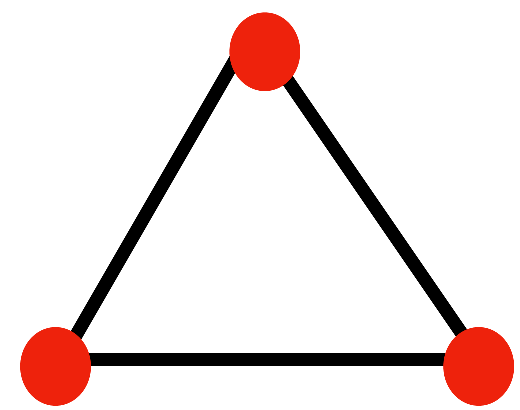

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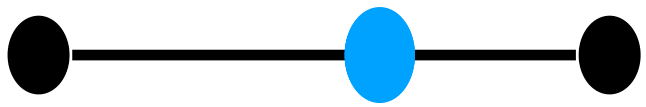

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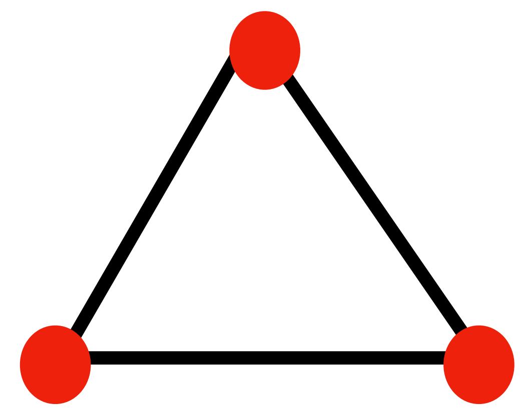

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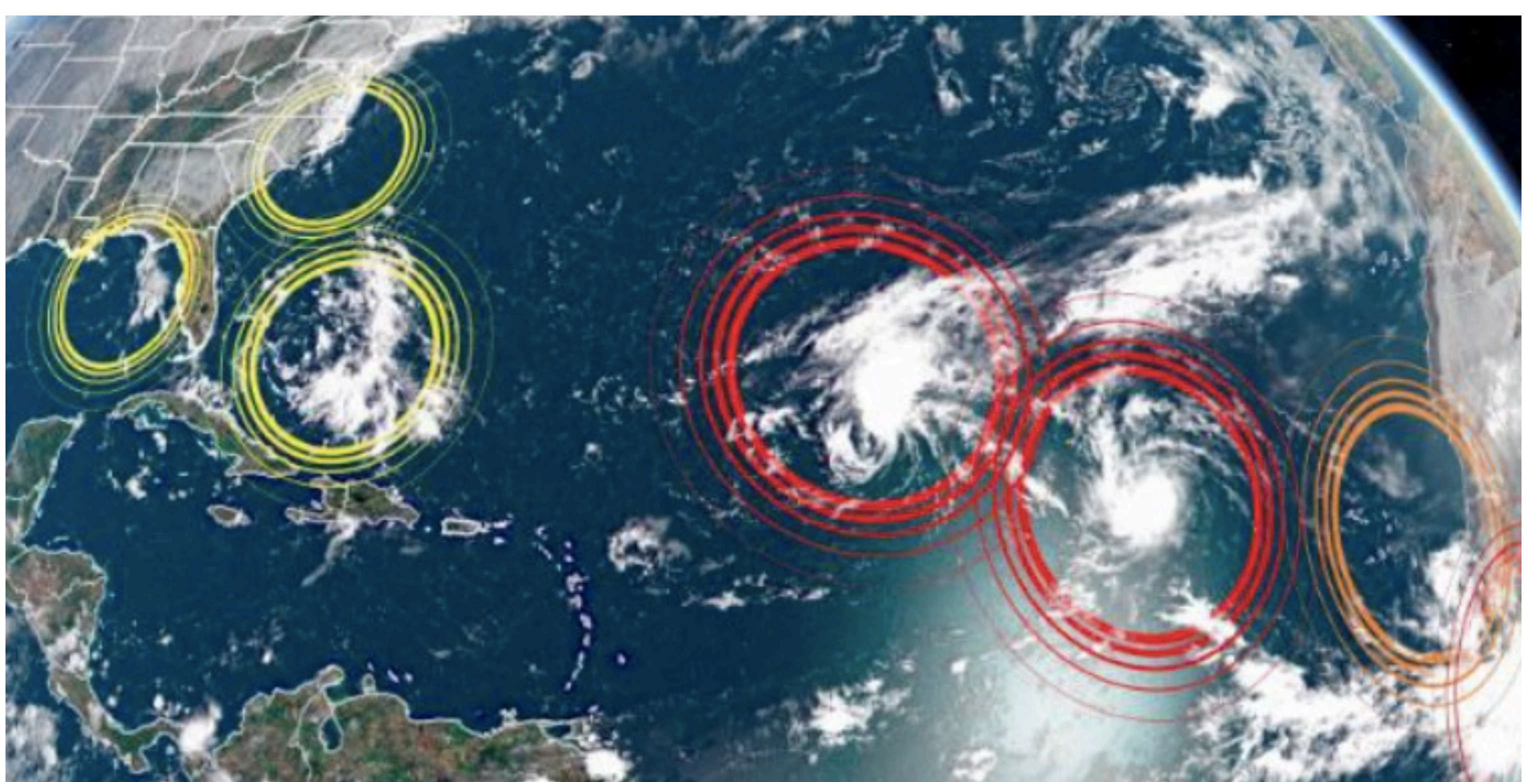

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A billion, or so, body central configuration

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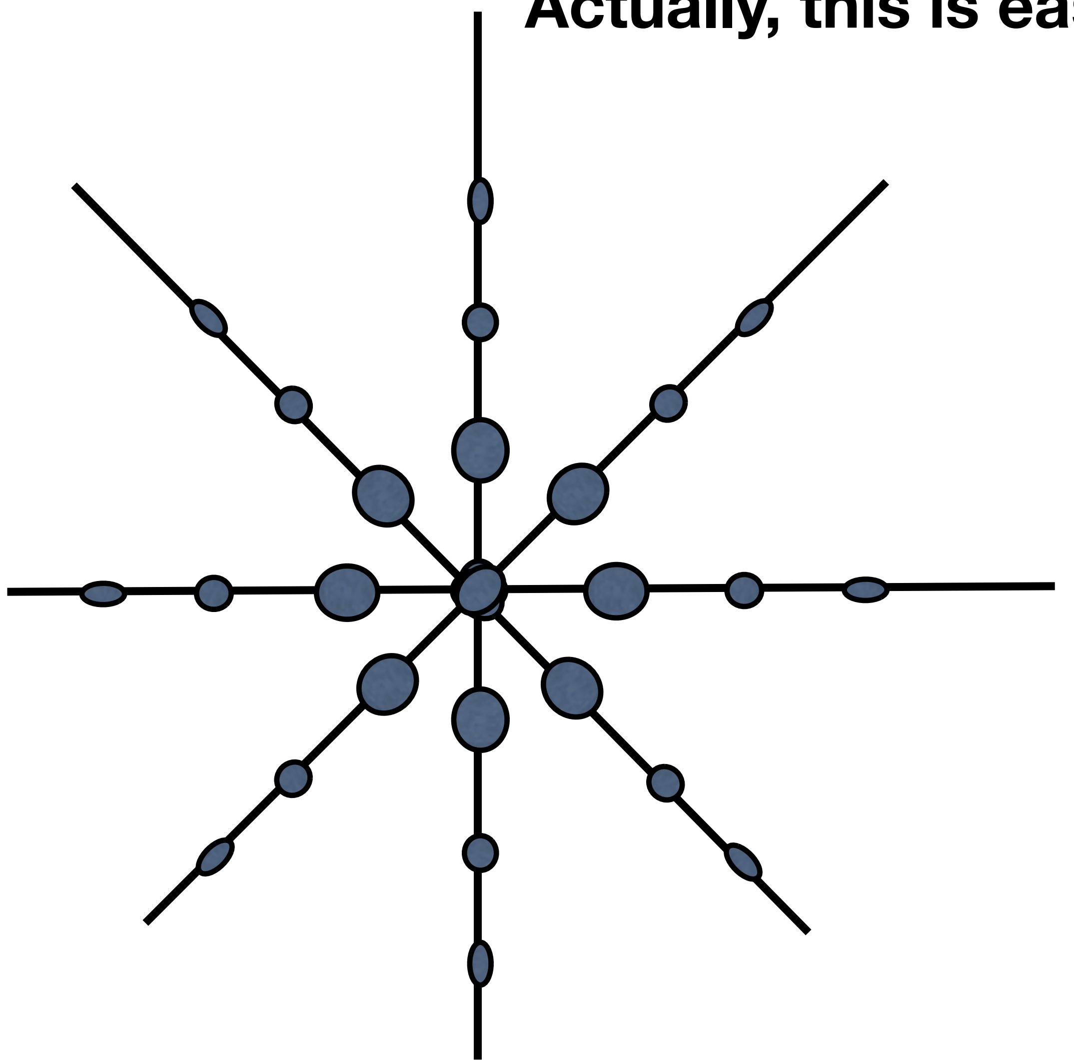

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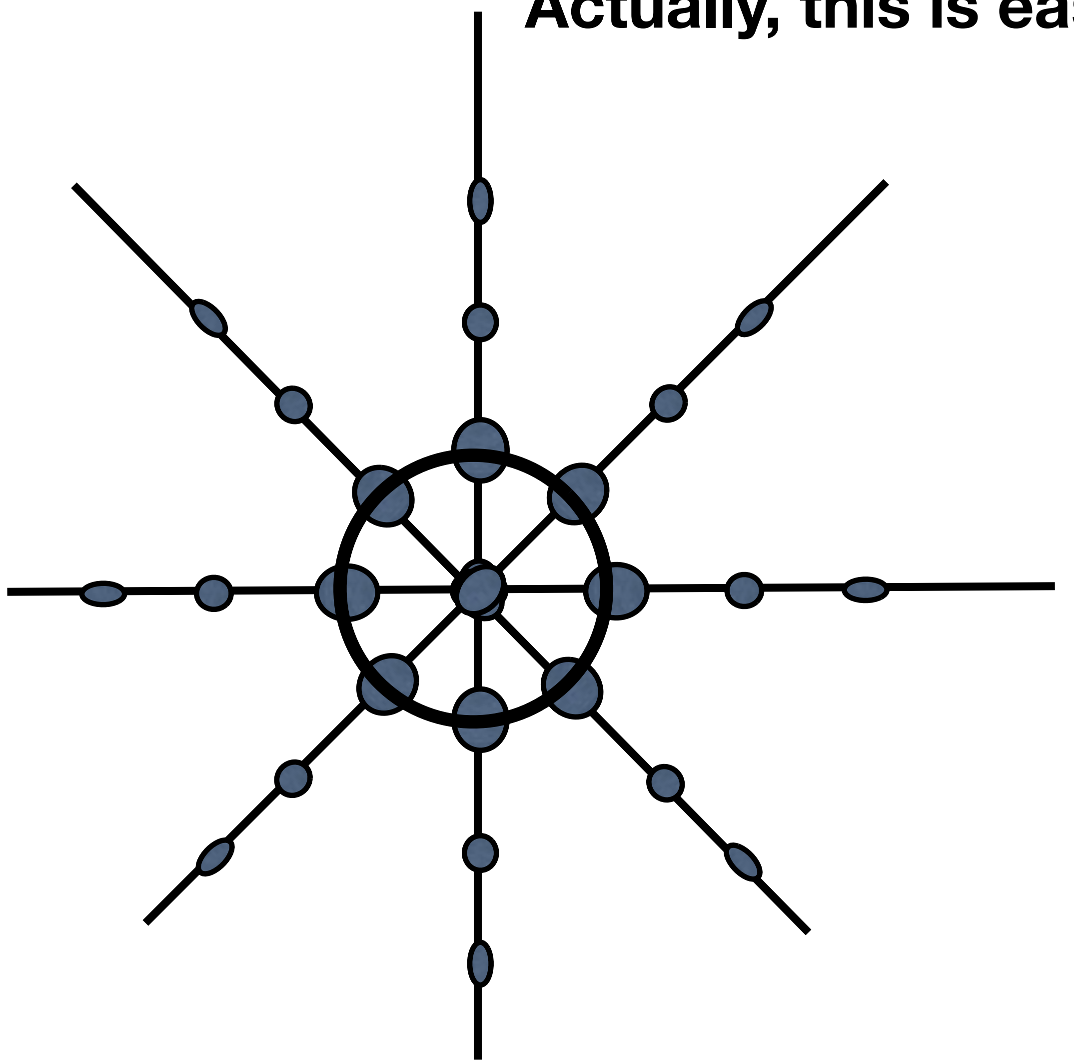

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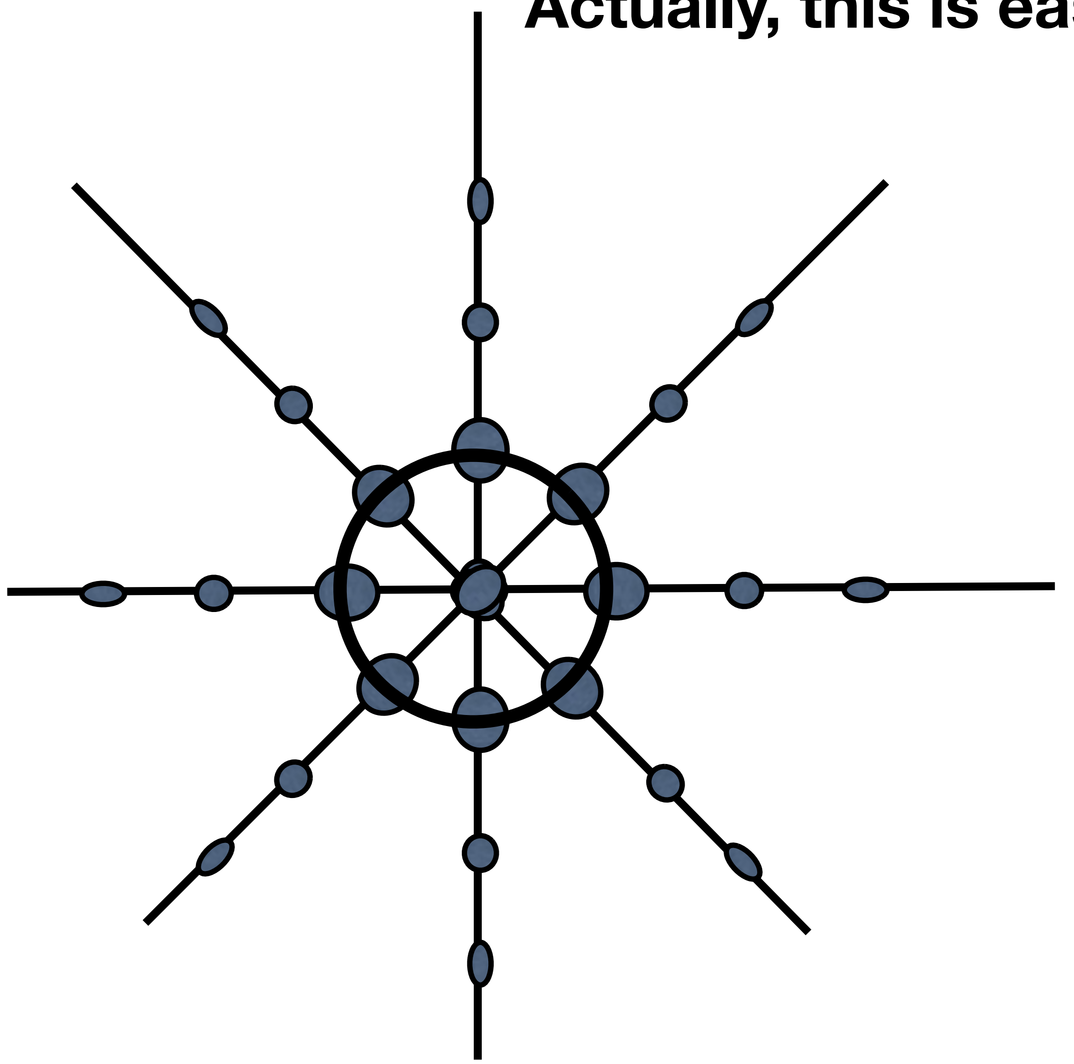

This is Maxwell's central configuration for rings of Saturn

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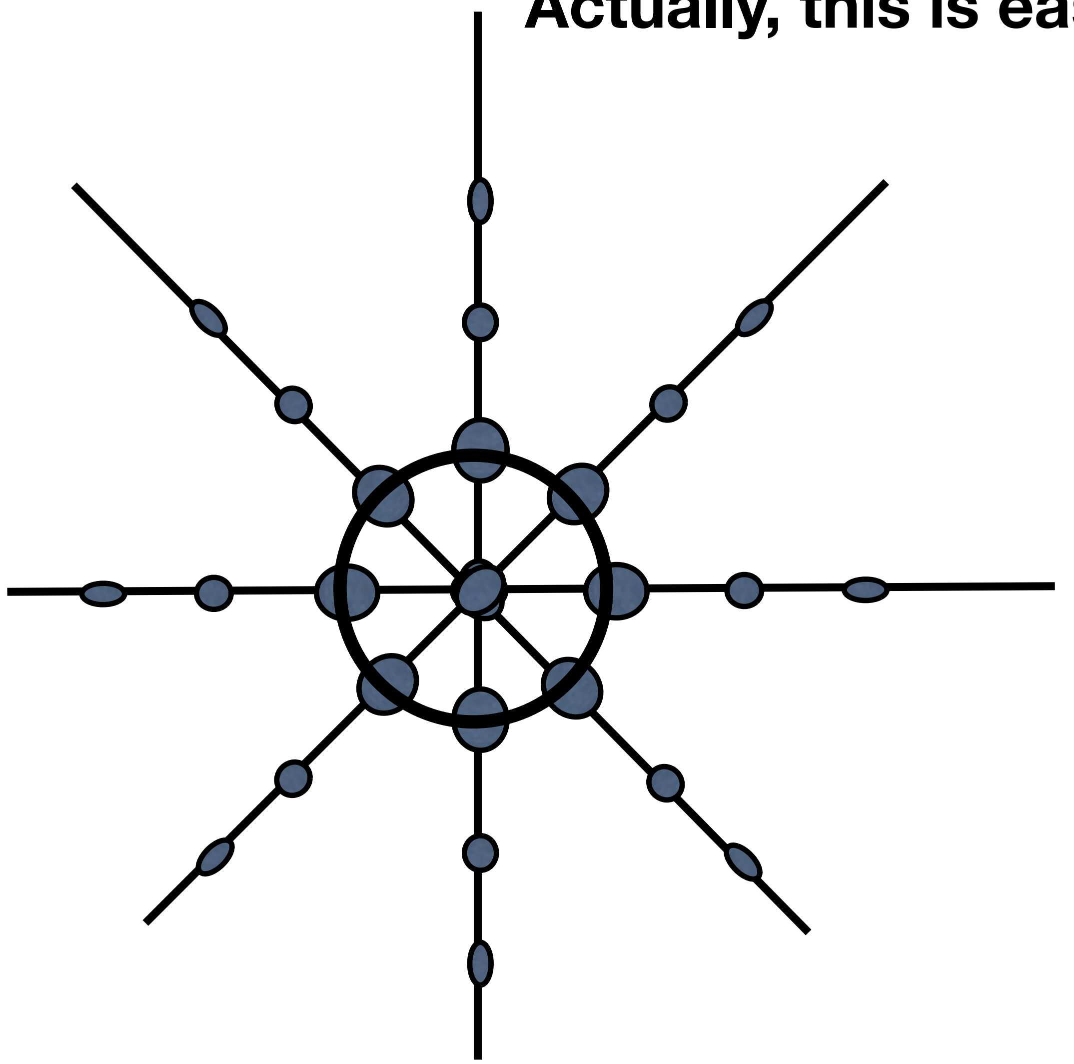

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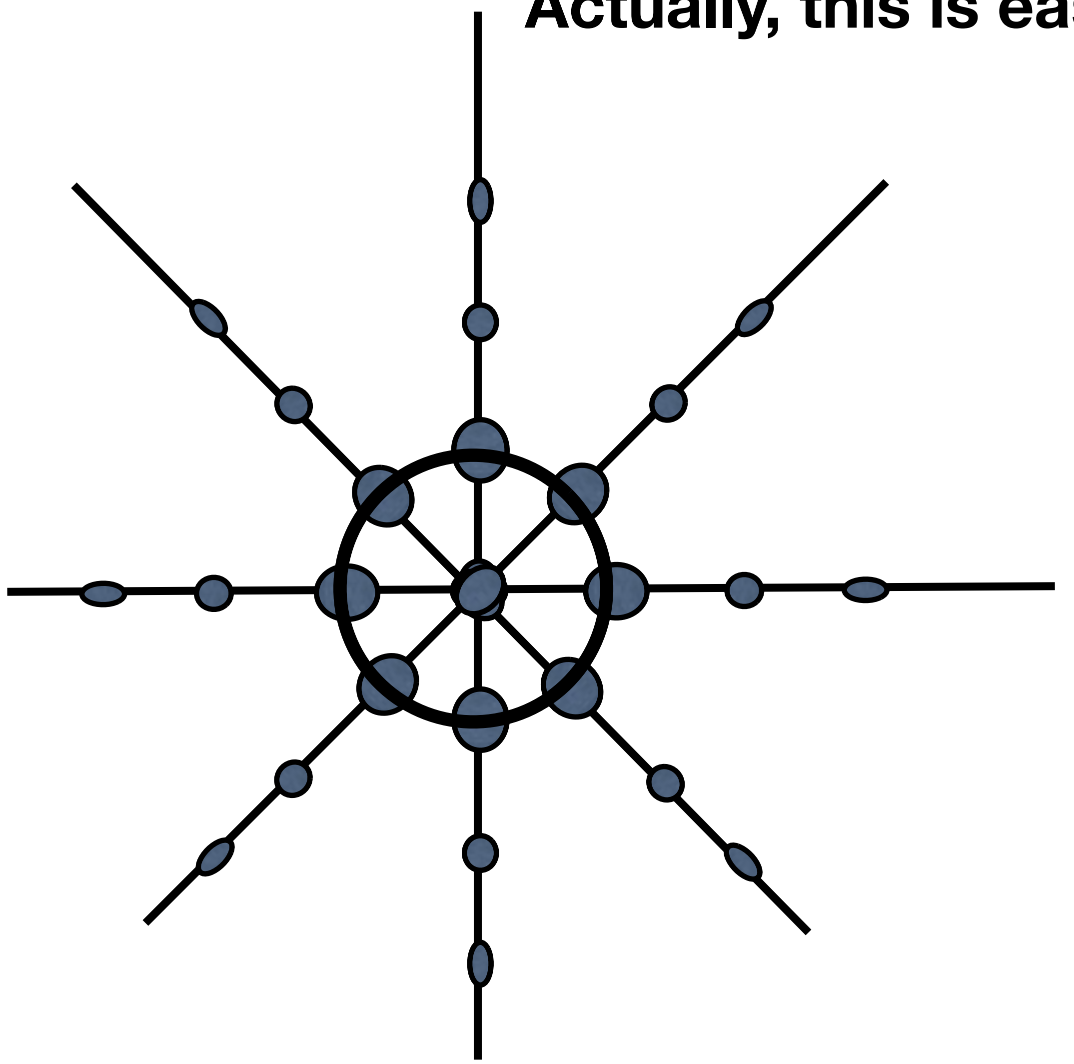

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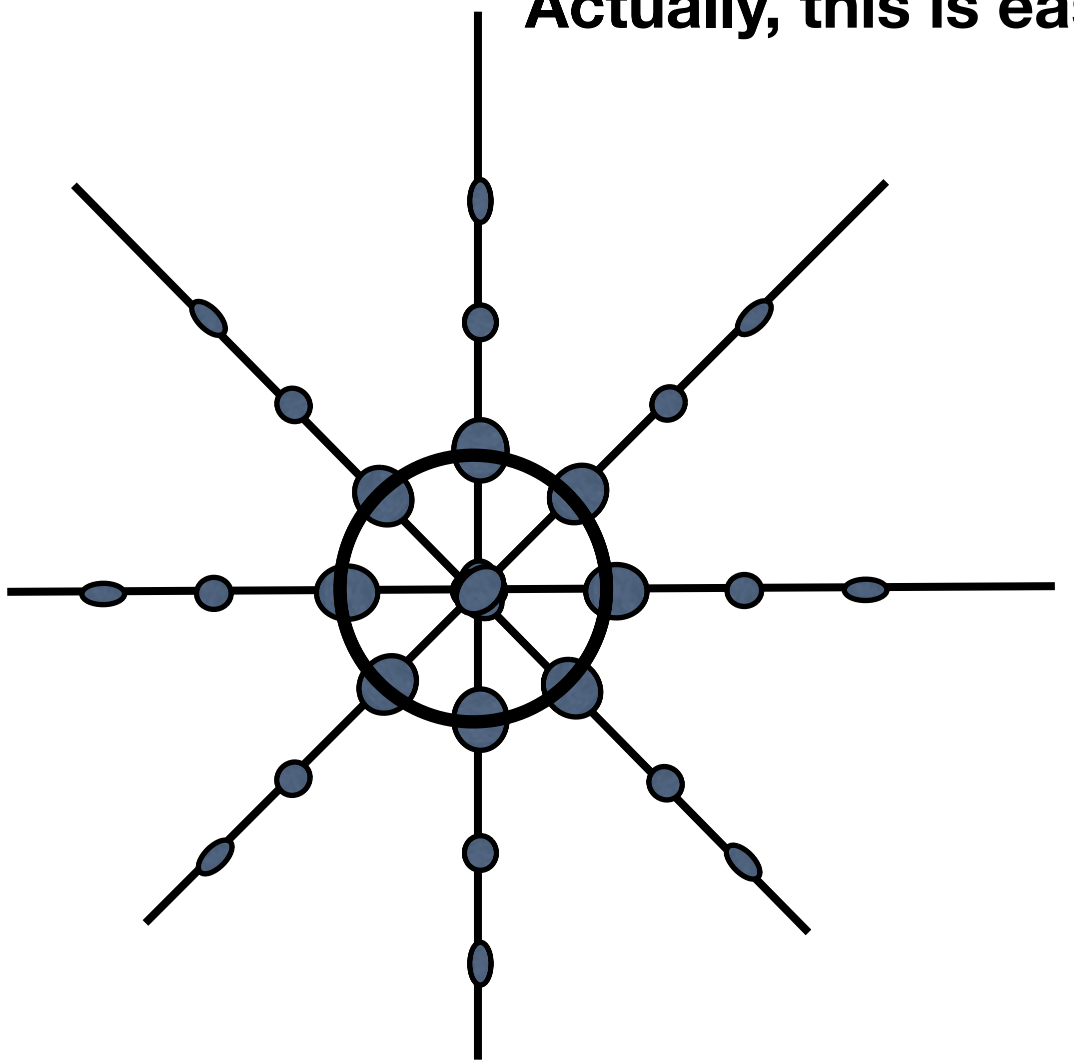

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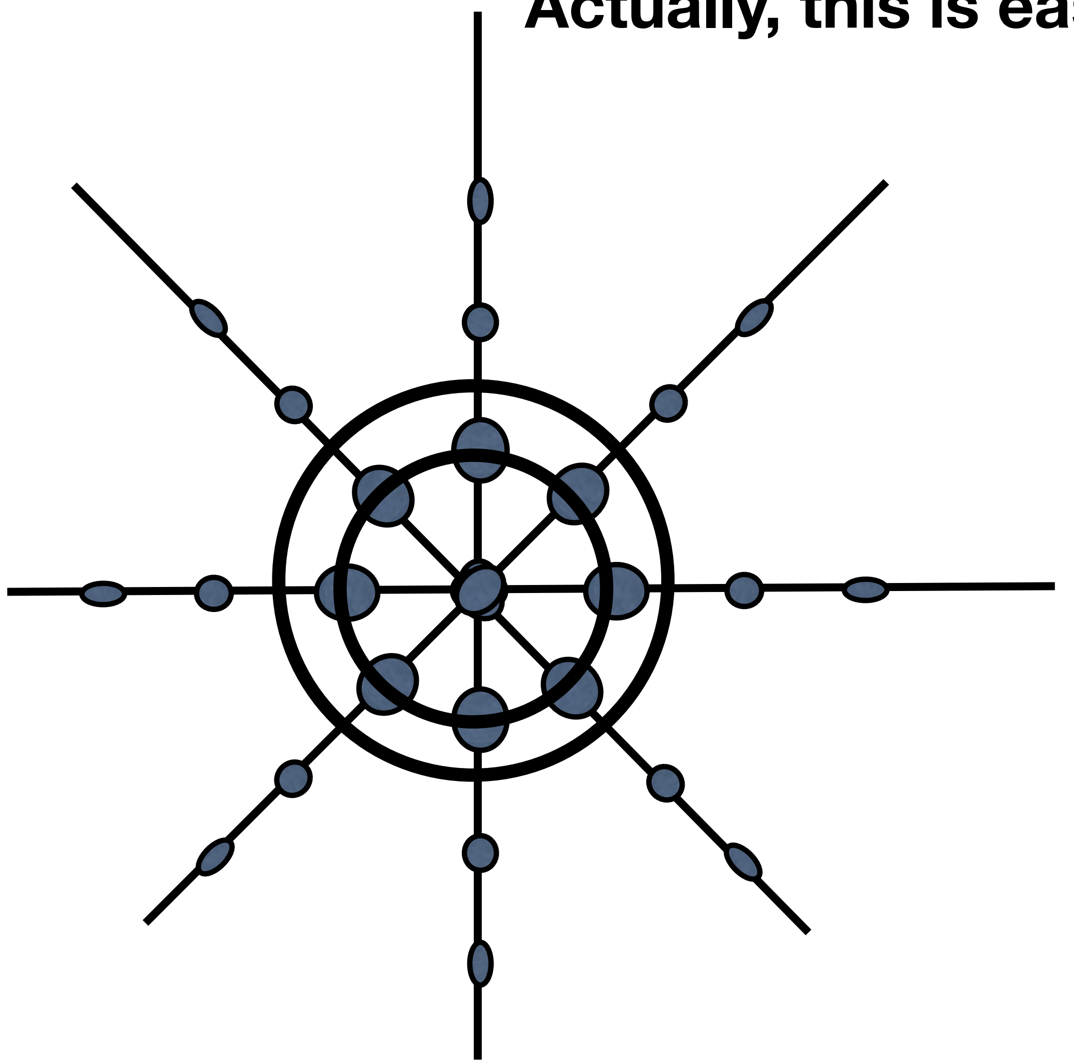

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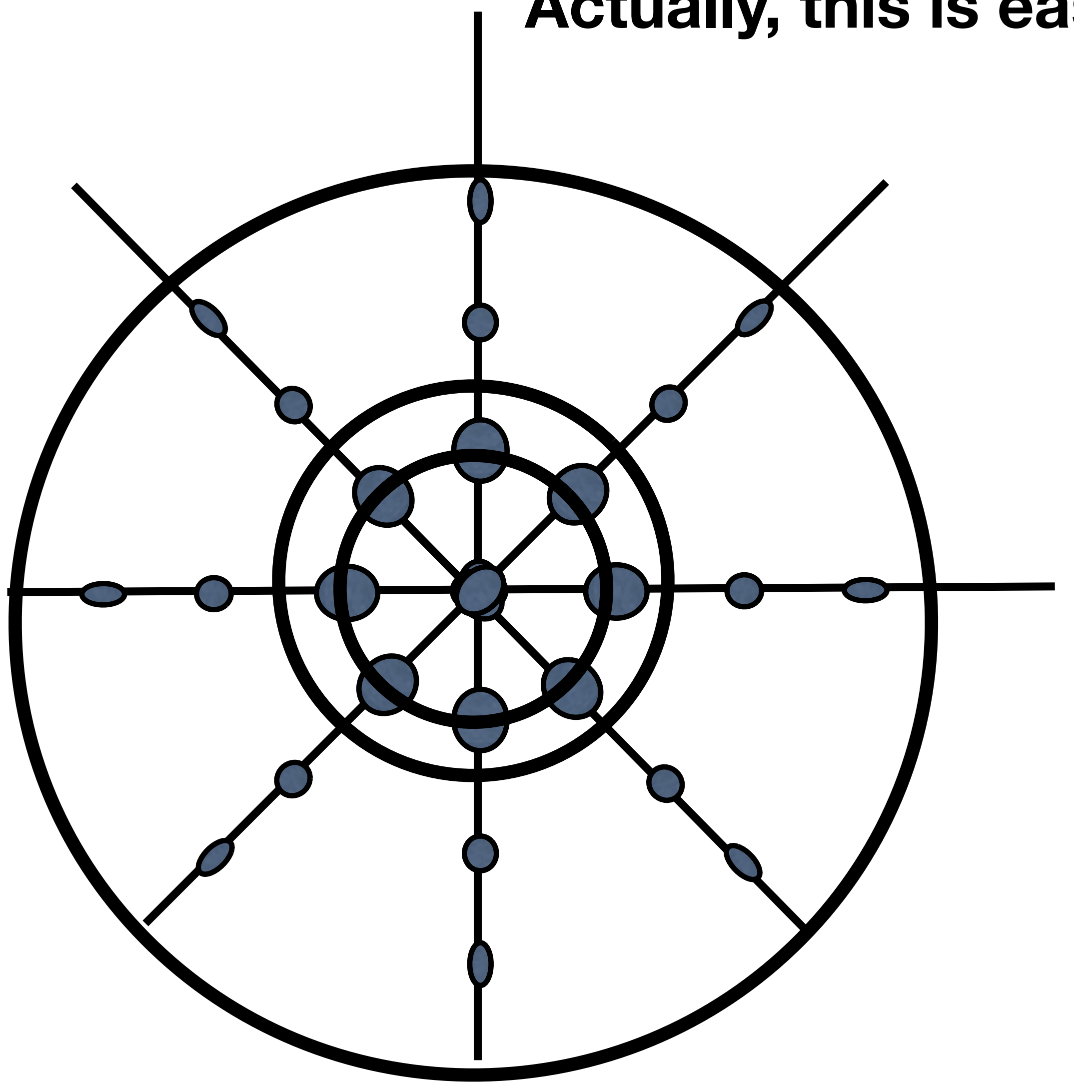

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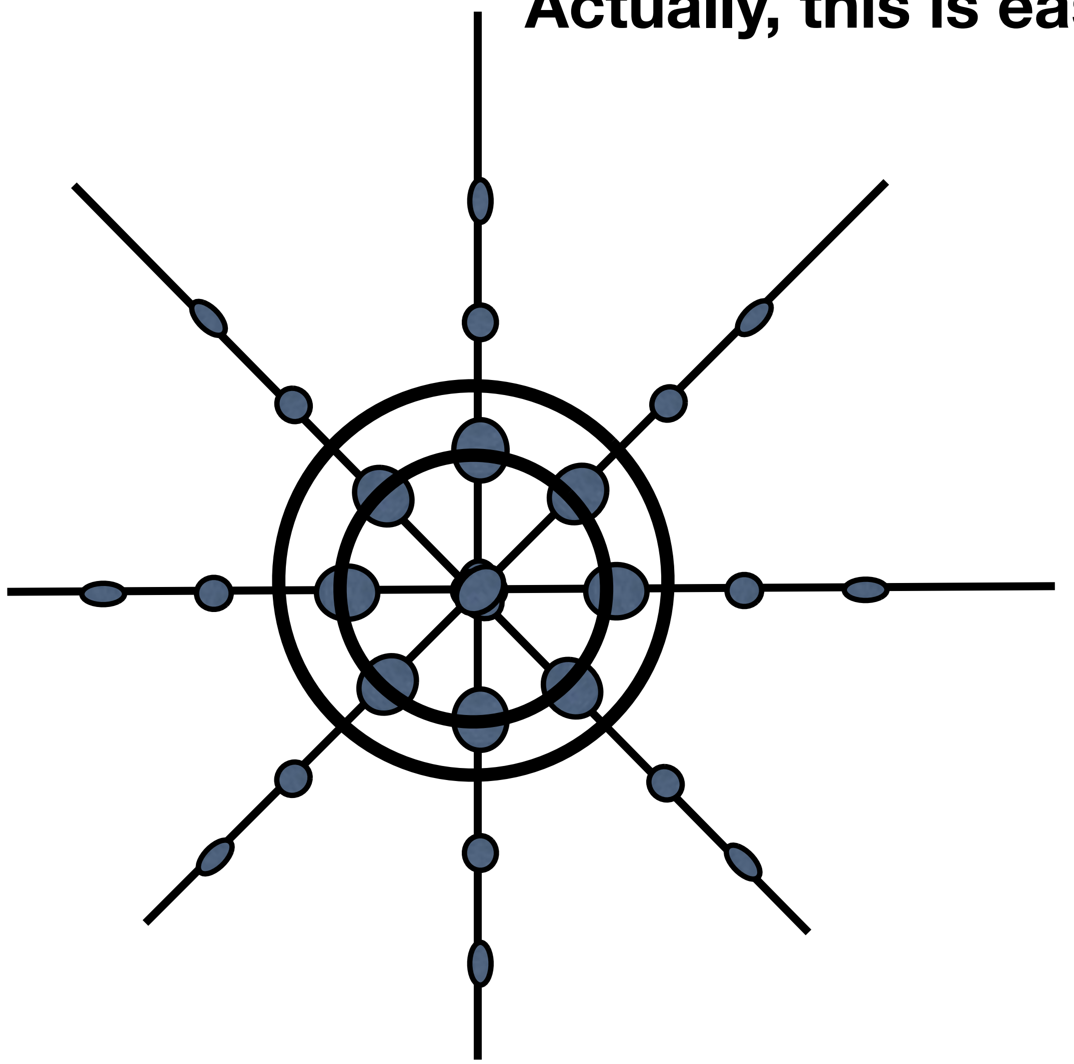

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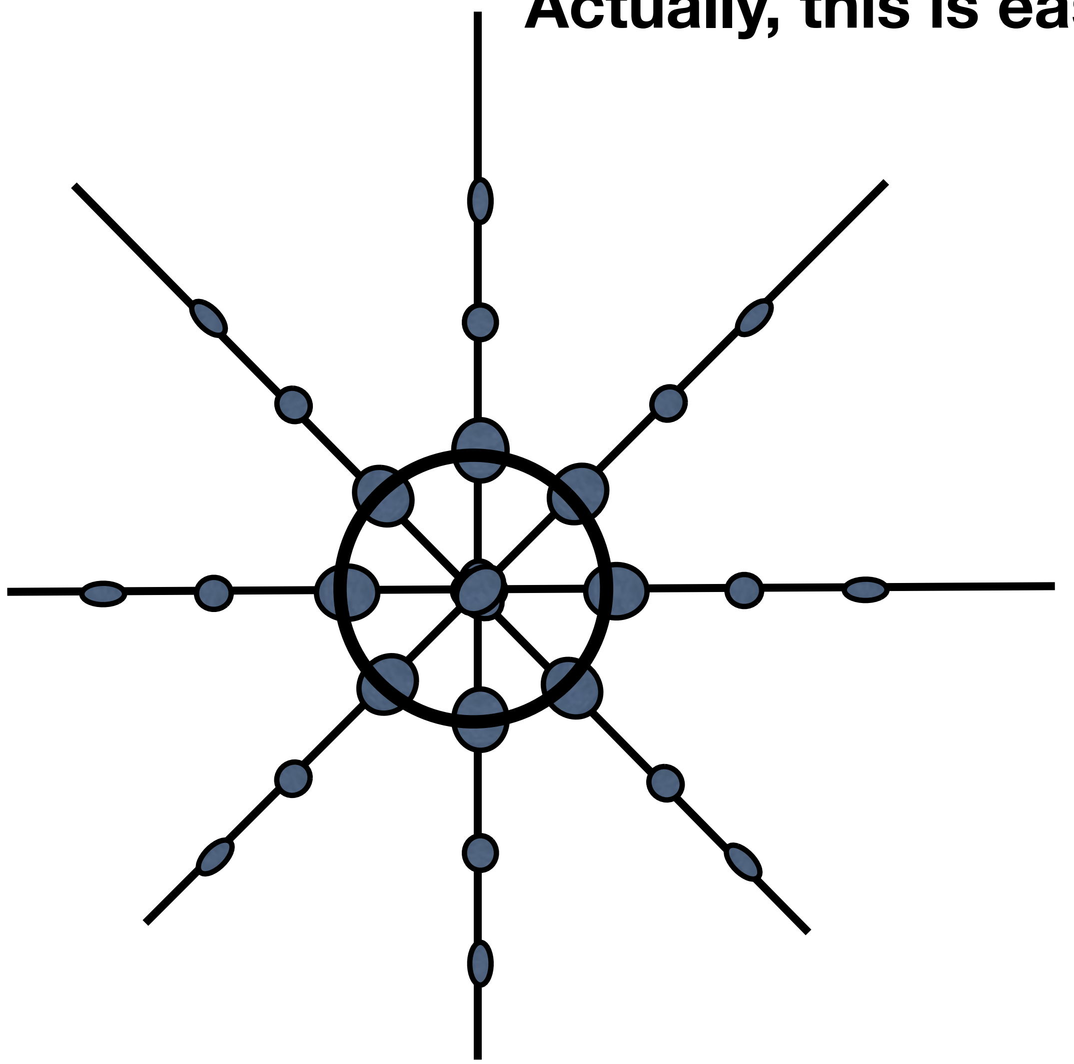

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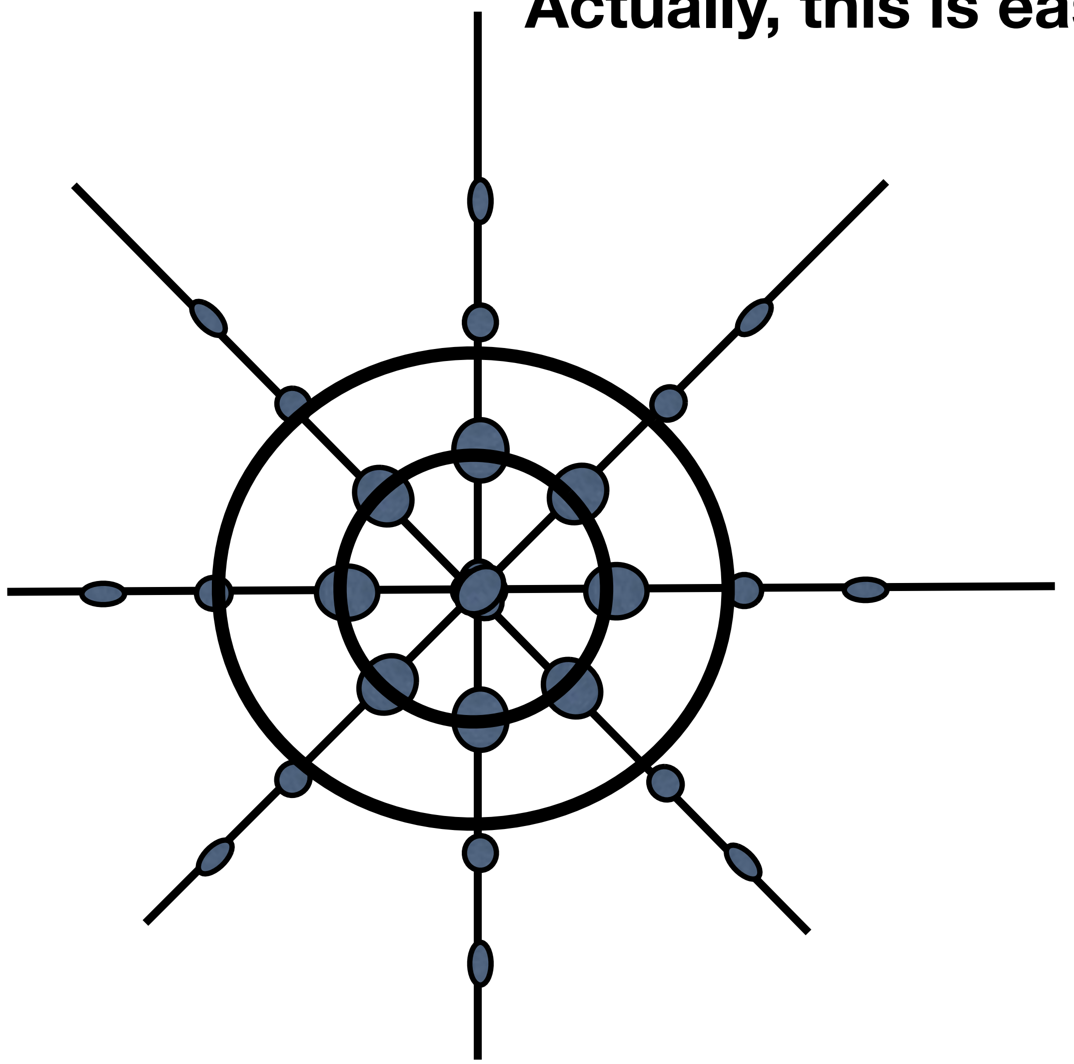

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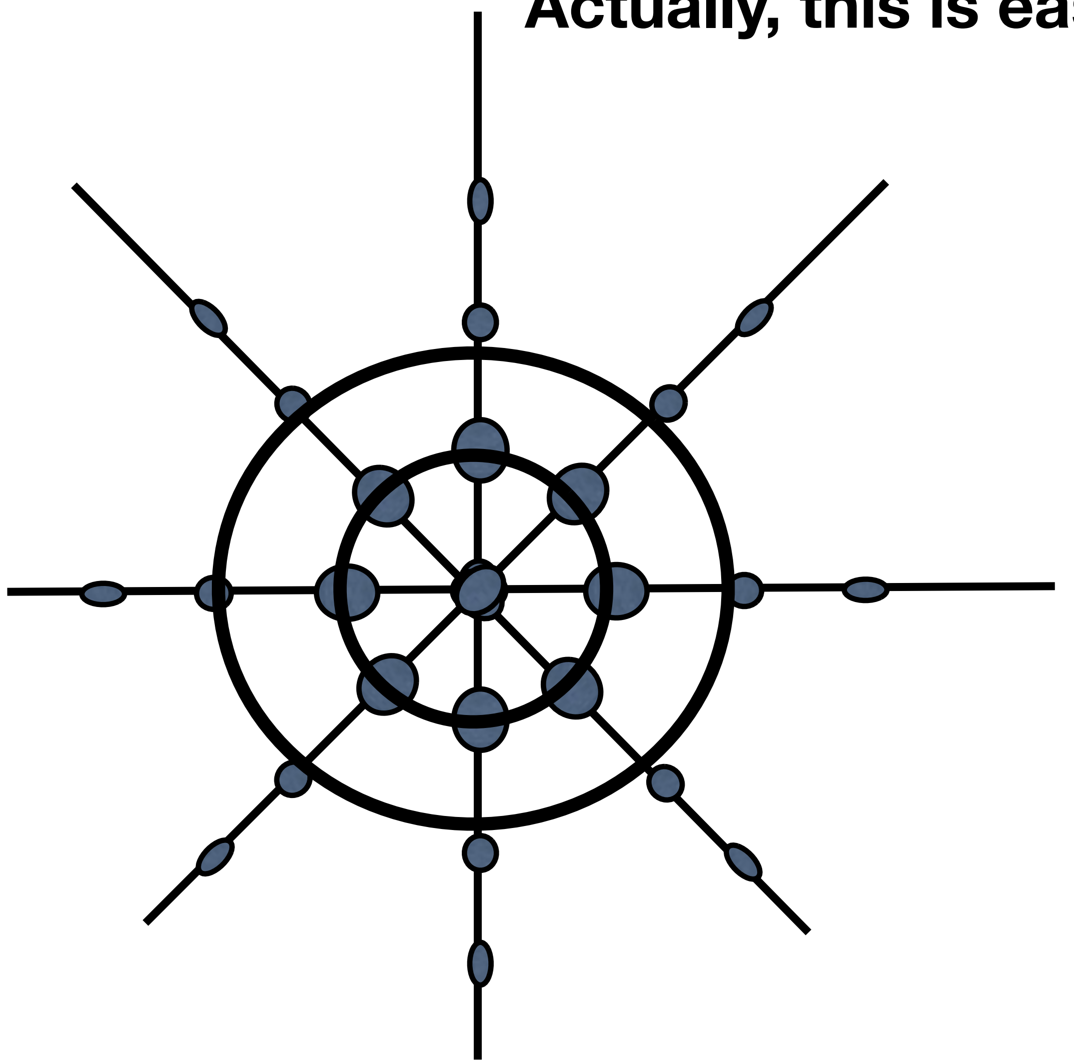

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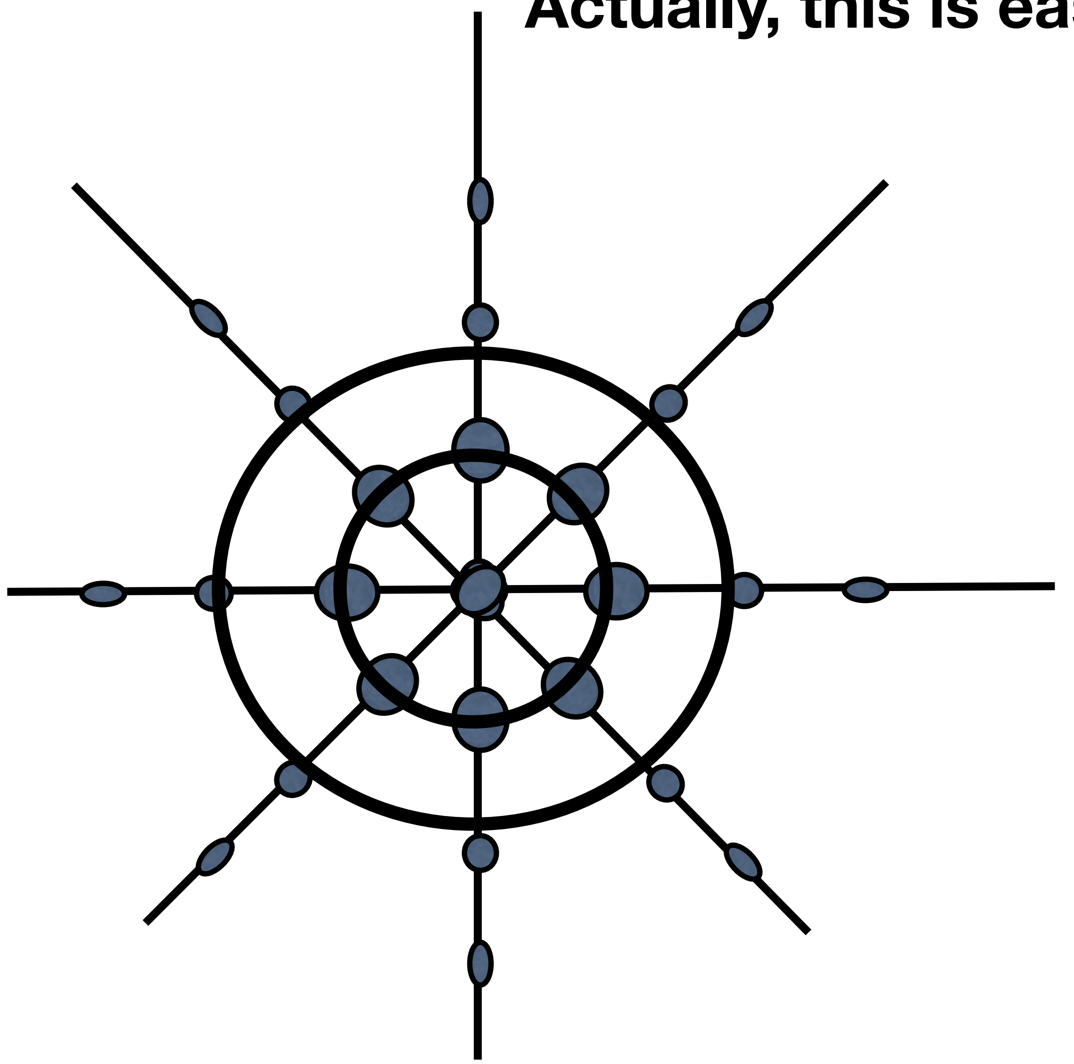

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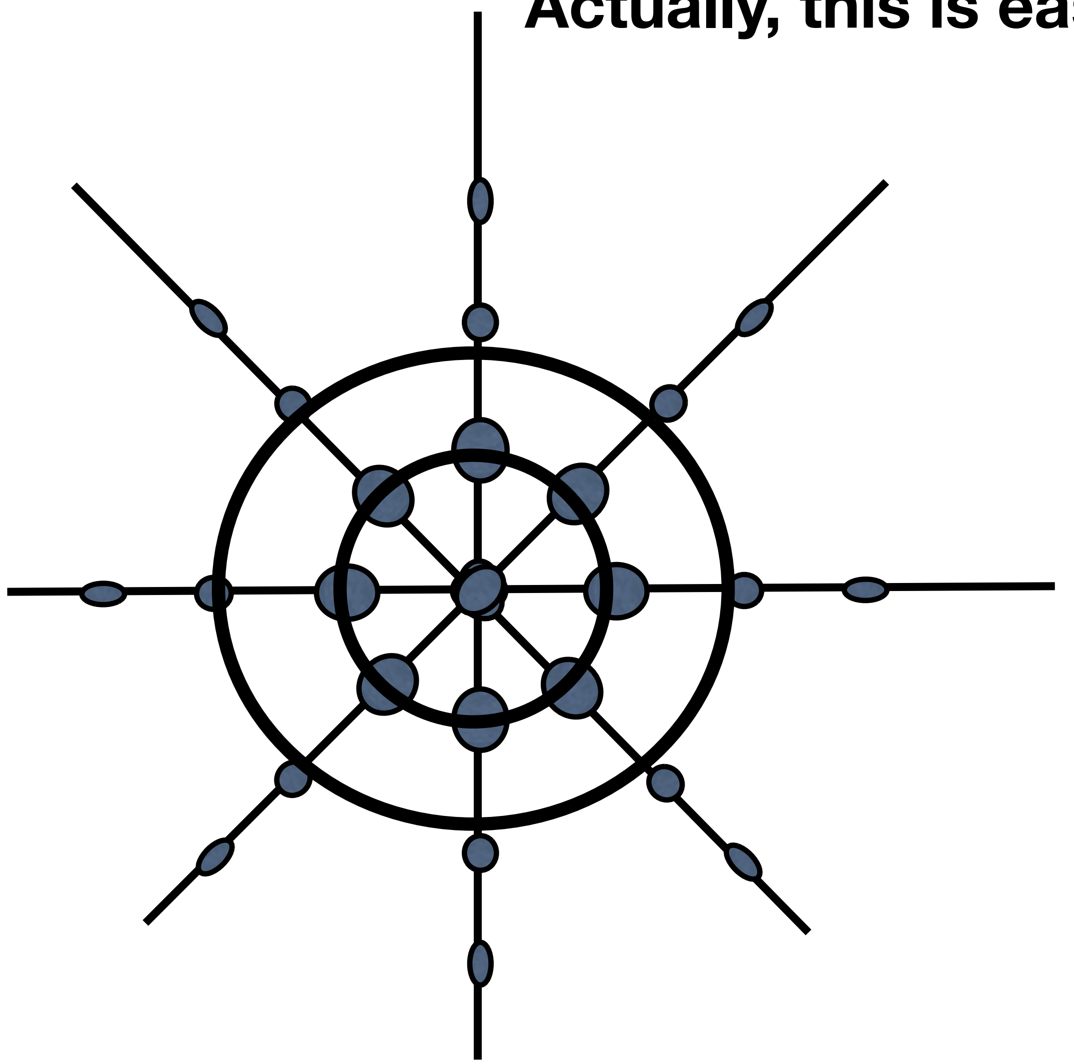

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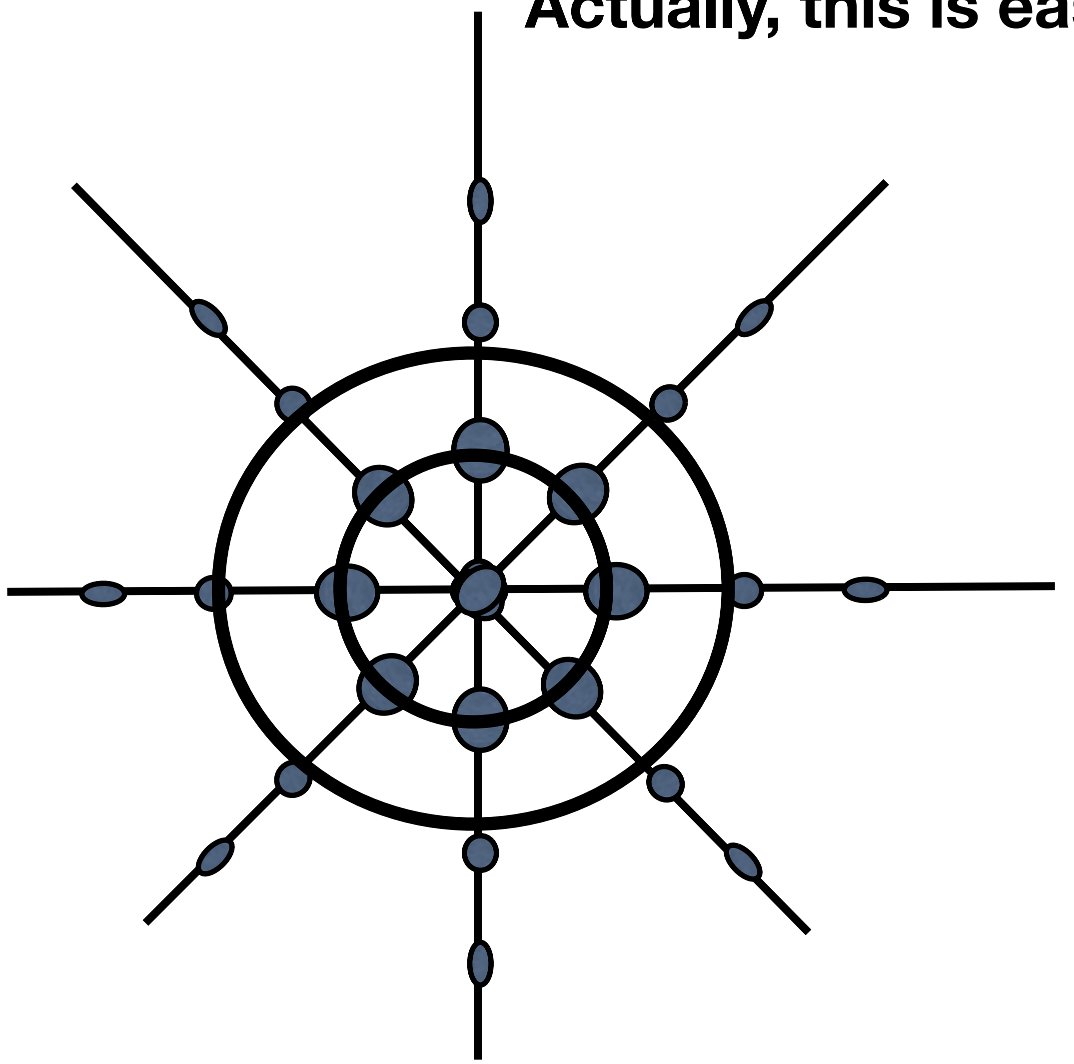

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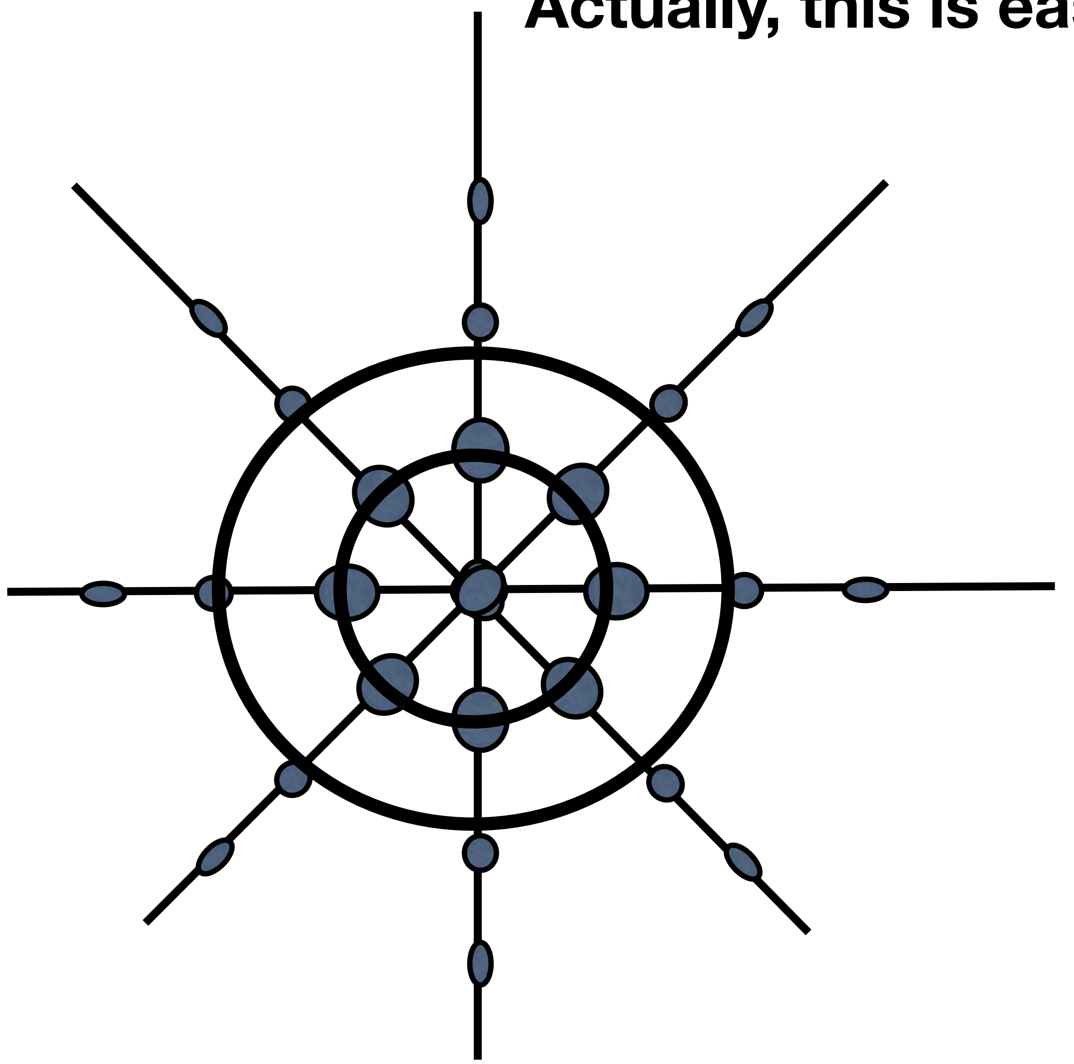

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This is Maxwell's central configuration for rings of Saturn

Two rings?

$$(\underbrace{\lambda_1 - \lambda_2}_{= 0}, \ldots, \lambda_1 - \lambda_n)$$

Continue

prove existence of distances by fixed point theorems,

or by Moulton's approach, or by analysis

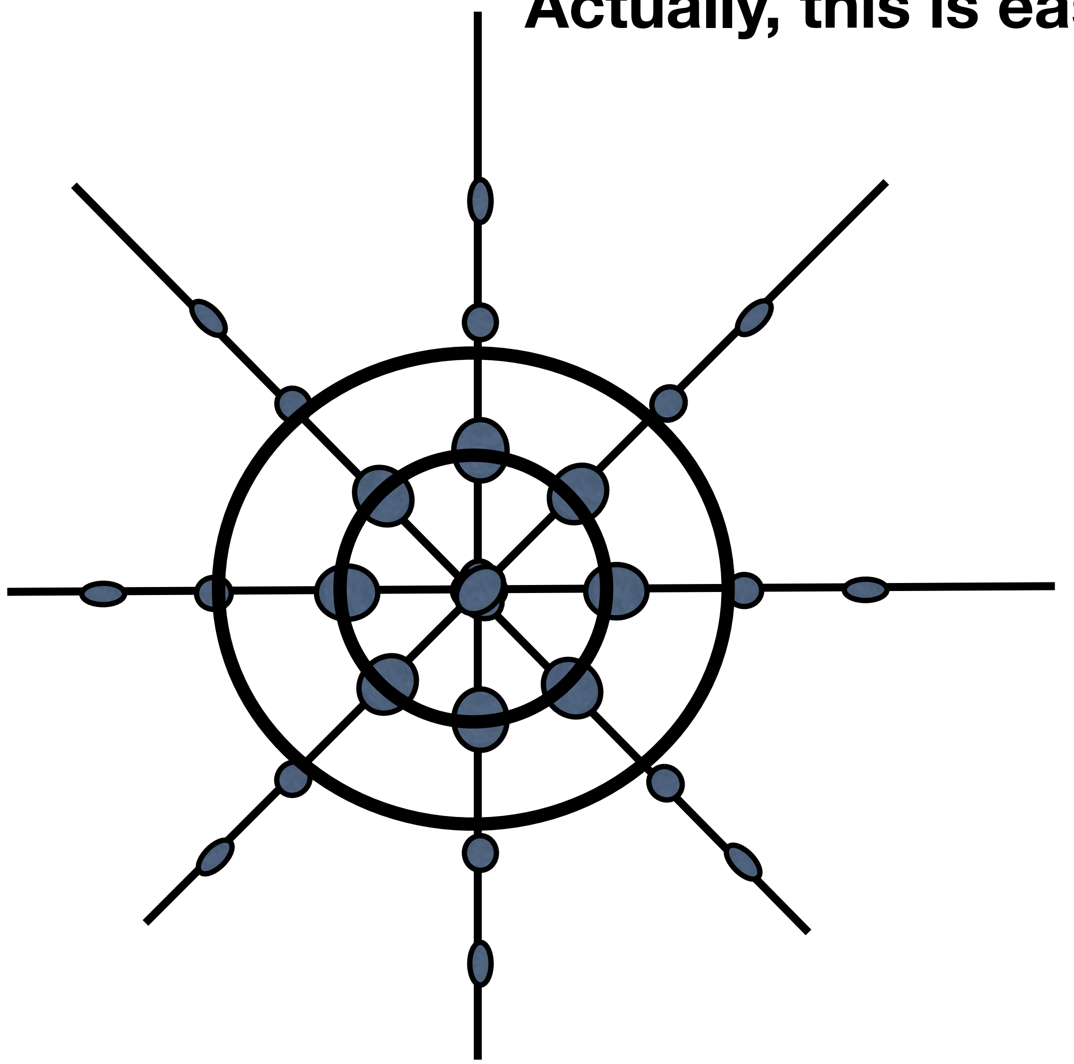
or by .....

A billion, or so, body central configuration

$$\mathbf{r}_j'' = \lambda_j \mathbf{r}_j$$

All λ_j are equal

Actually, this is easy



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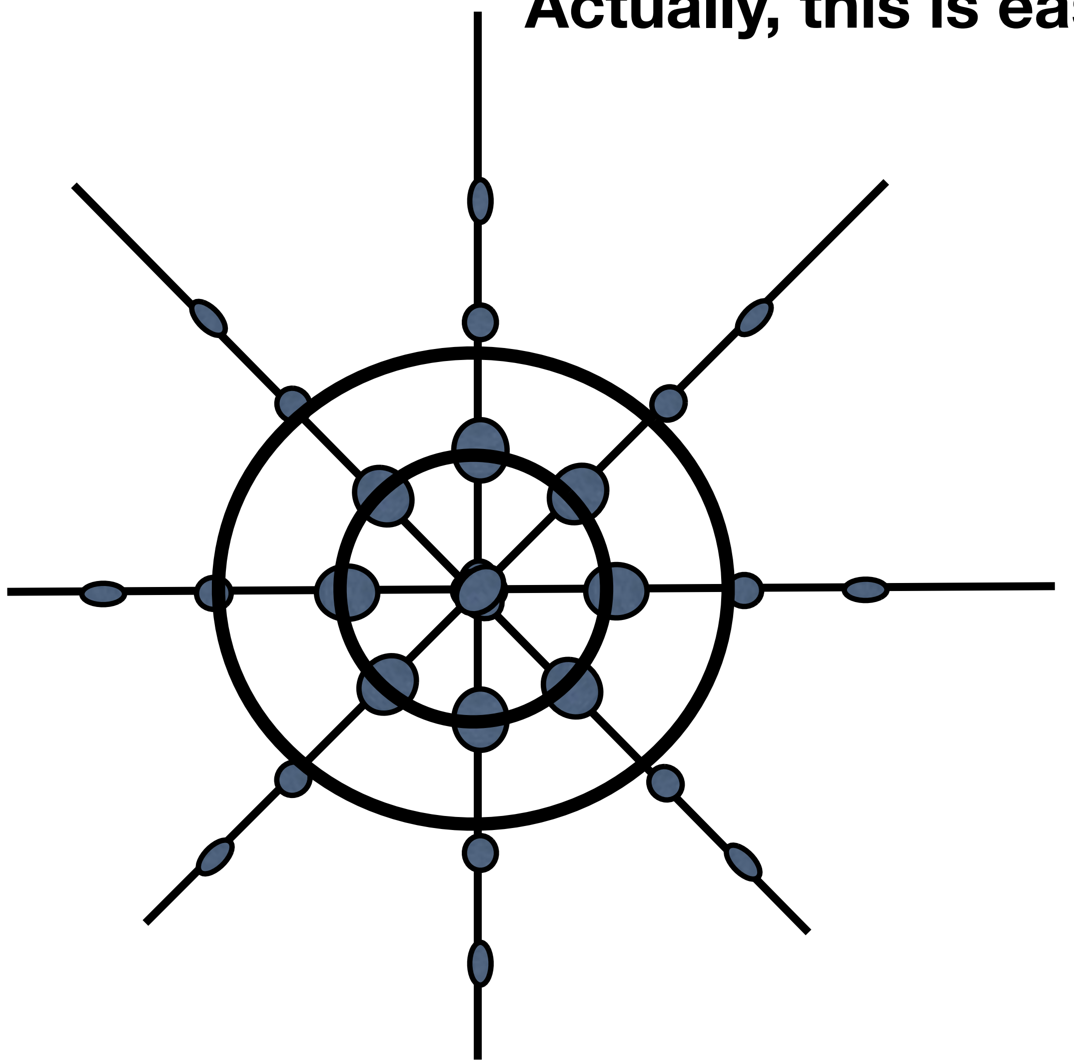

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$$v_{rot,j} = D\, r_j$$

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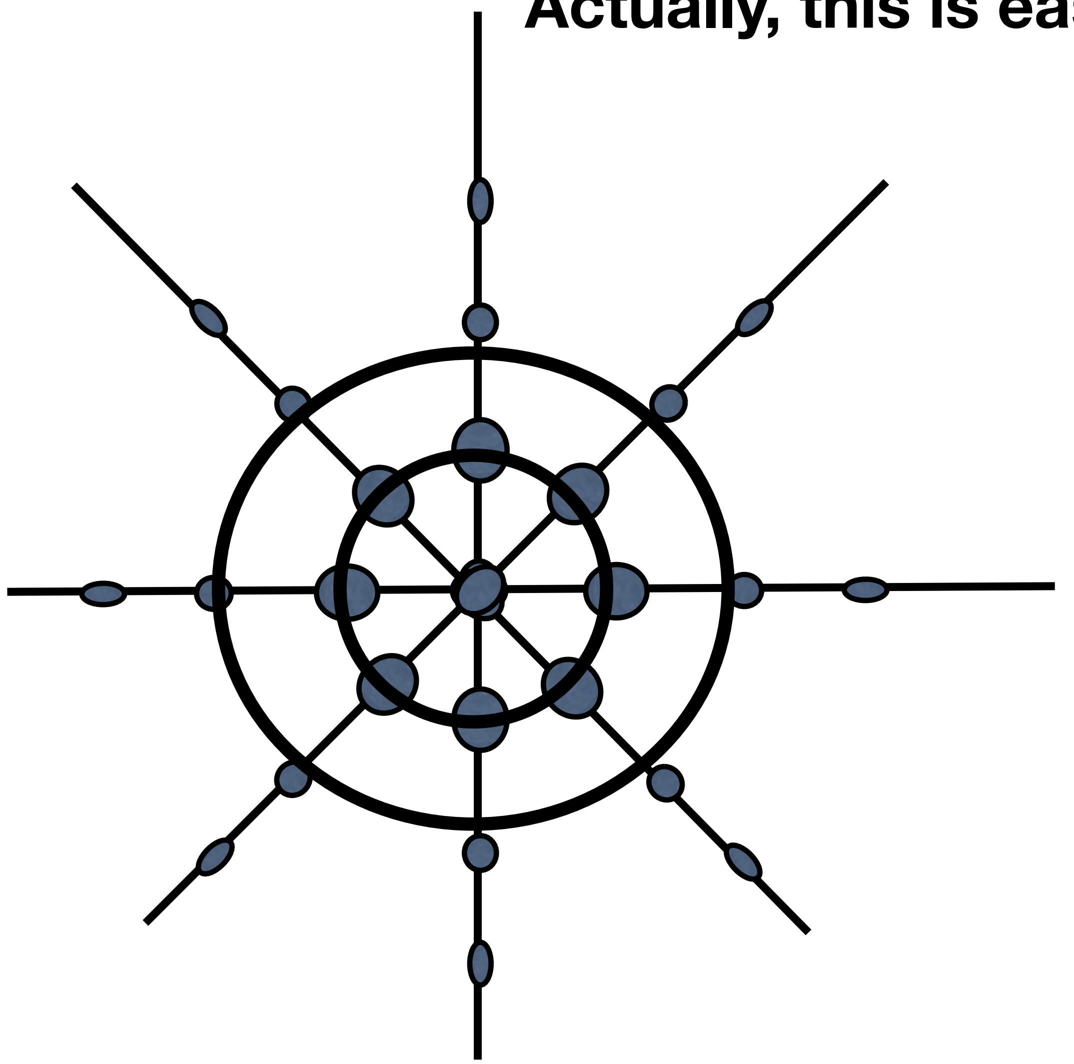

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$$M(r) = \frac{r v_{rot}^2}{G} \qquad v_{rot,j} = D\, r_j$$

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A billion, or so, body central configuration $\mathbf{r}_j'' = \lambda_j \mathbf{r}_j$

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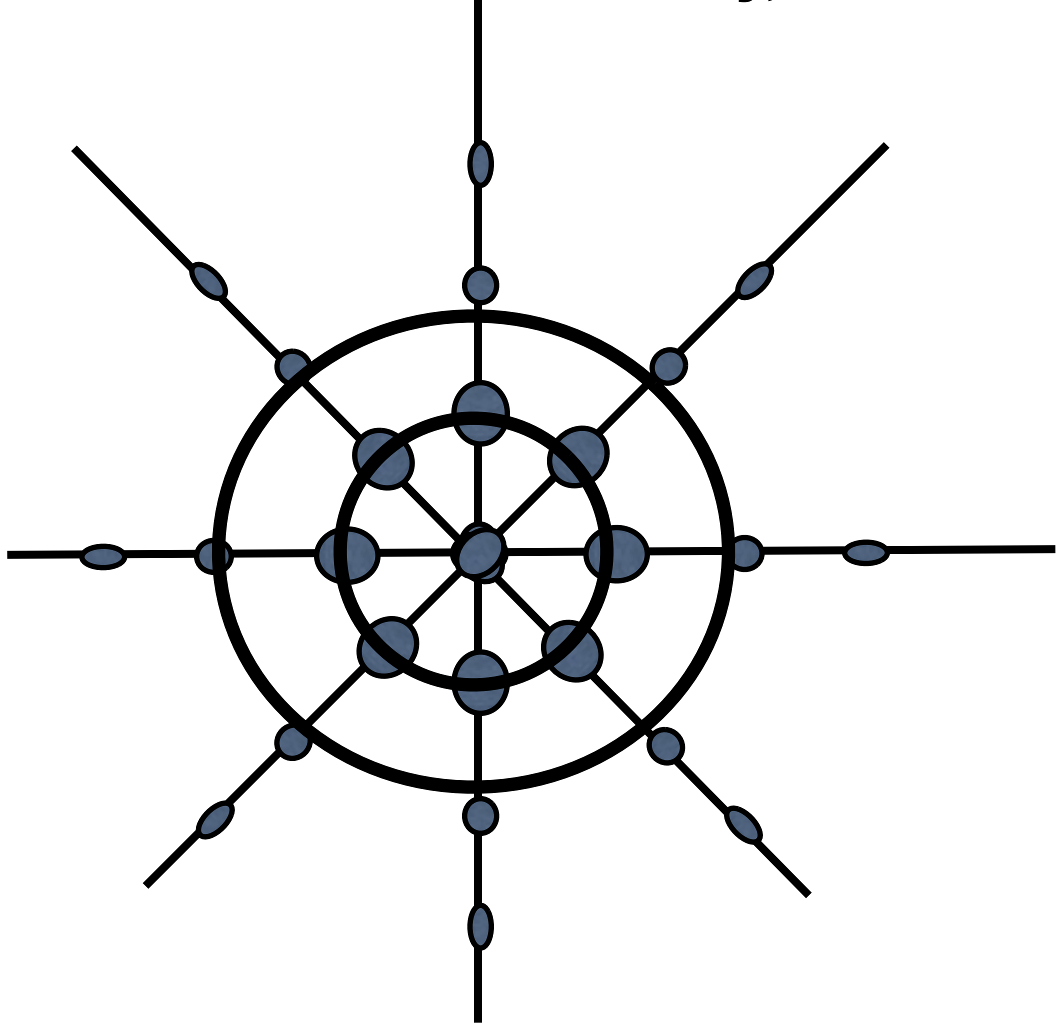

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$$M(r) = \frac{r v_{rot}^2}{G} \qquad v_{rot,j} = D\, r_j \qquad M(r) \sim D\, r^3$$

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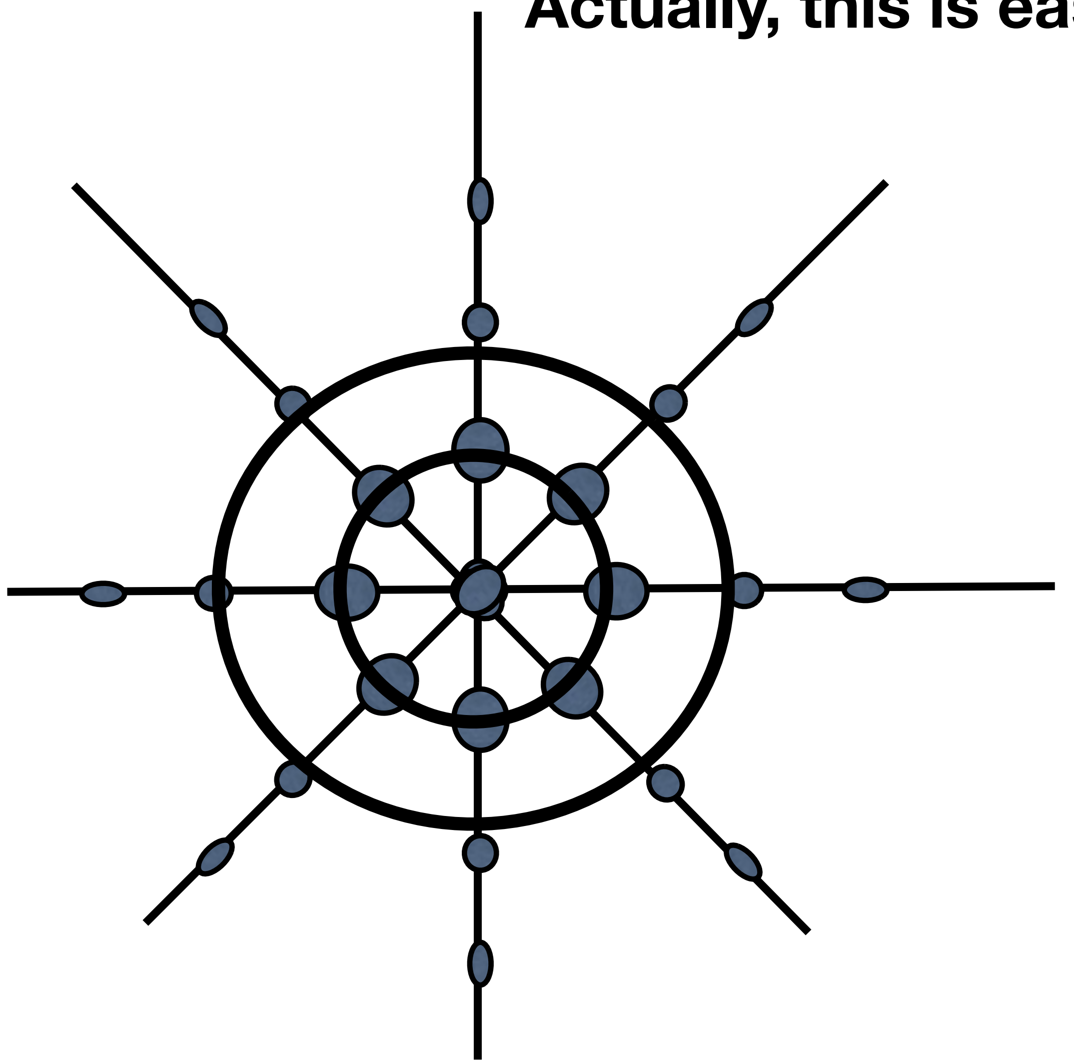

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Wrong! What is missing is angular momentum

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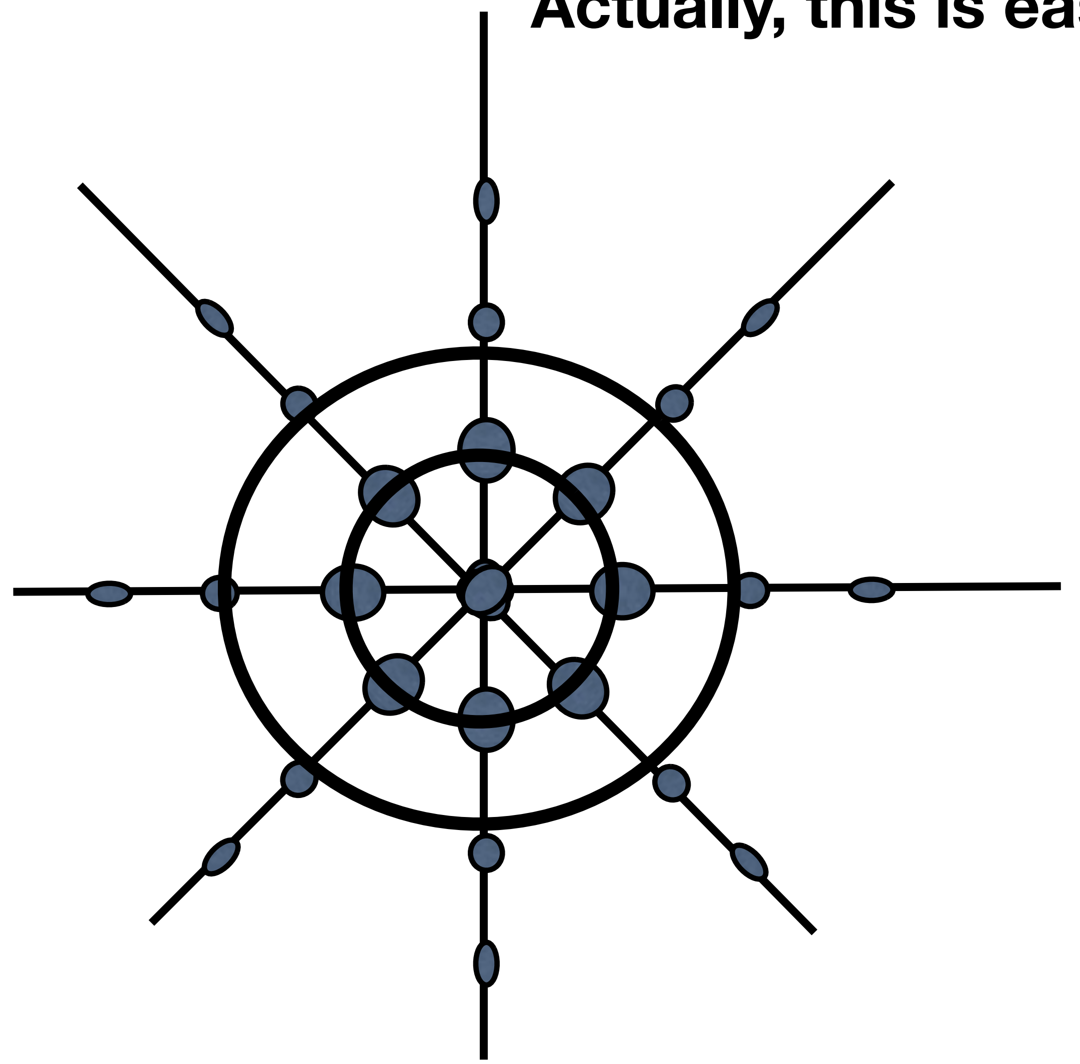

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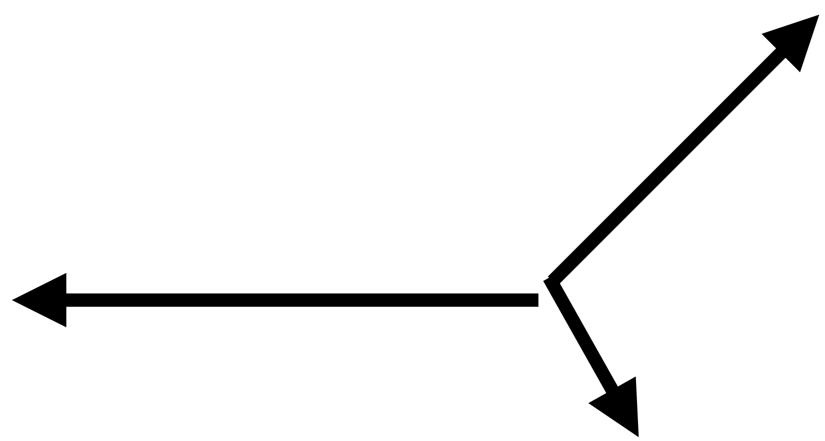
Wrong! What is missing is angular momentum So, next I indicate how to include it—for <u>any</u> force law!

Angular momentum via a velocity decomposition

Angular momentum via a velocity decomposition



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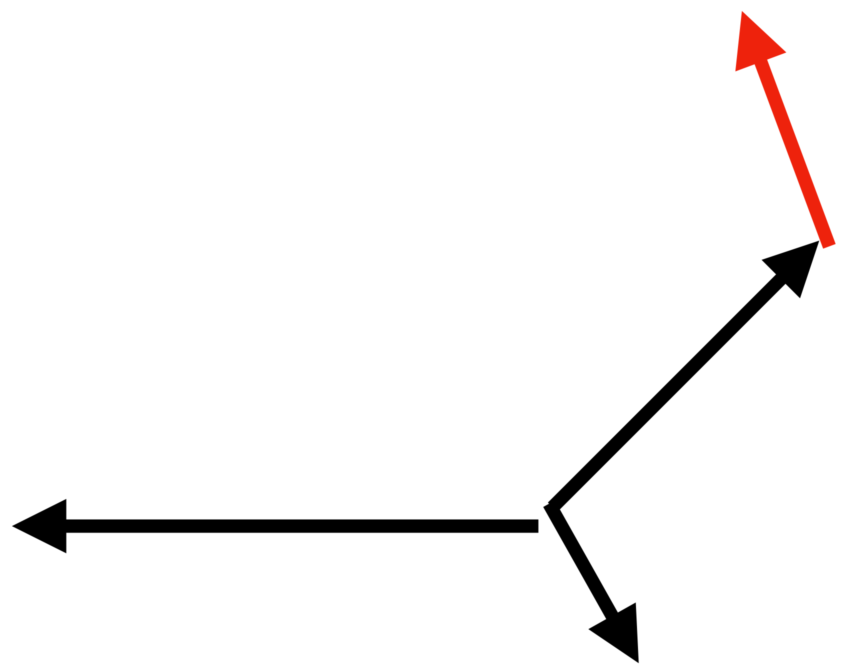

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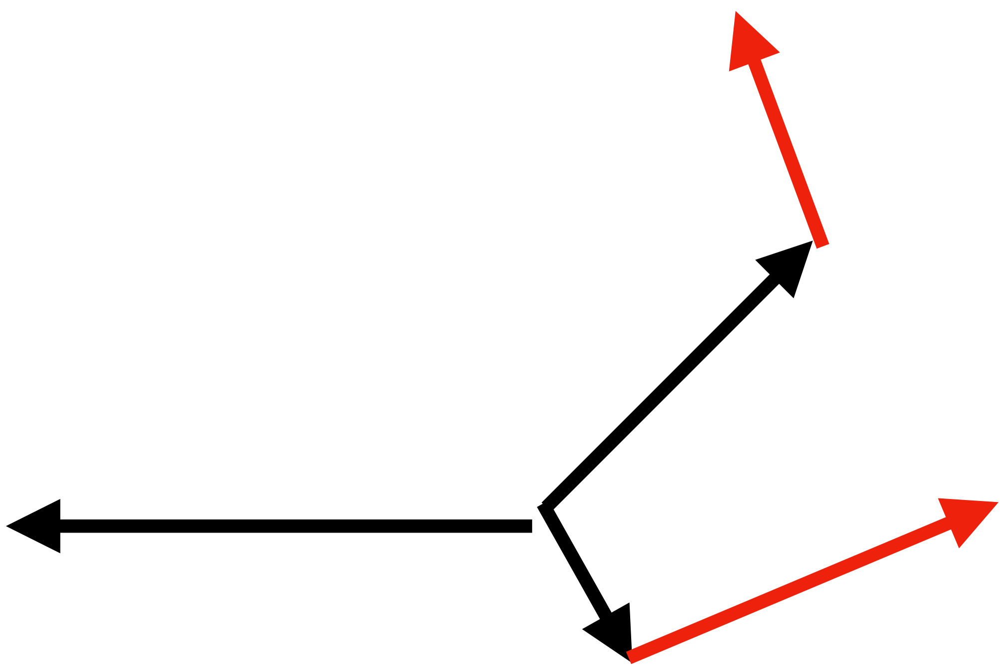

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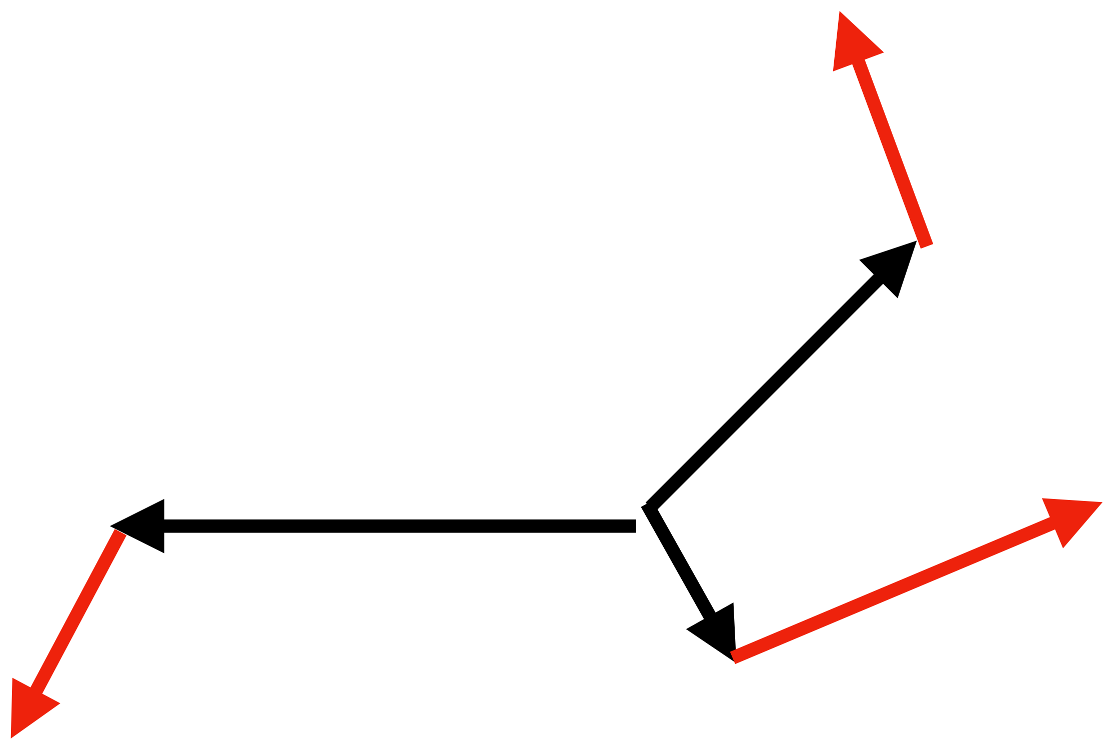

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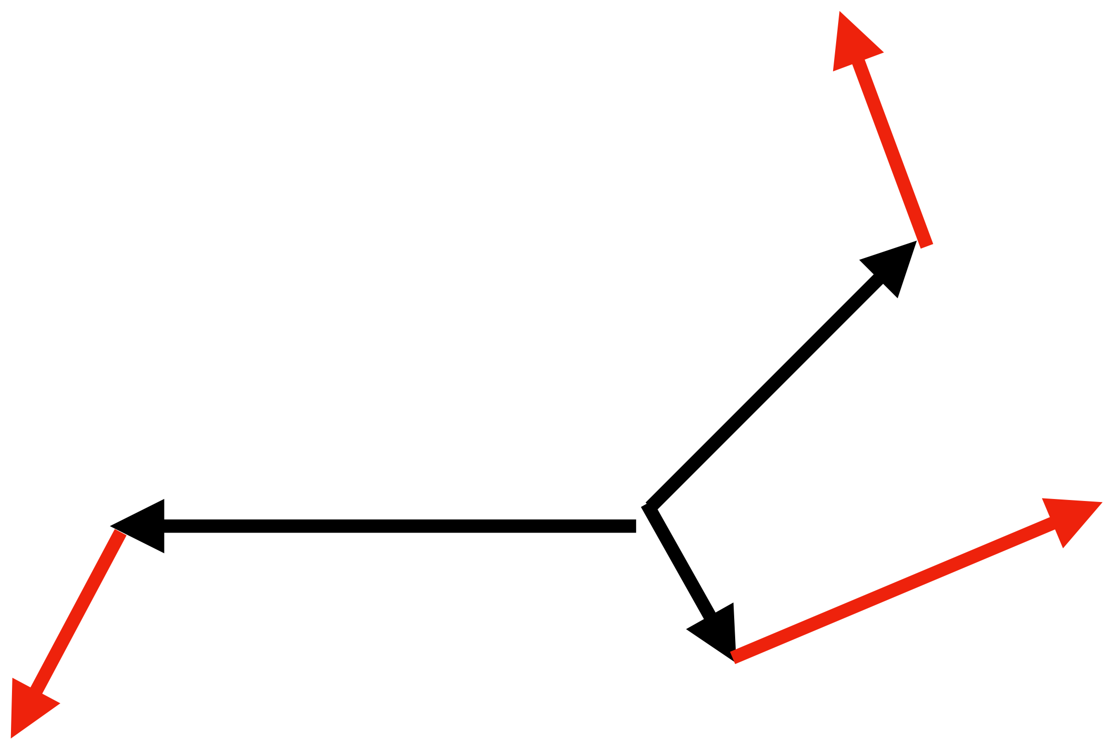
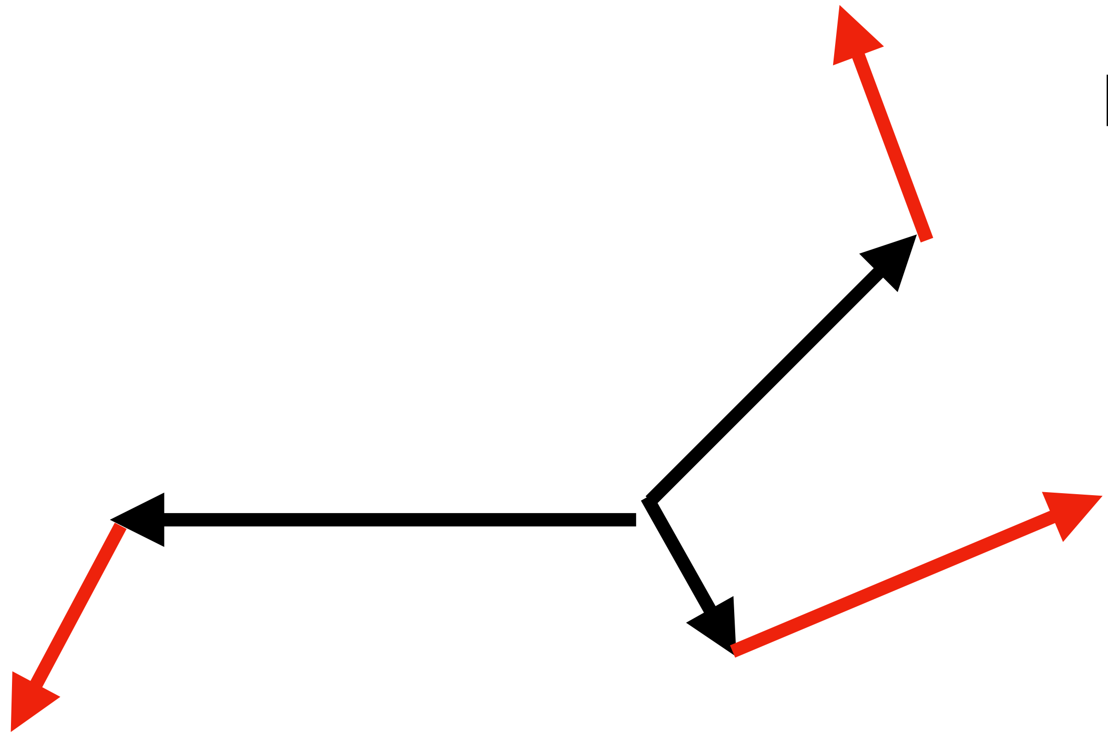


What can we say about impact of the velocities?

Angular momentum via a velocity decomposition

What can we say about impact of the velocities?
E.G., how will the shape change?



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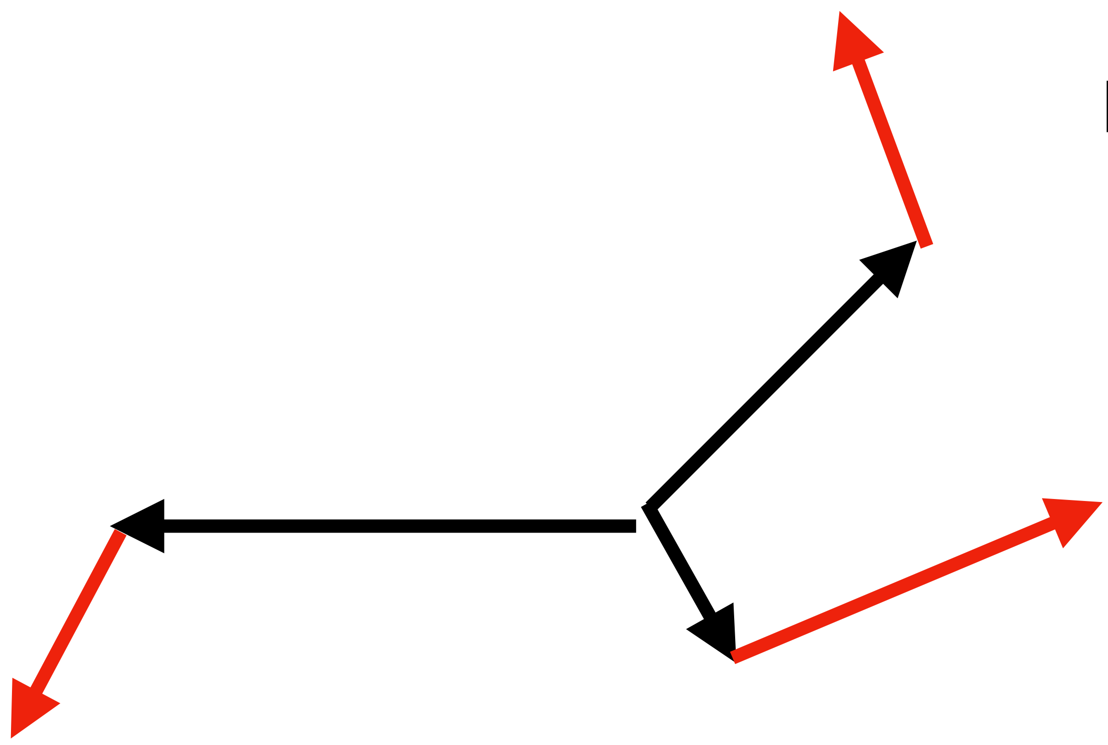

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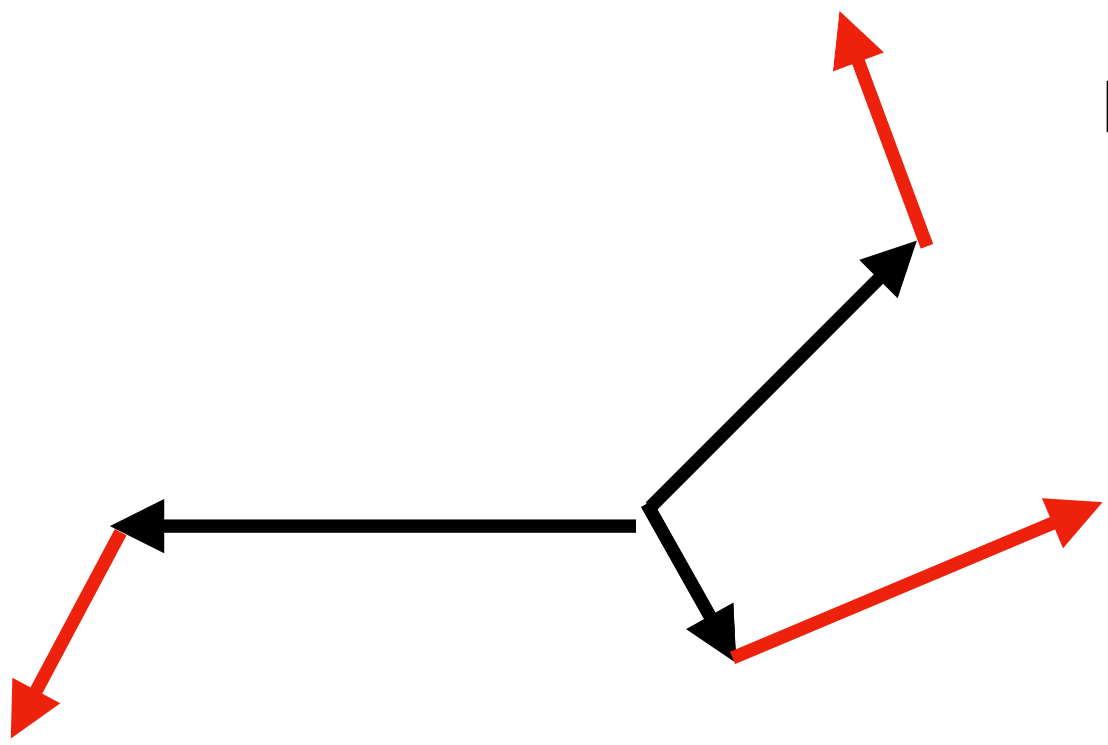
Only do it for rotation, but also holds for radial

Angular momentum via a velocity decomposition



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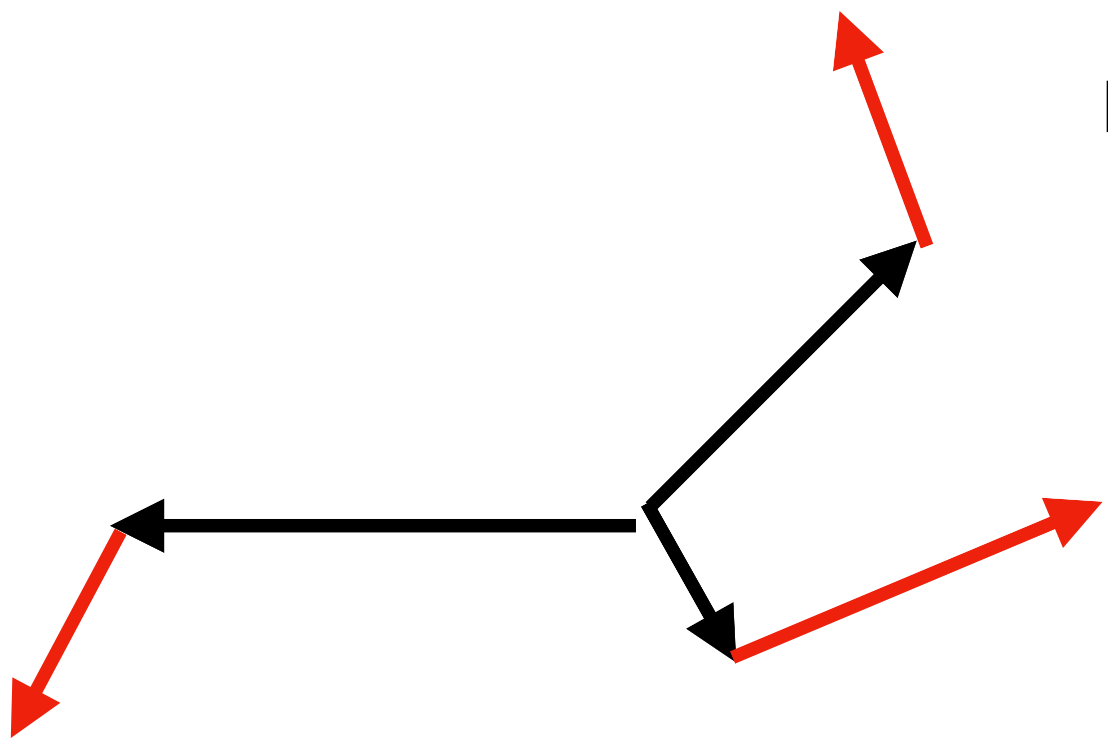

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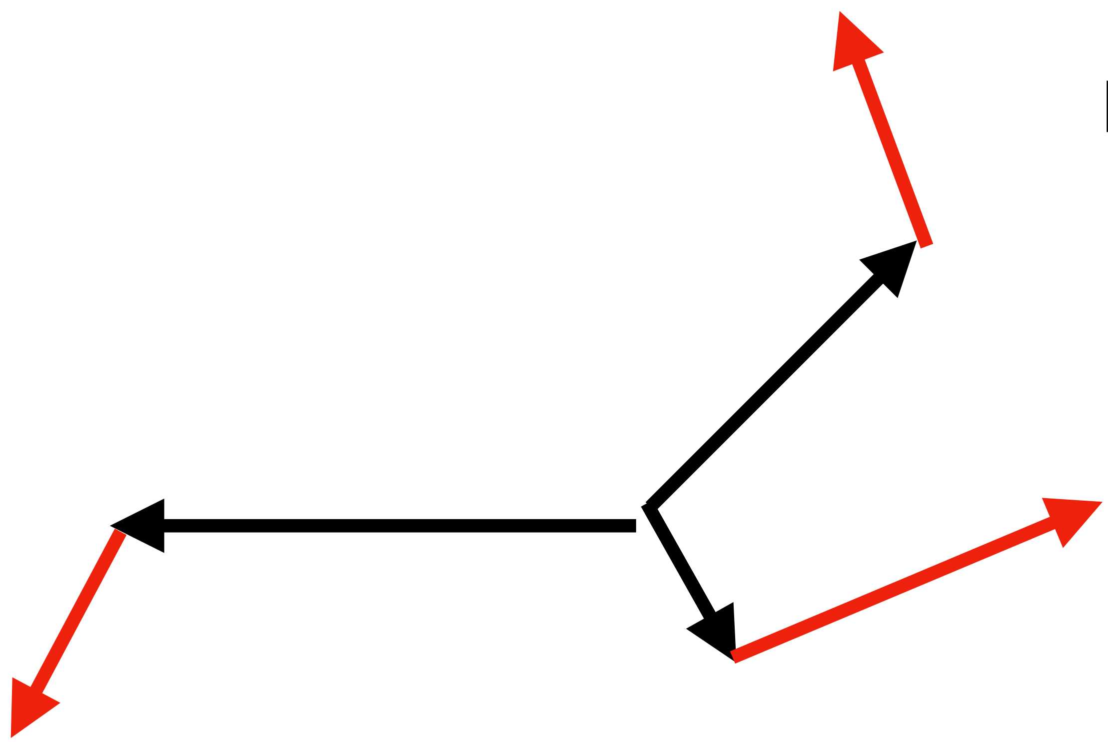
That is $s \; e_3 \times r_j$

Angular momentum via a velocity decomposition



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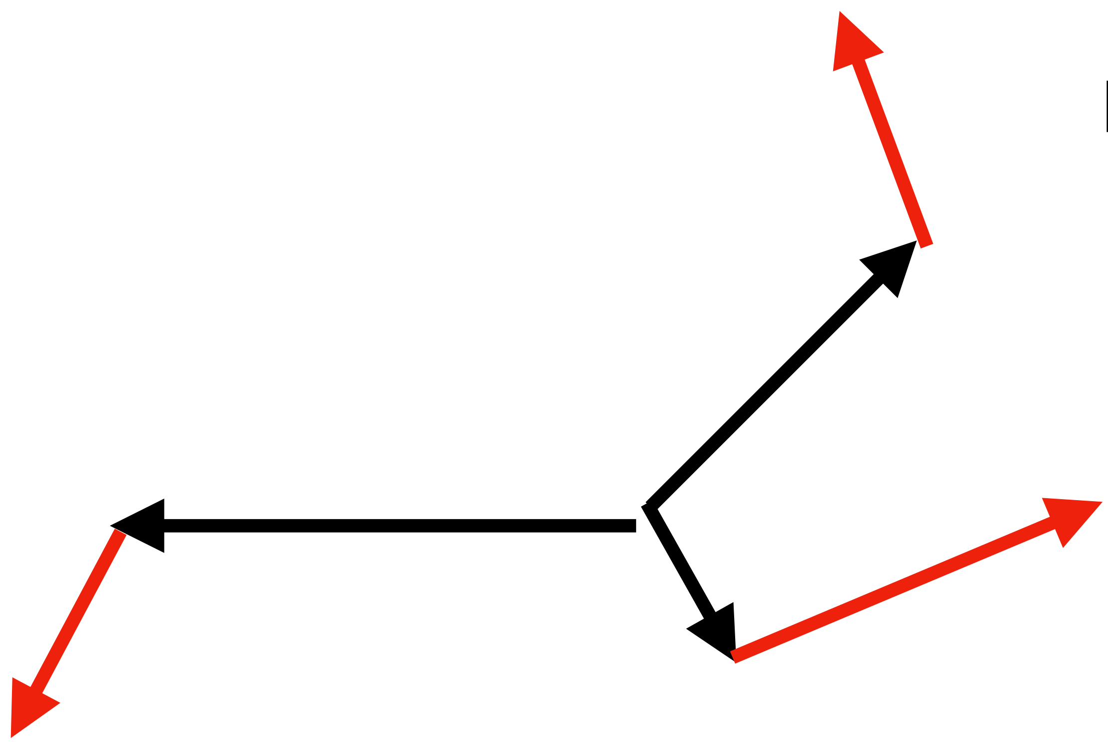

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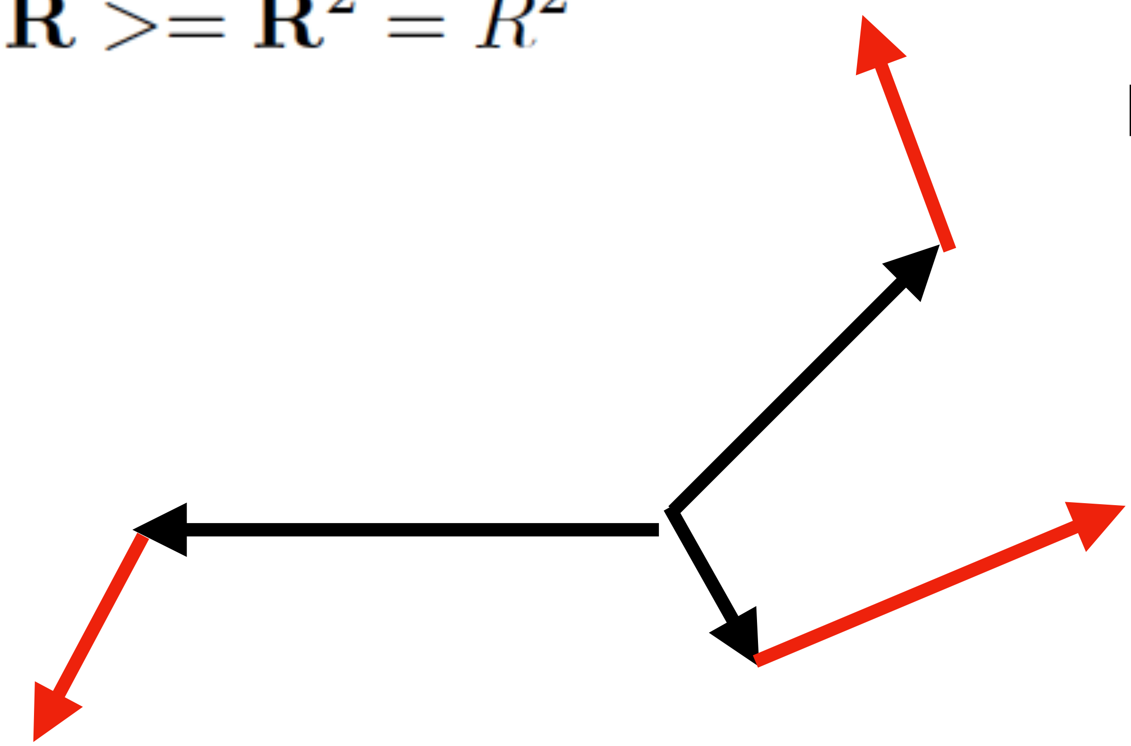

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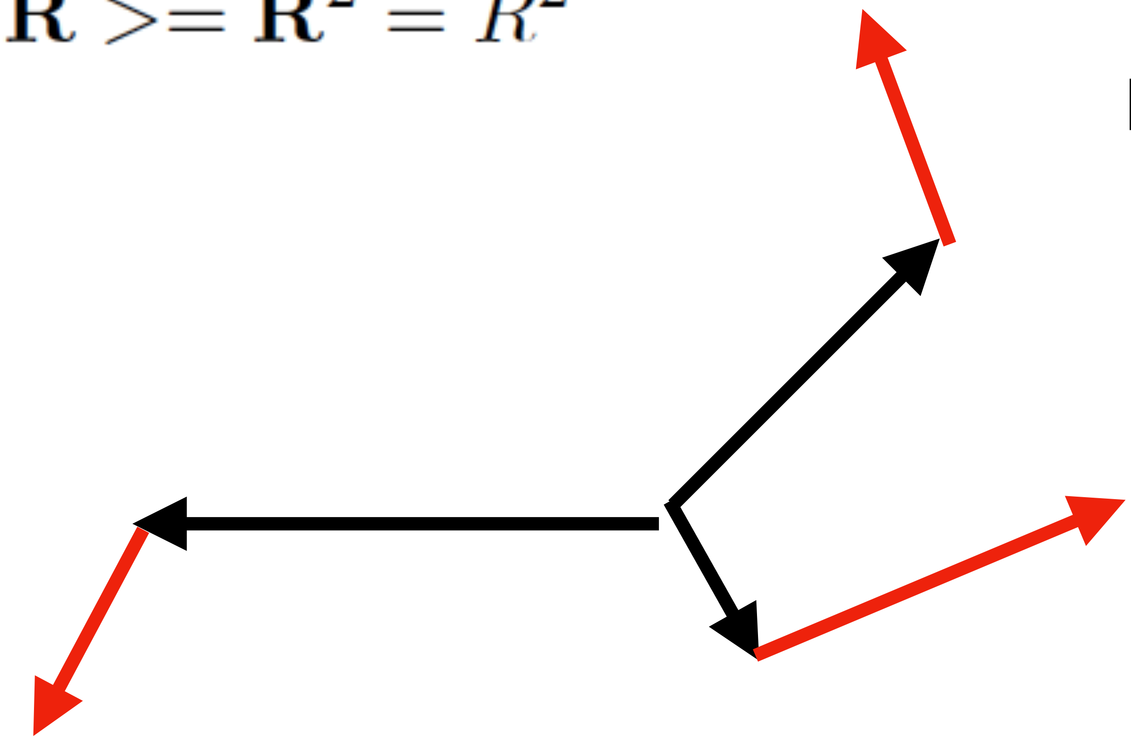

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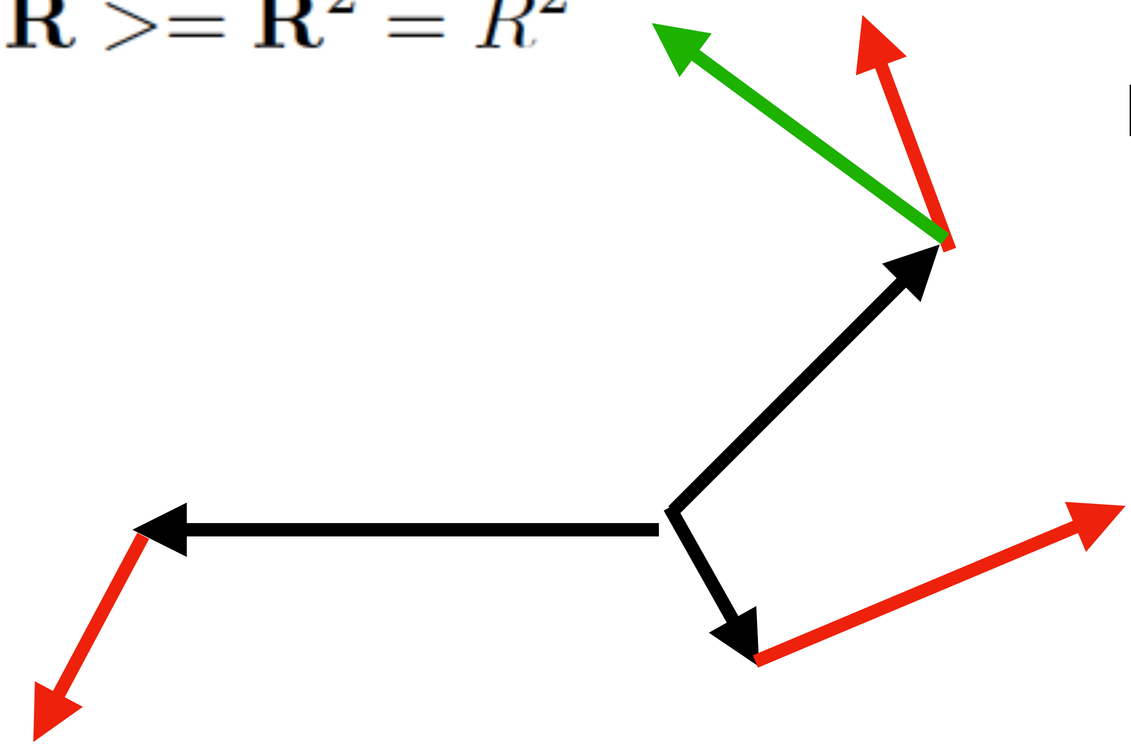

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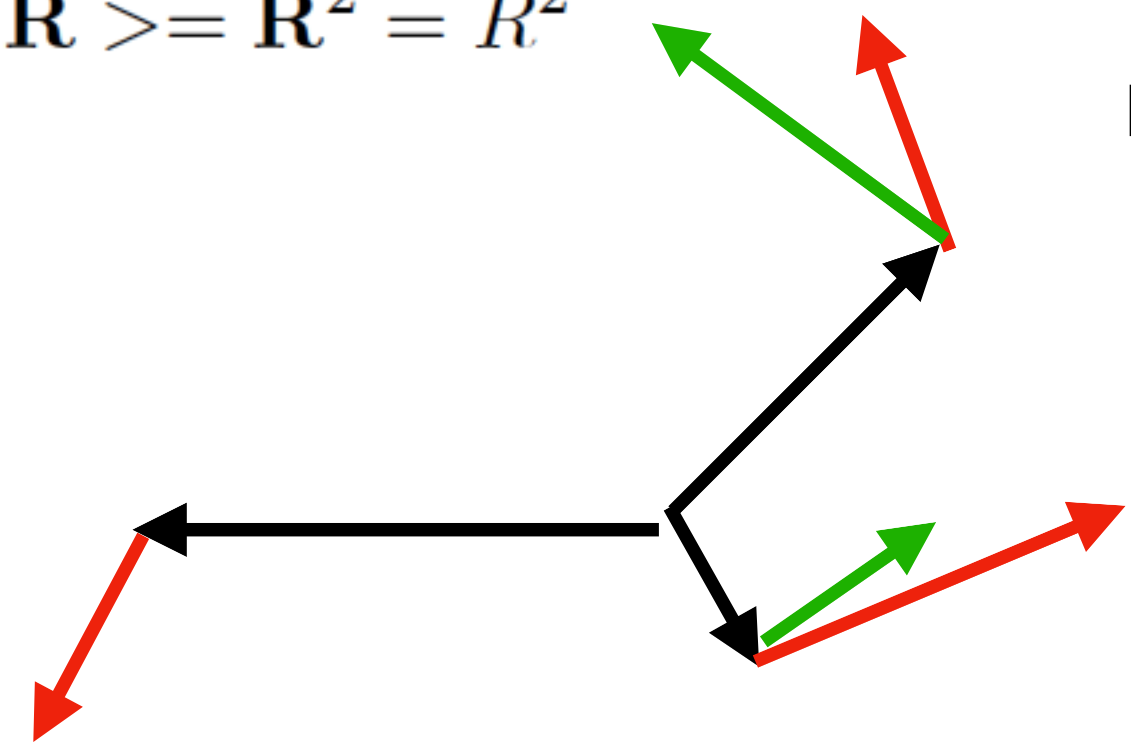

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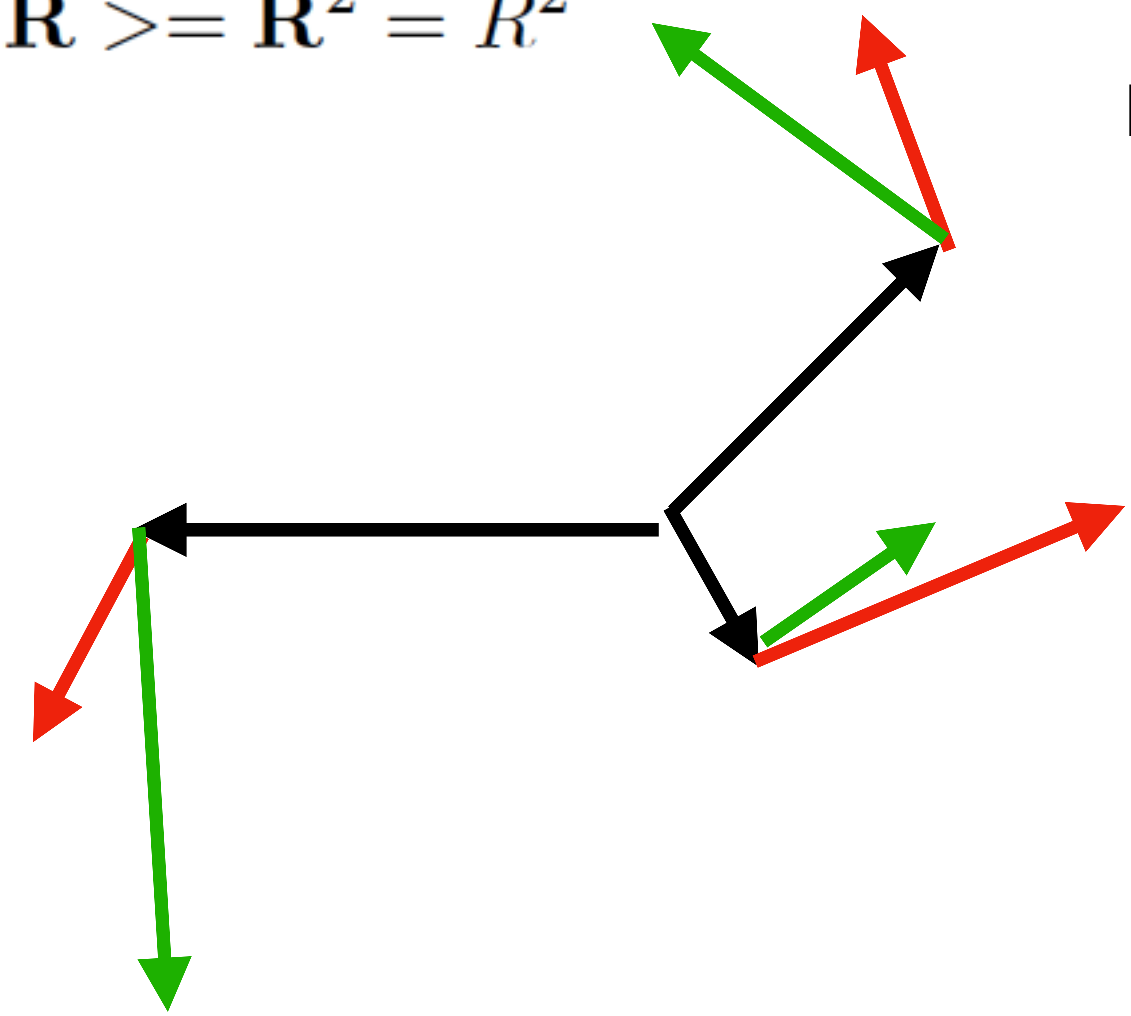

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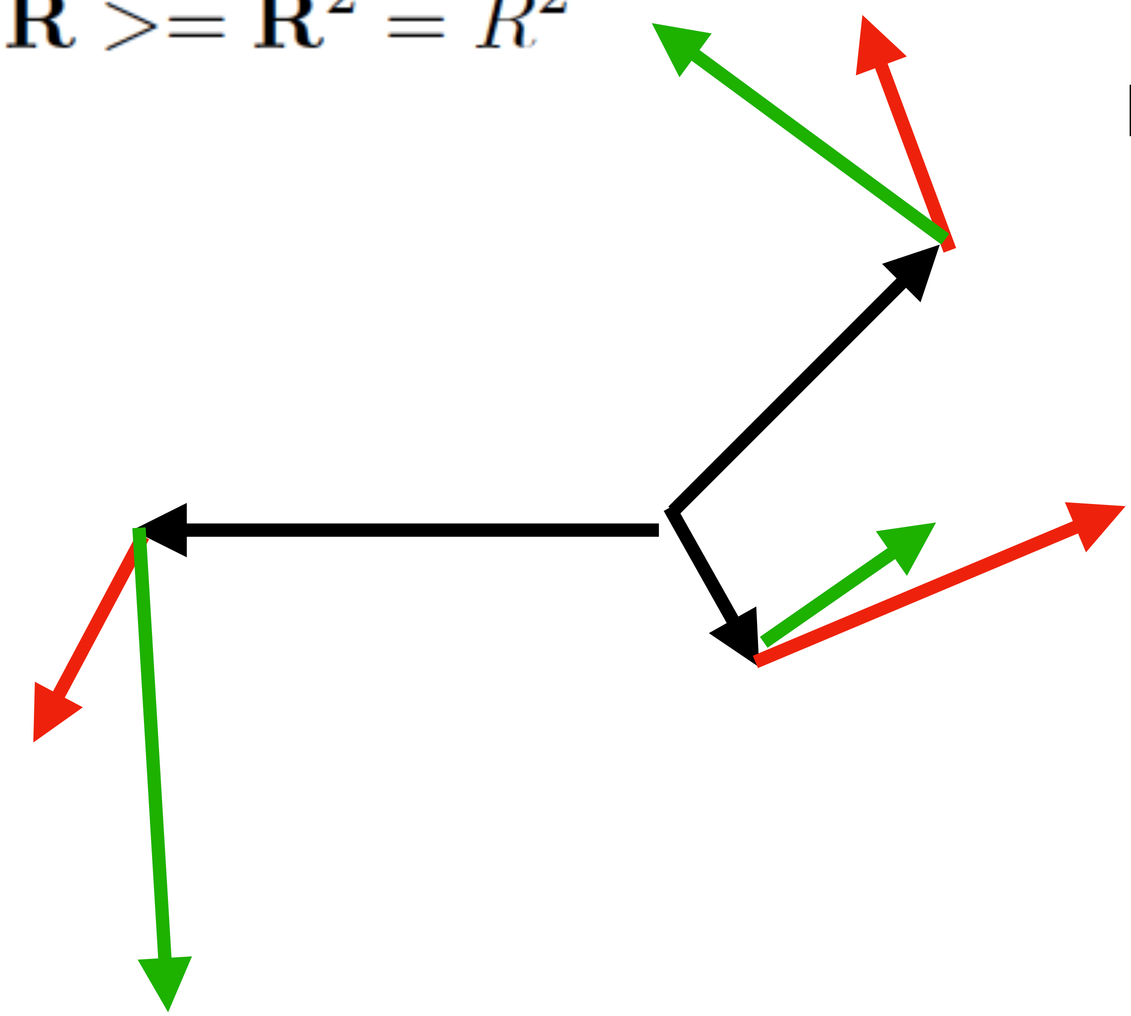

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Now find the rotational velocity that changes the shape

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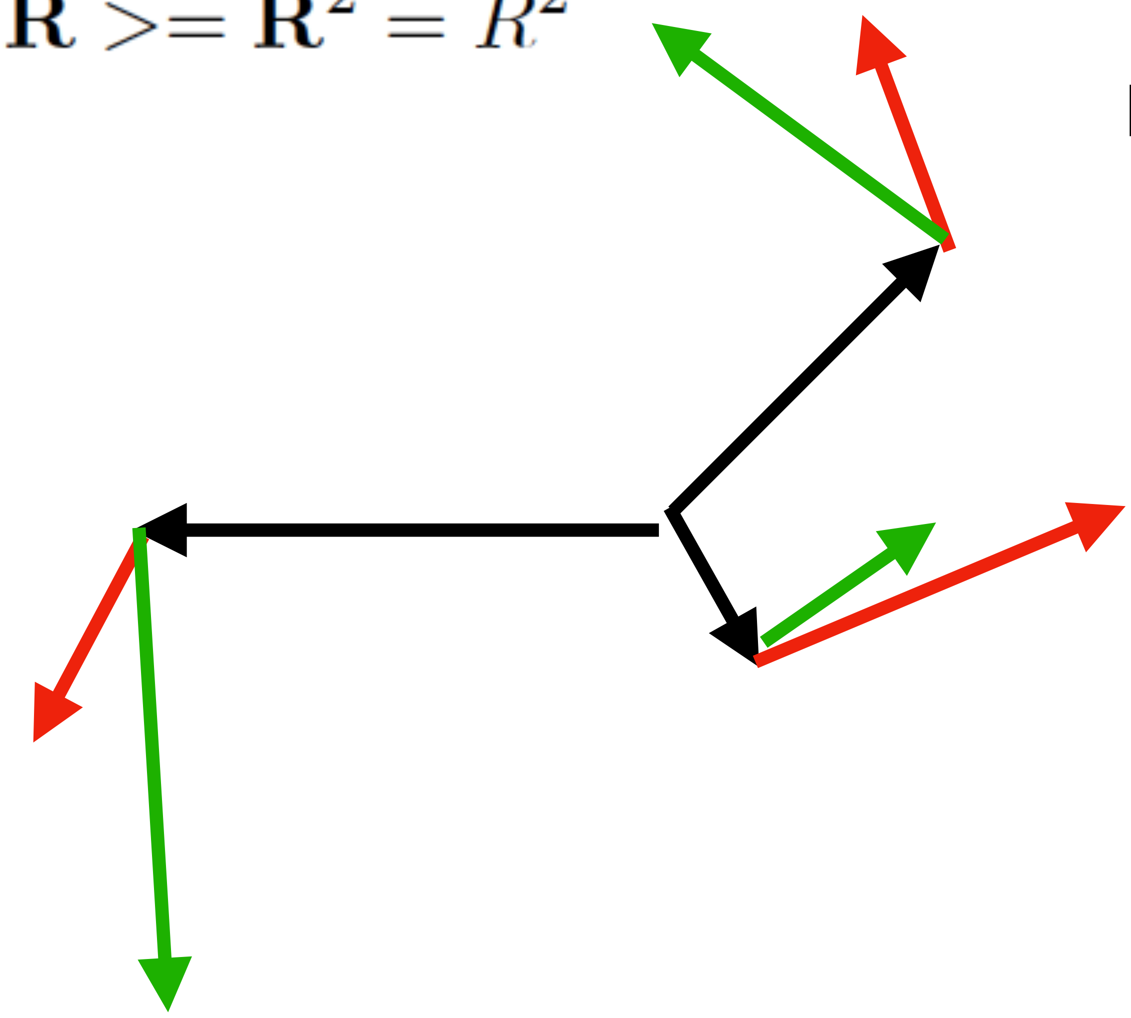

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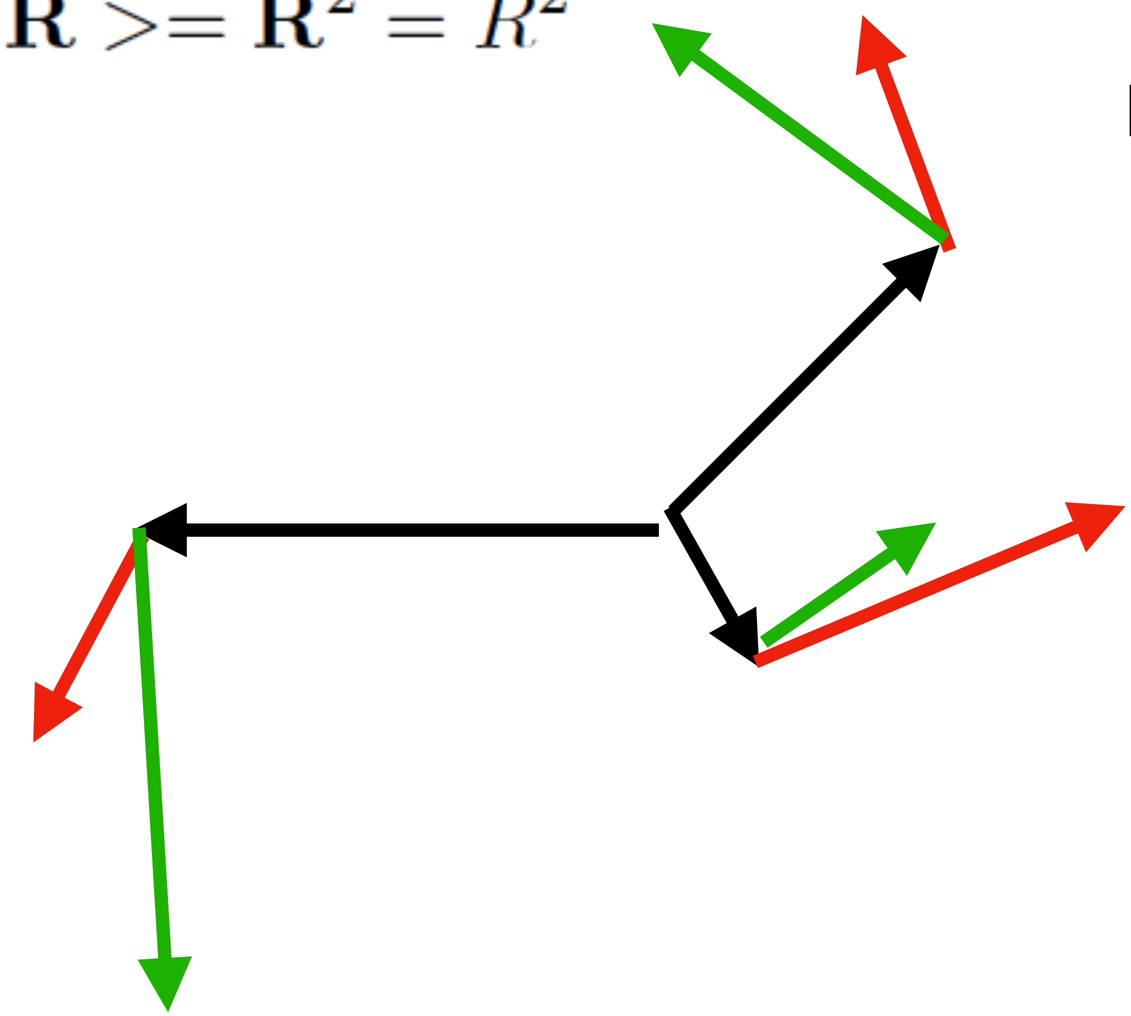

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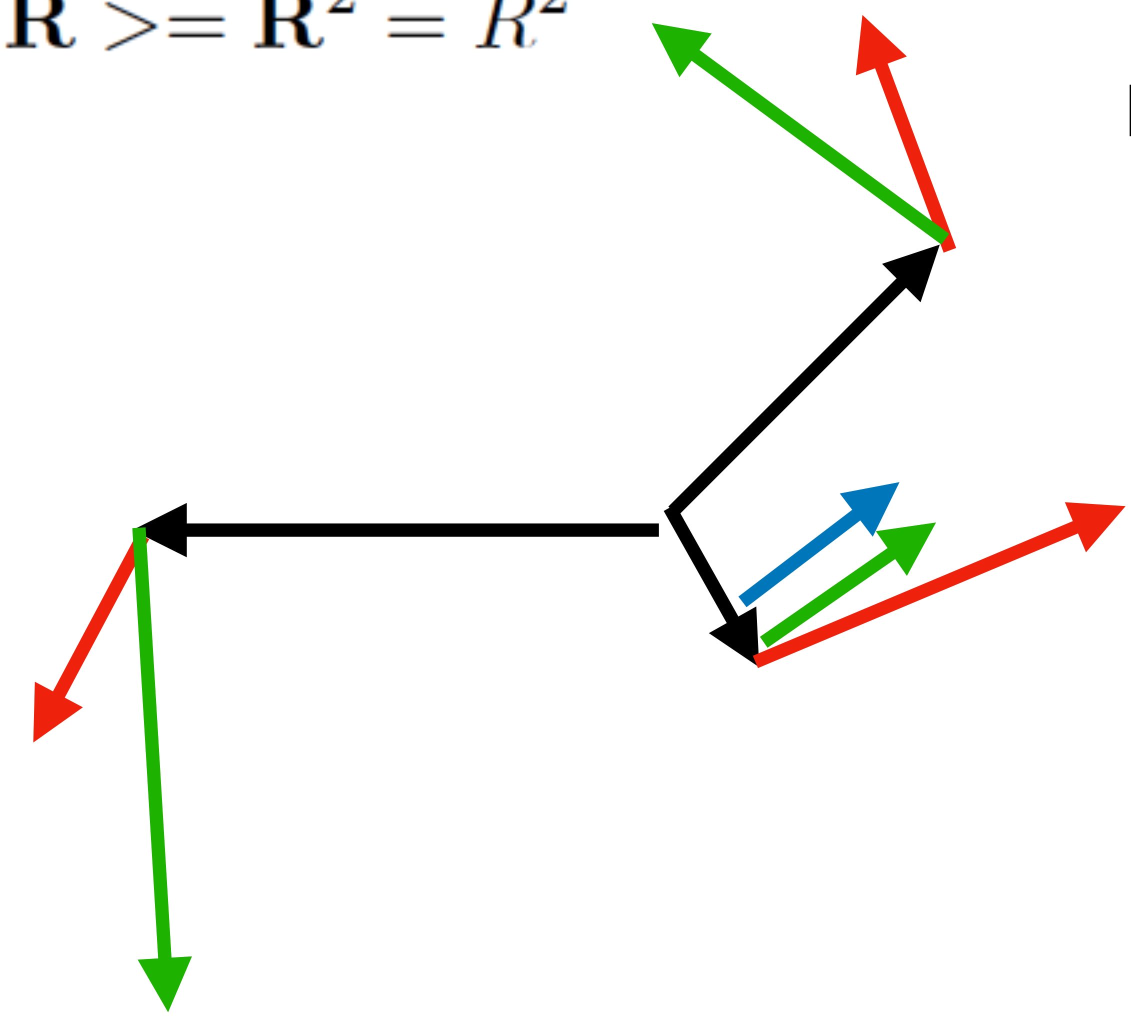

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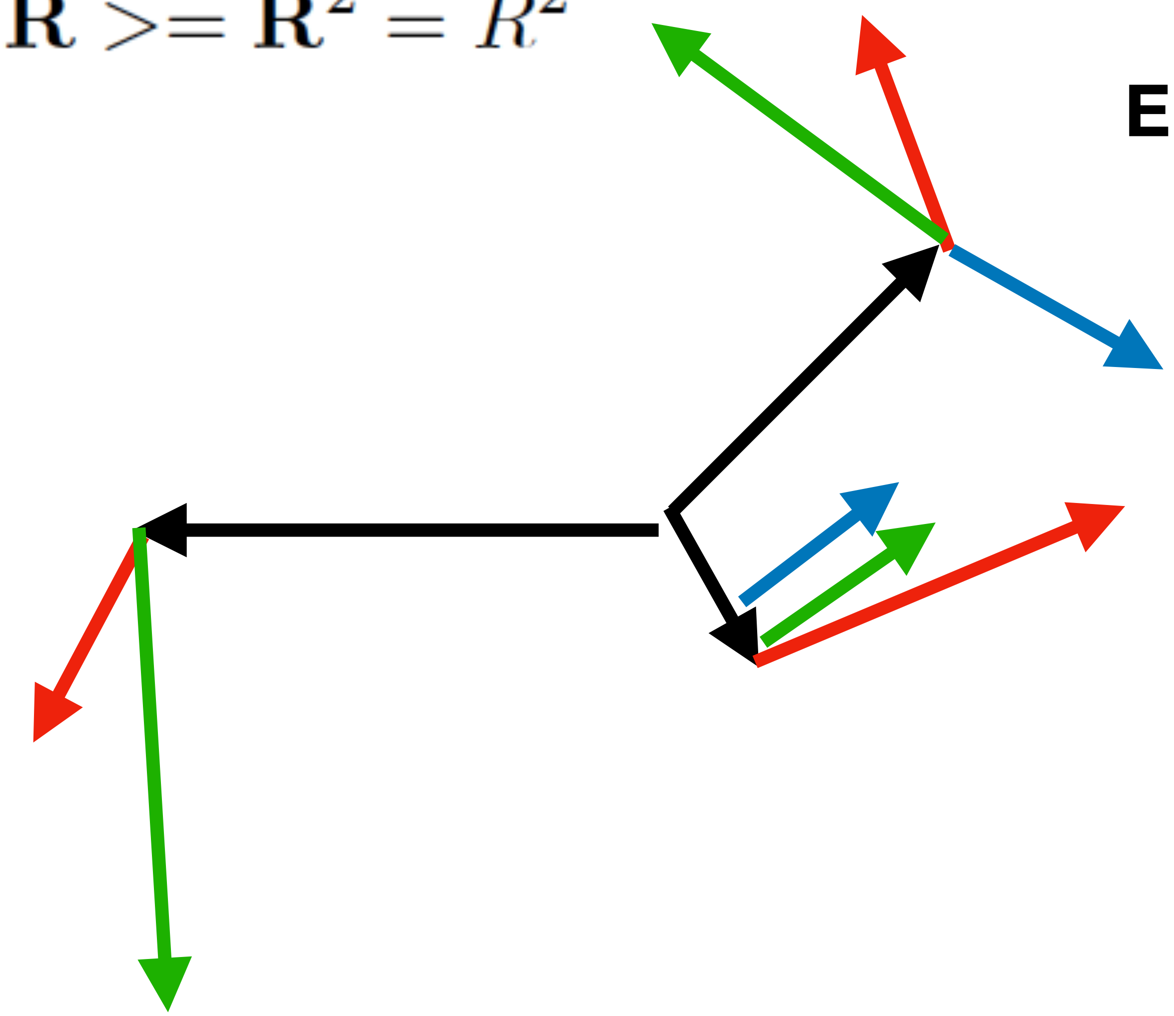

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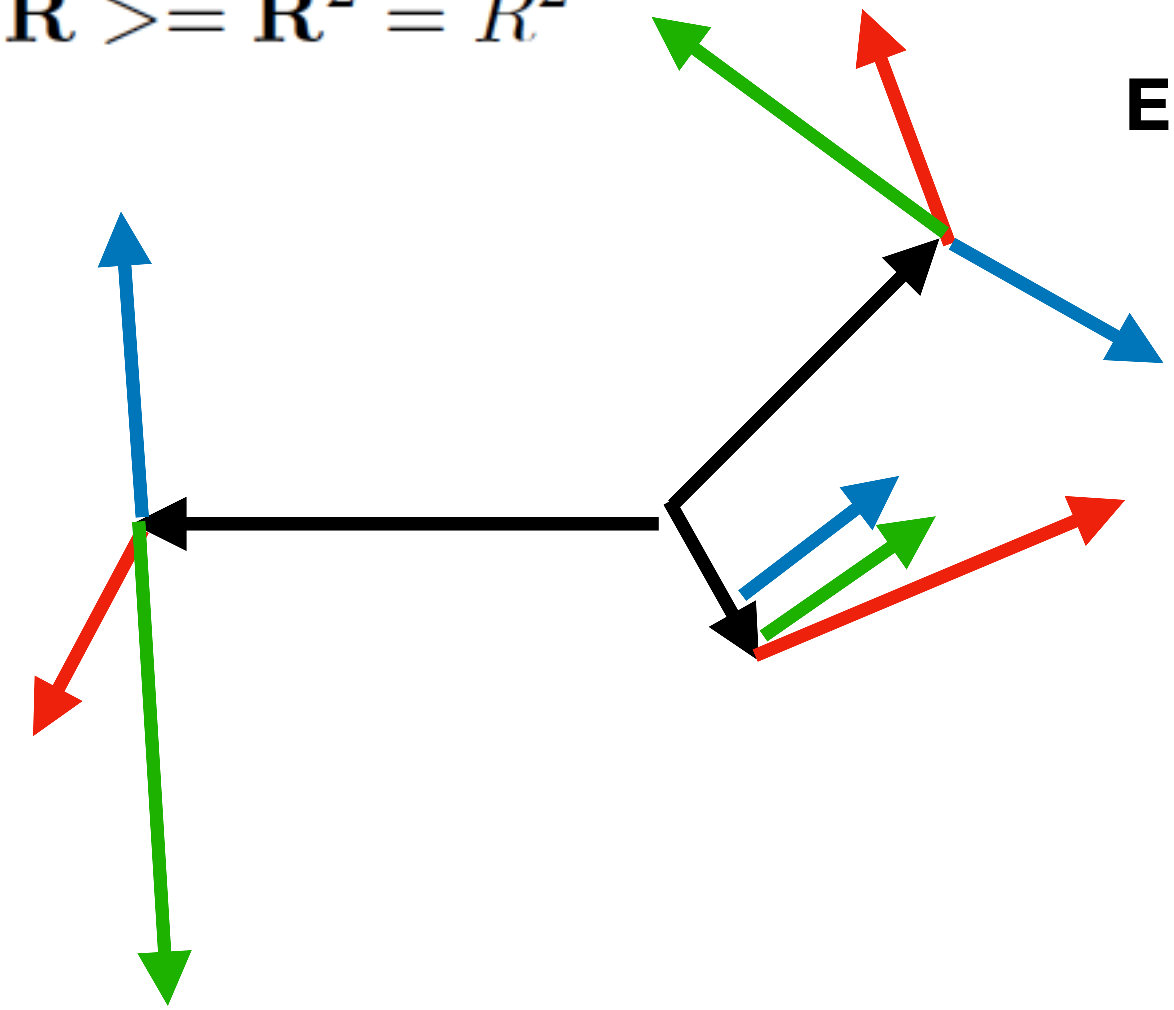

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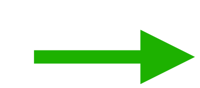
This is each body's rotational velocity minus the 

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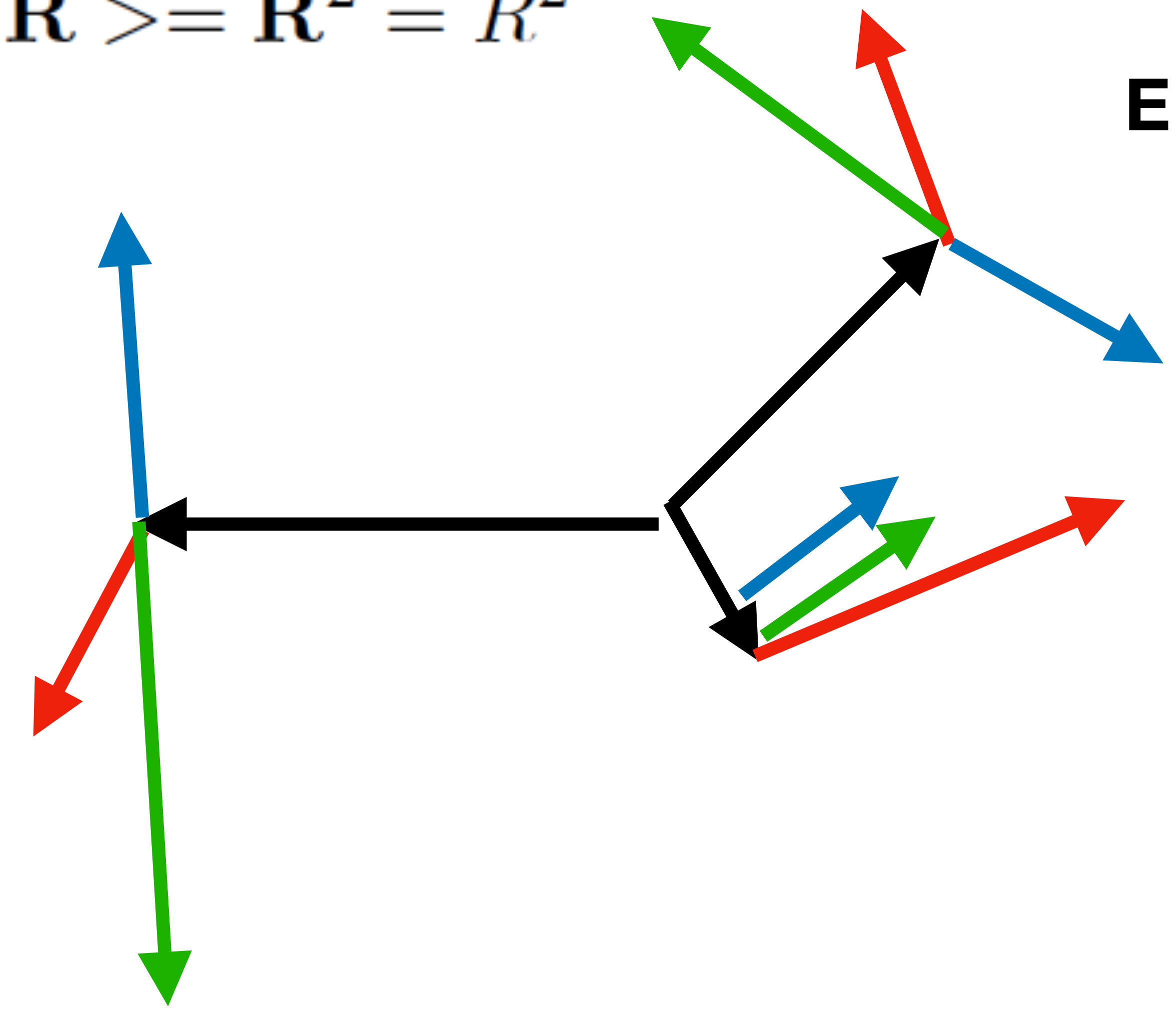

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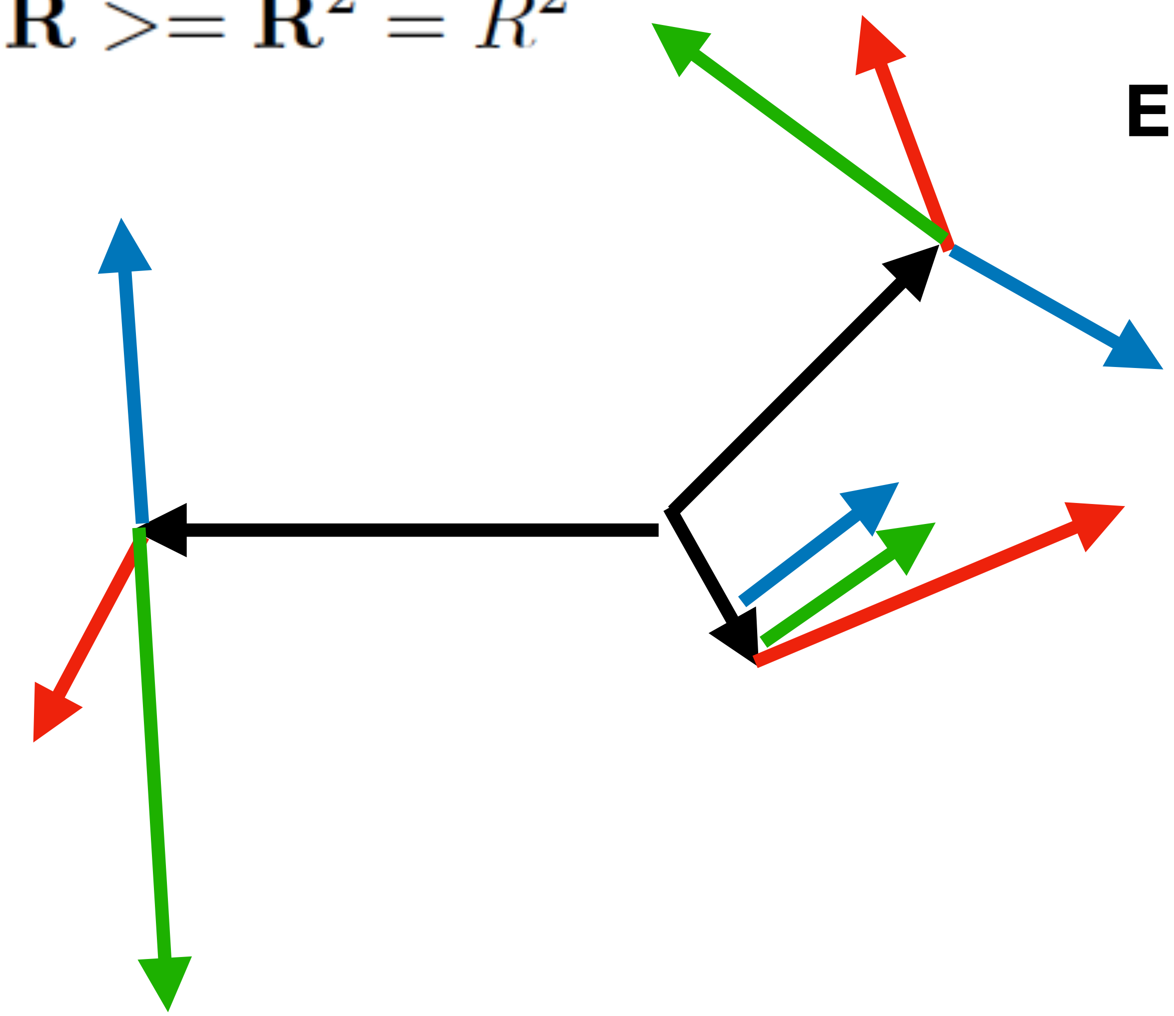

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The green and blue vectors are orthogonal!

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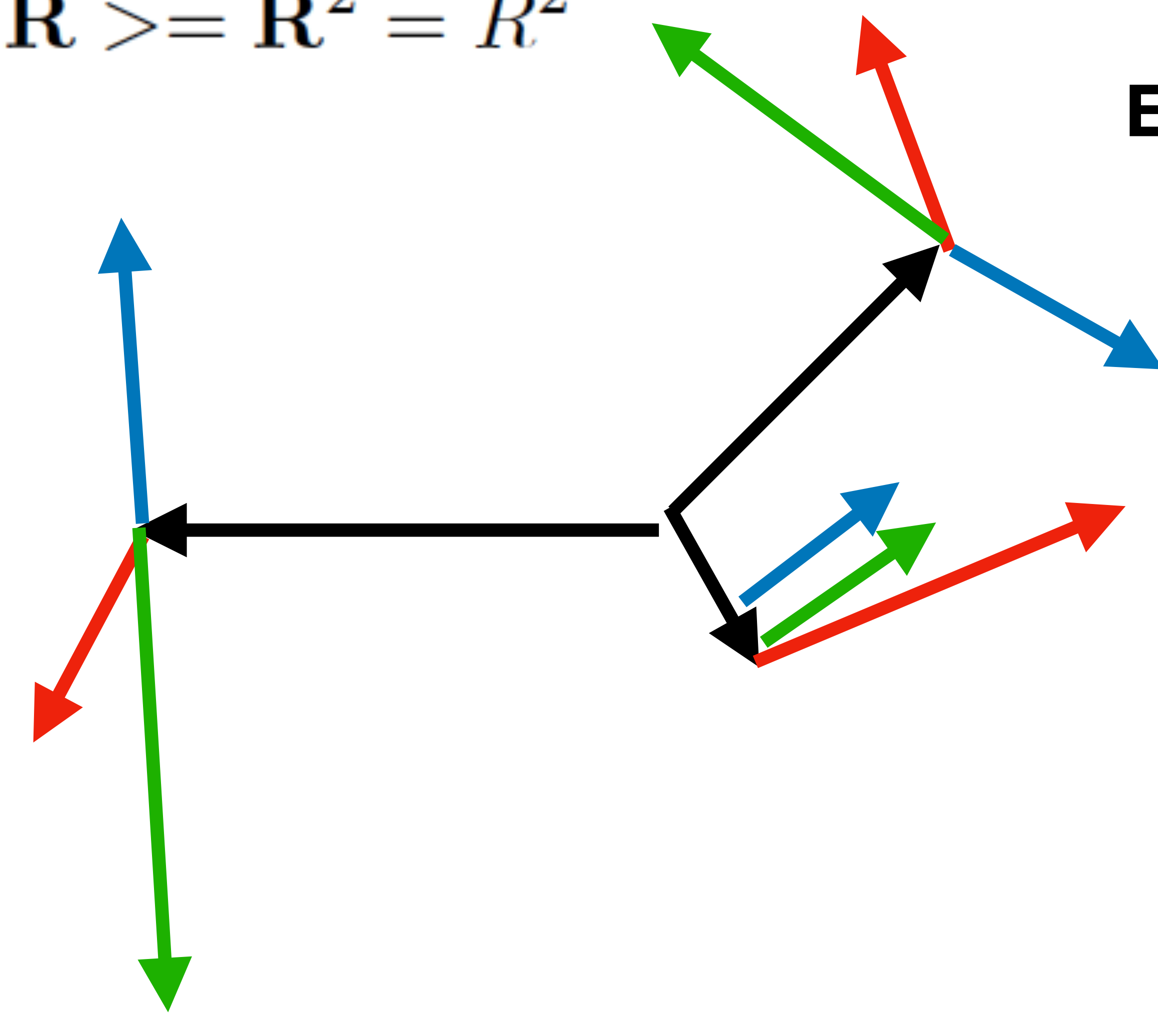

Theorem:
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$$\sum_{j=1}^{n} m_j r_j w_{j,rot,conf} = 0$$

First find what should be each body's rotational velocity so that the configuration remains unchanged.
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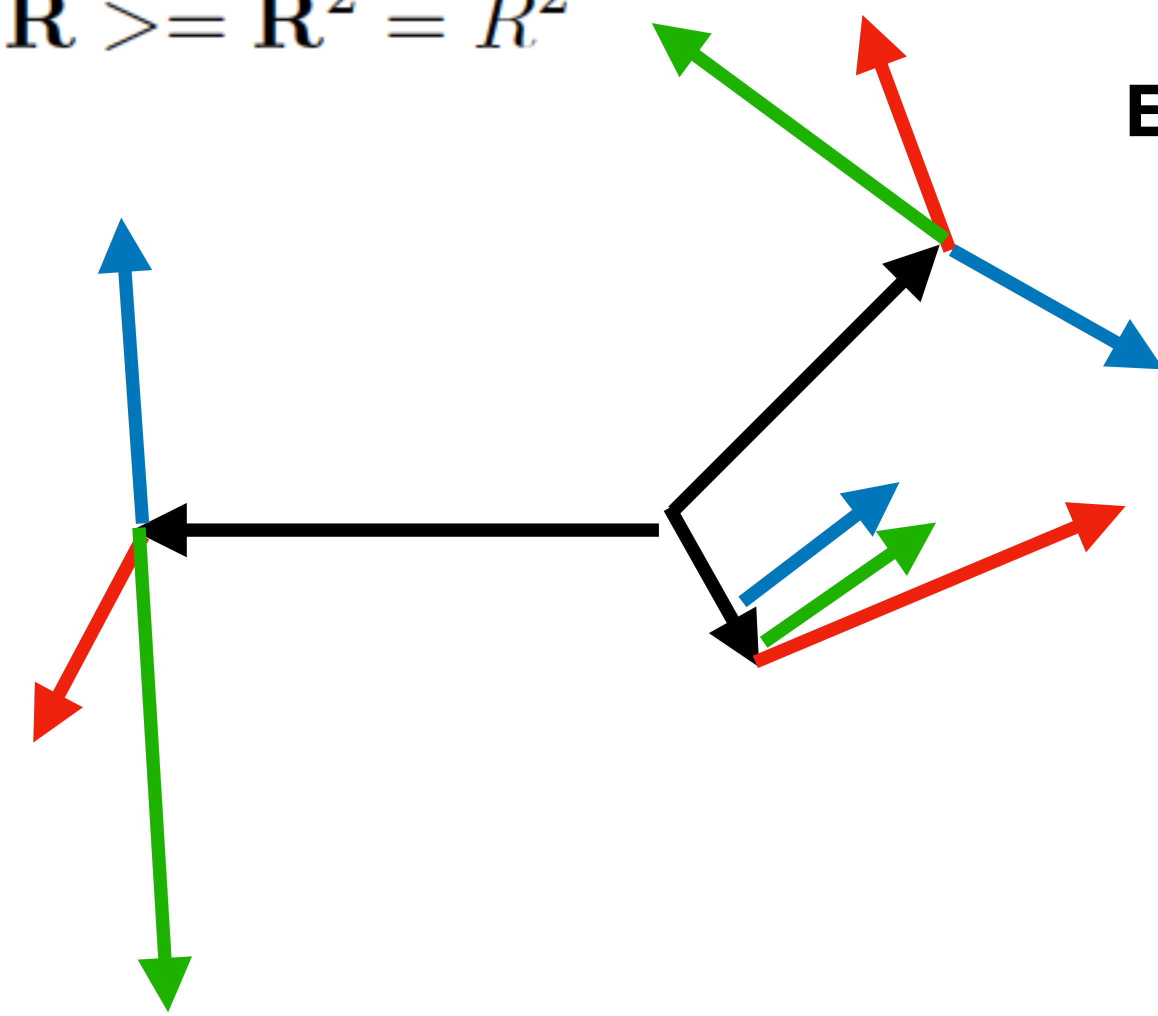

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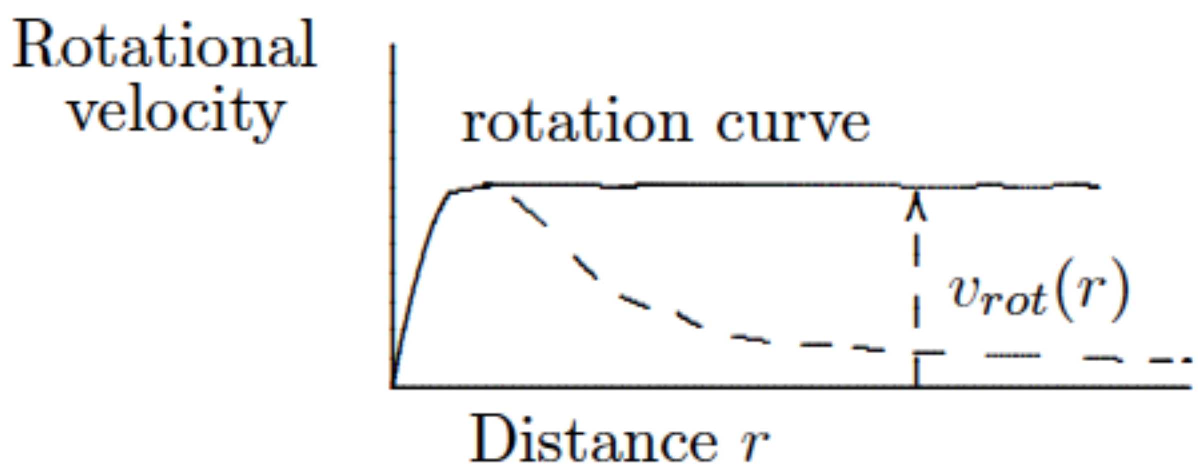
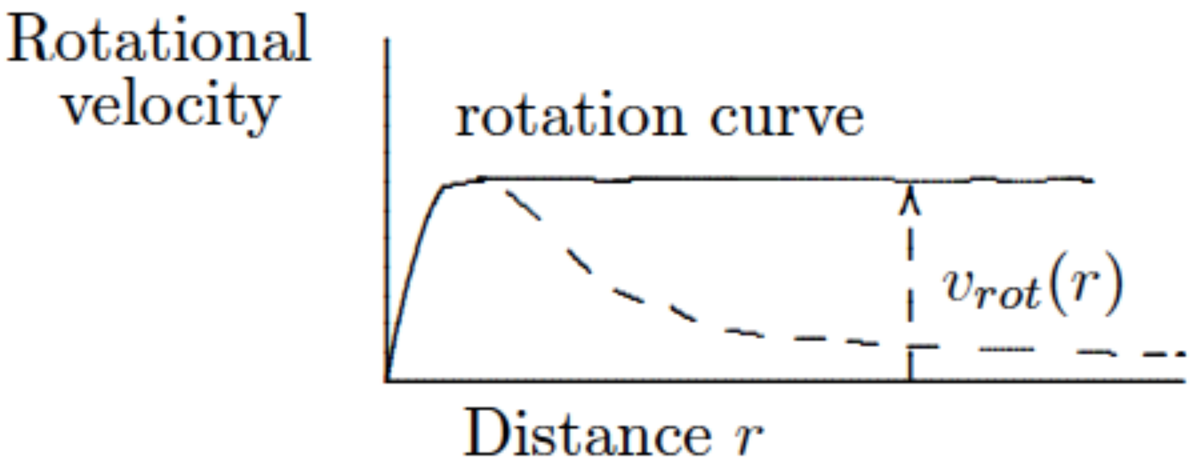
This is each body's rotational velocity minus the

Galaxy rotation problem

Galaxy rotation problem



Galaxy rotation problem



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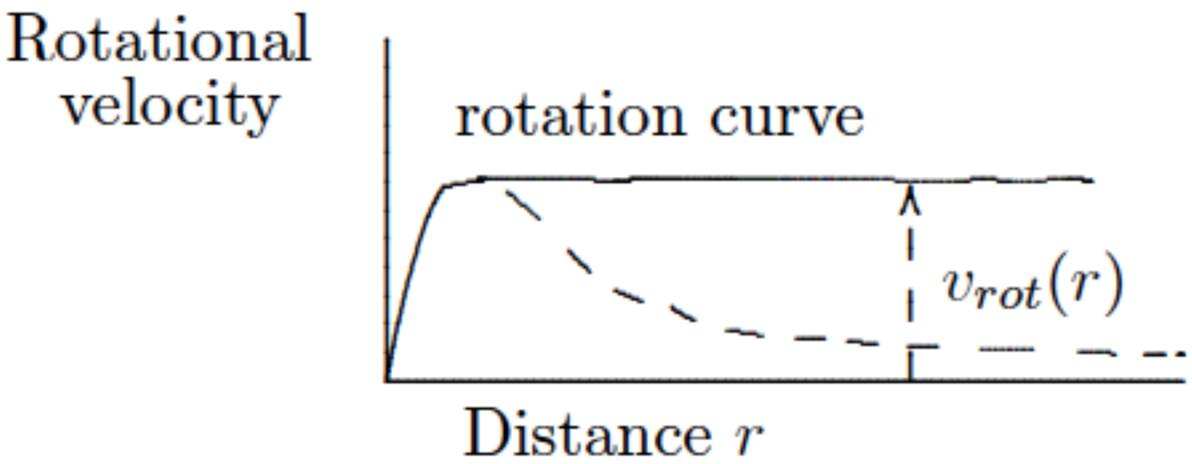
Galaxy rotation problem



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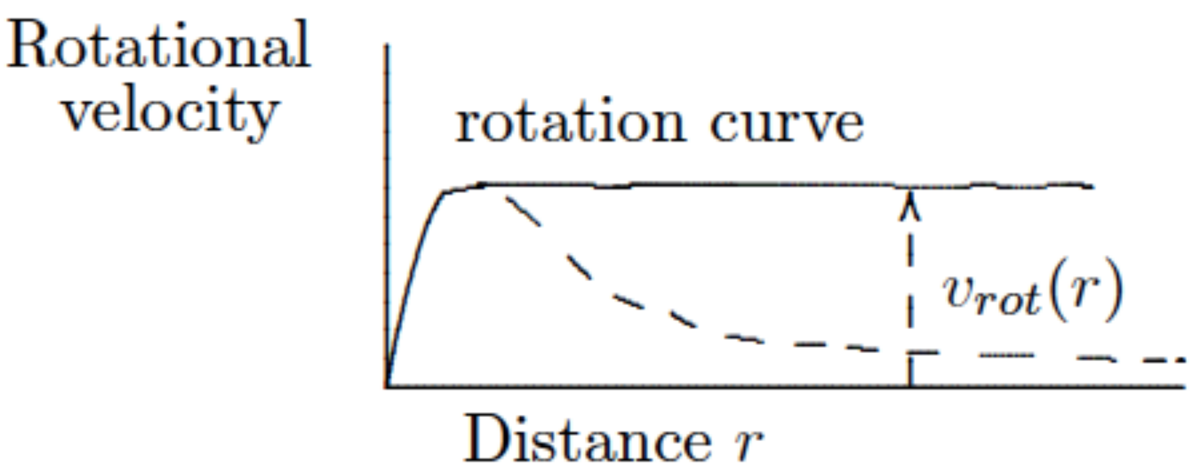

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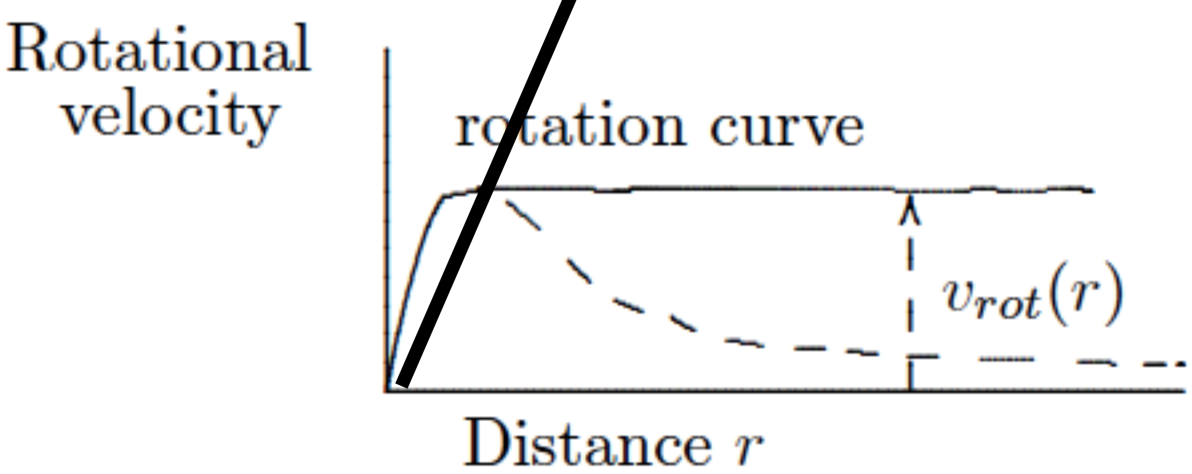

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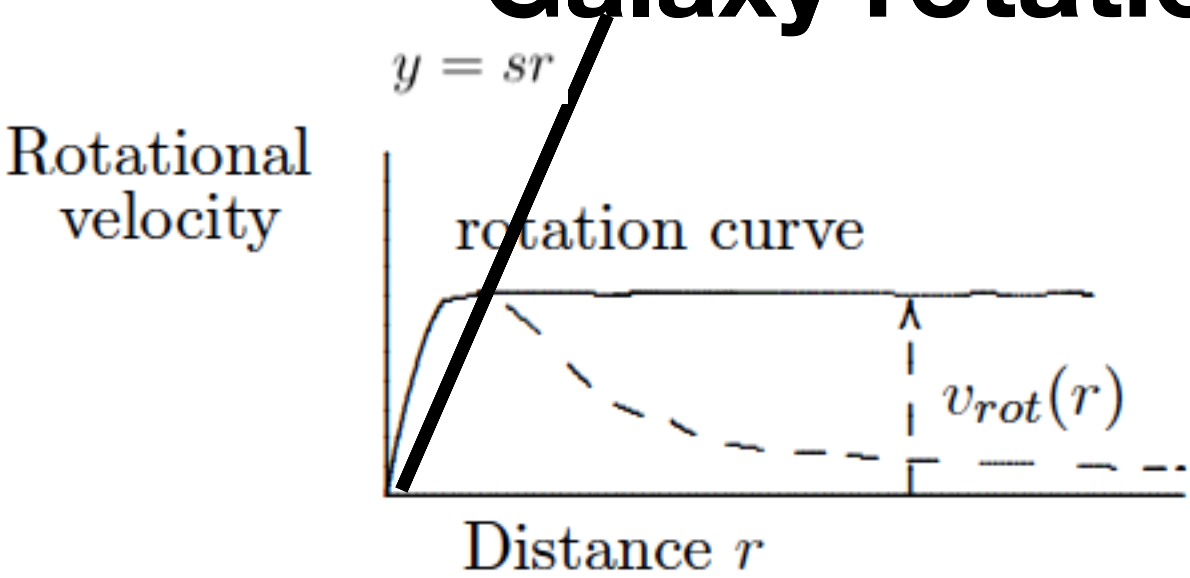

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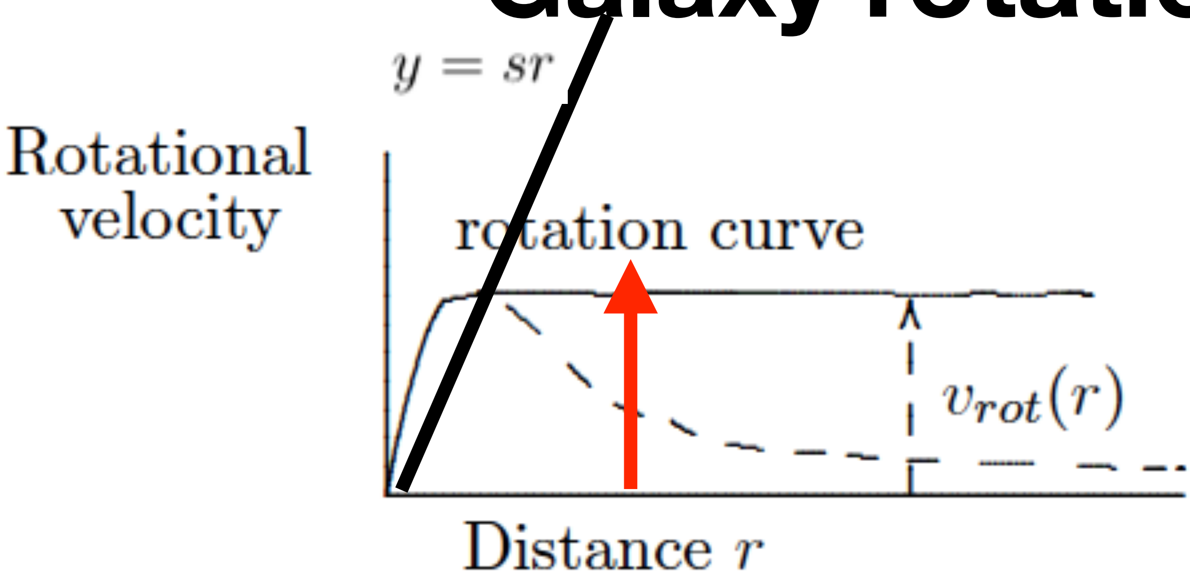

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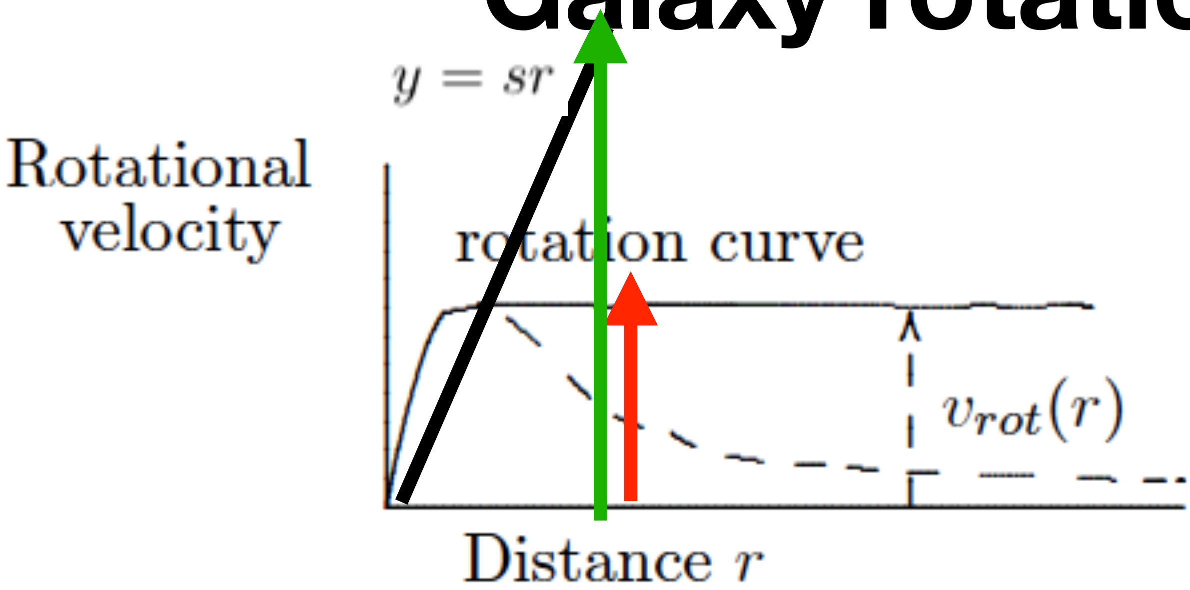

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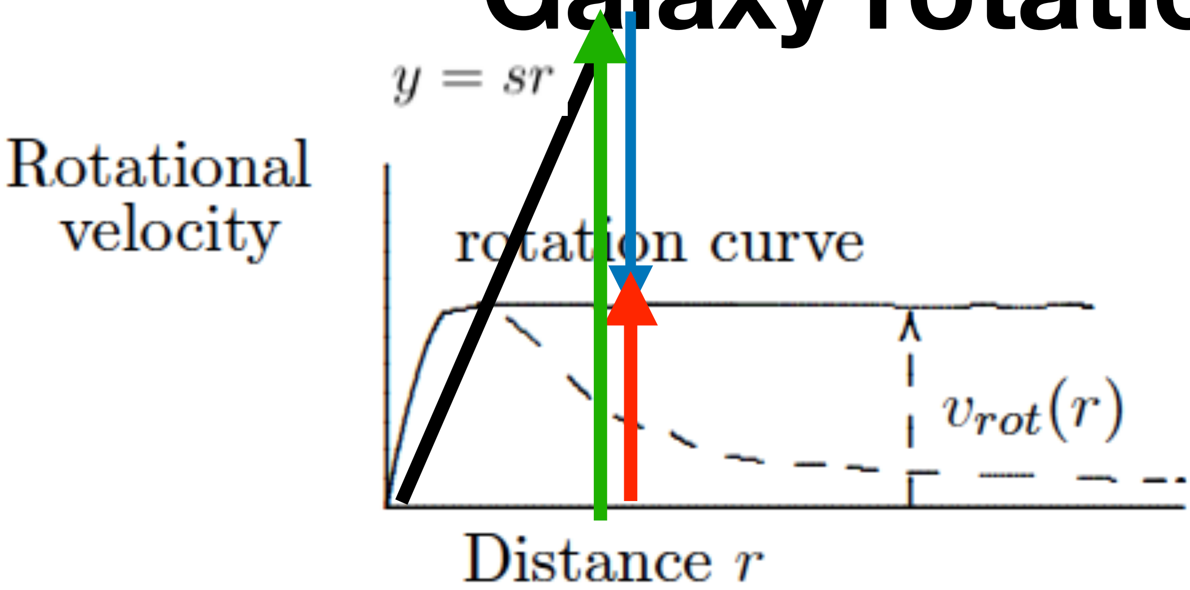

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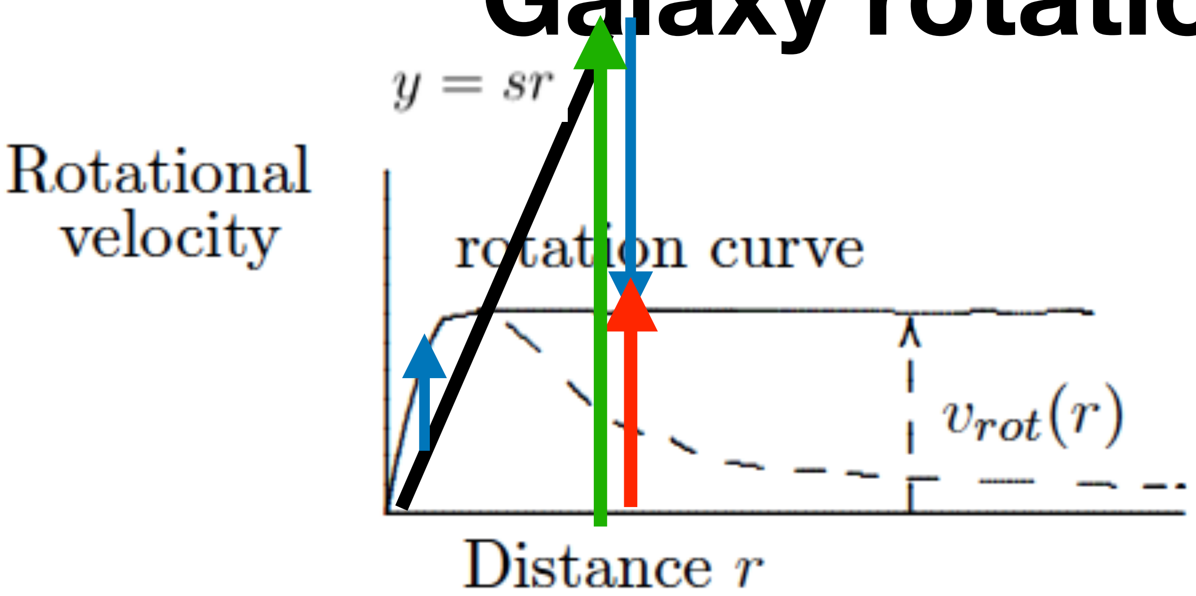

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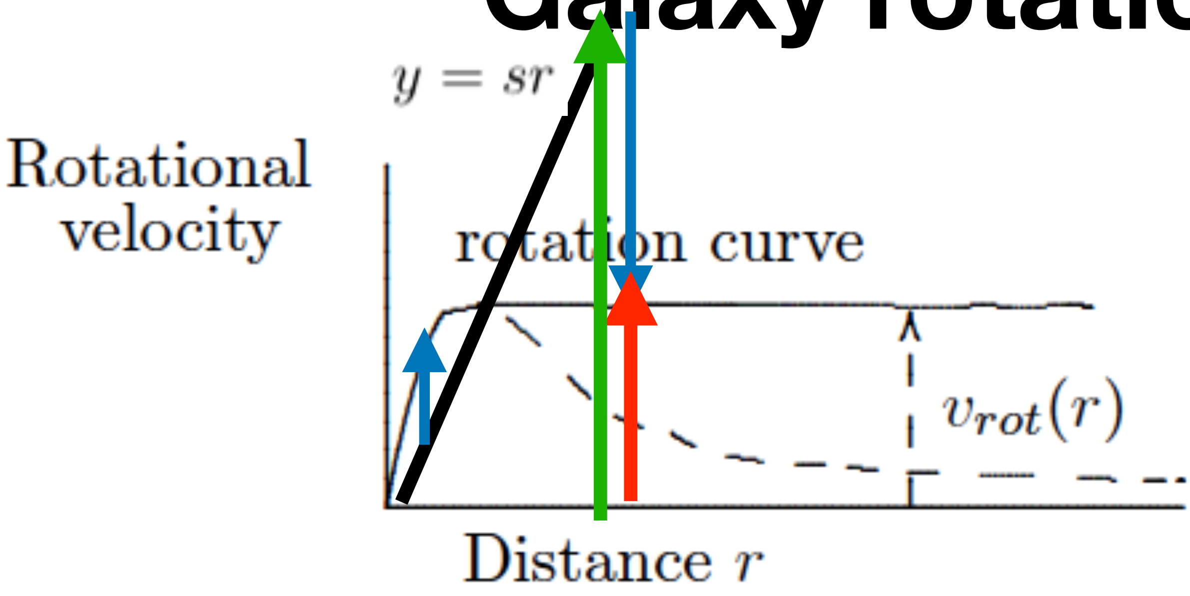

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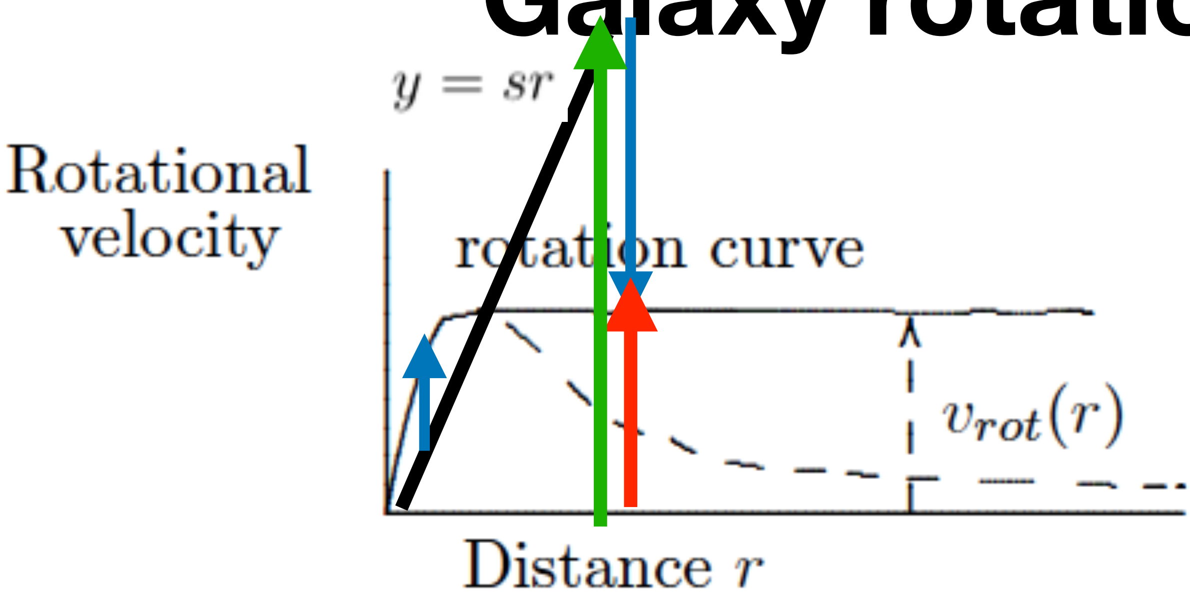

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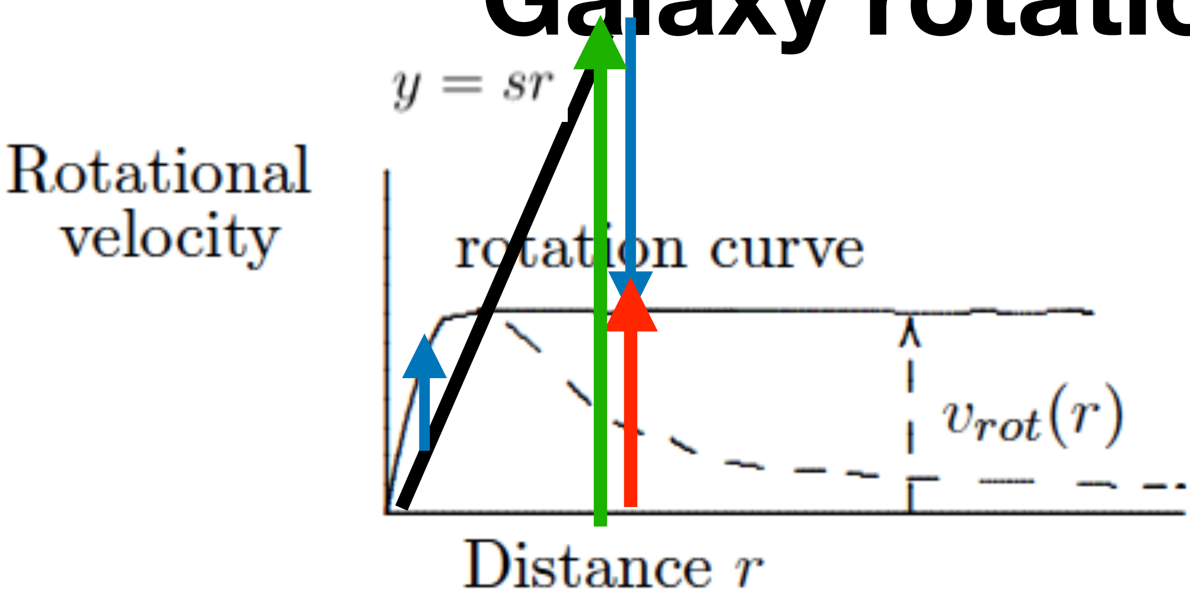

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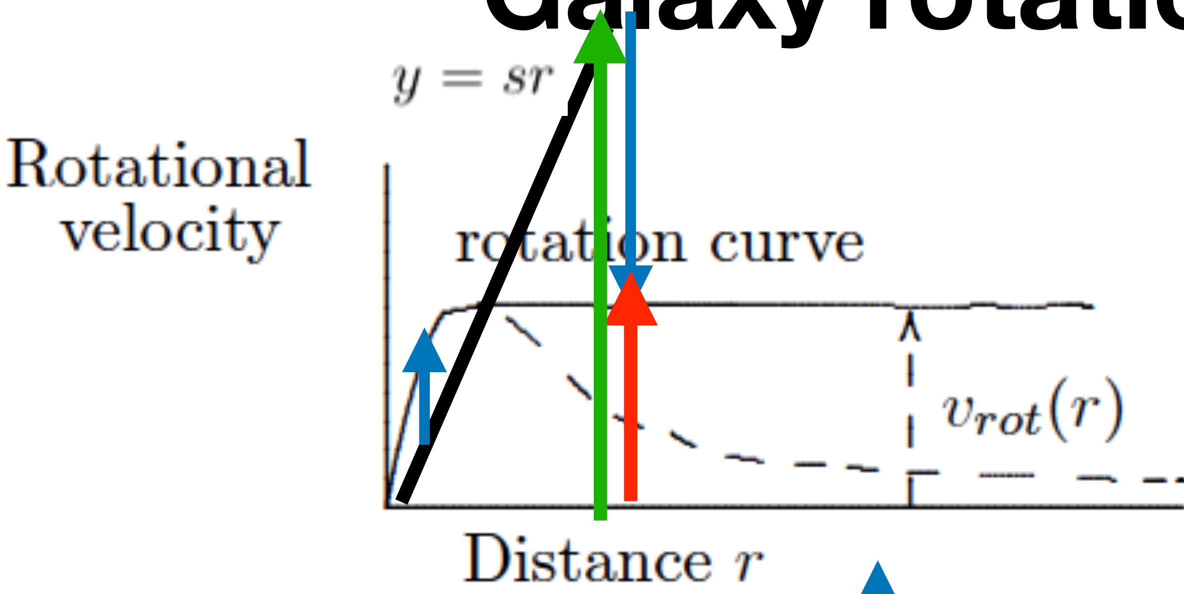

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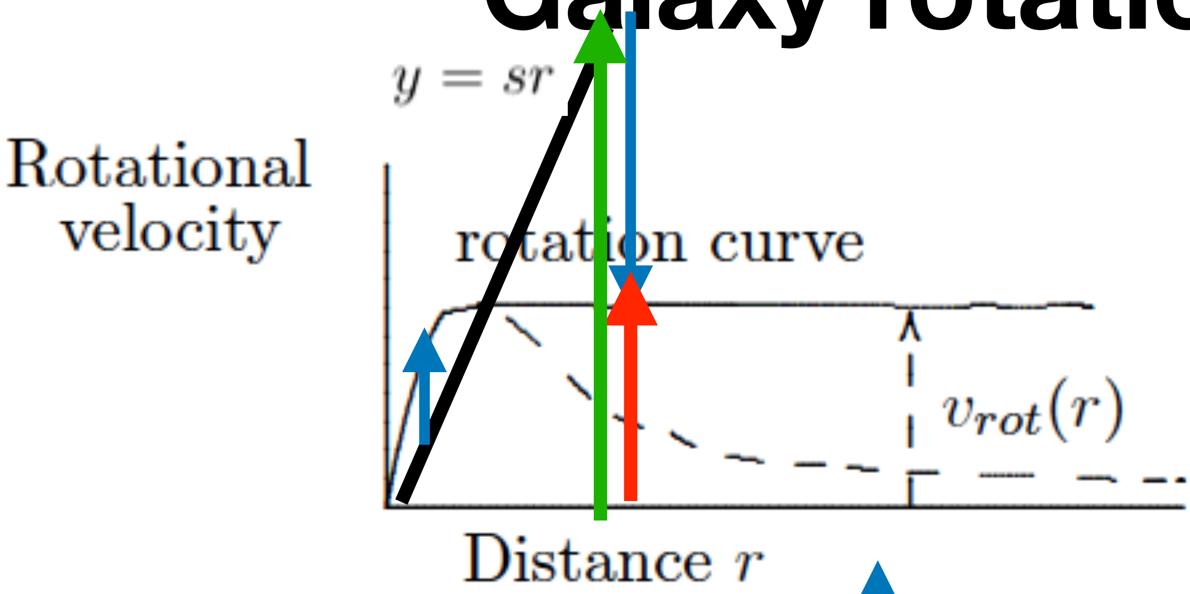

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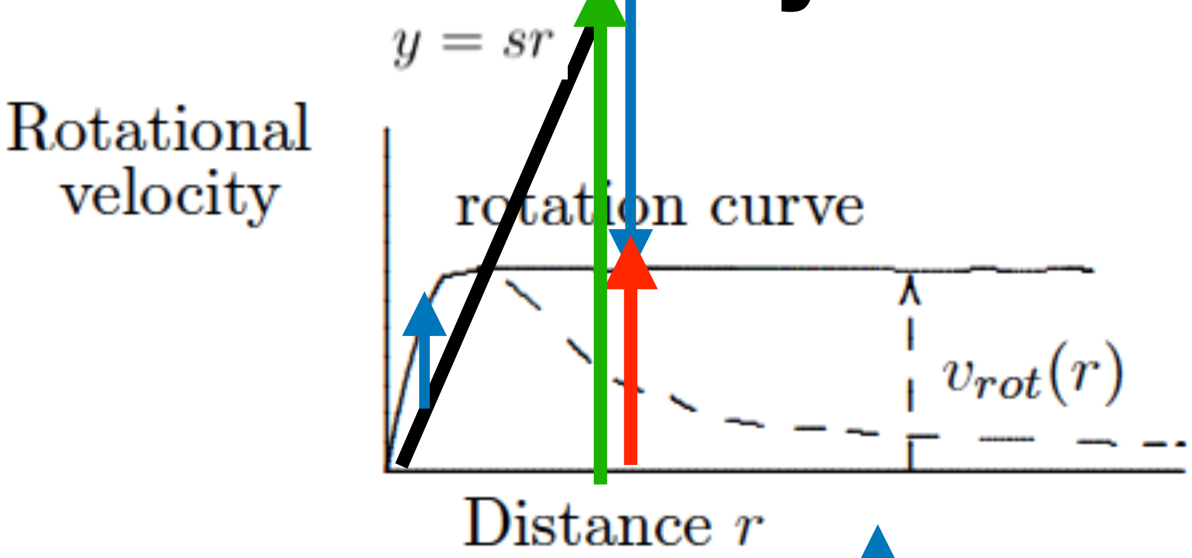

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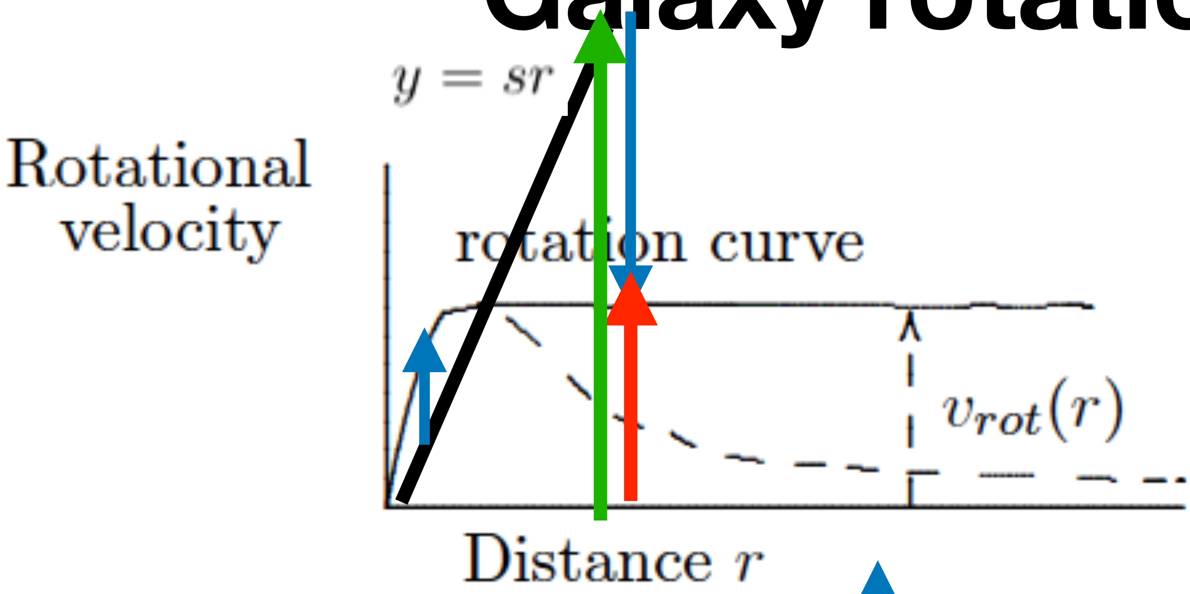
Theorem

Galaxy rotation problem



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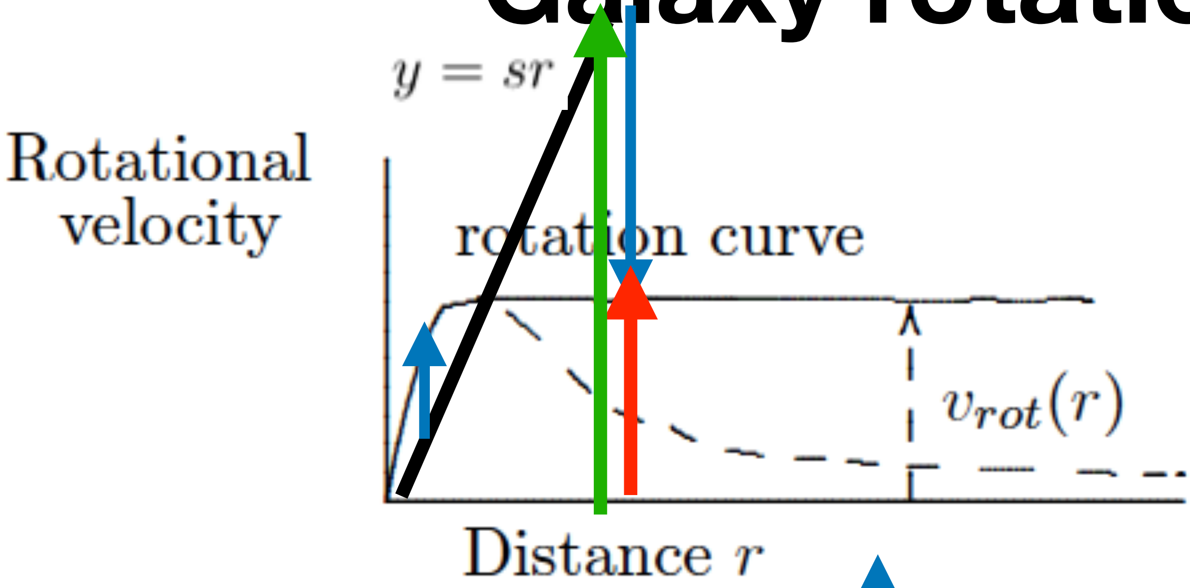
Theorem
For s defined defined by given values and a rotation curve, another rotation curve cannot be smaller in any region with positive masses

Galaxy rotation problem



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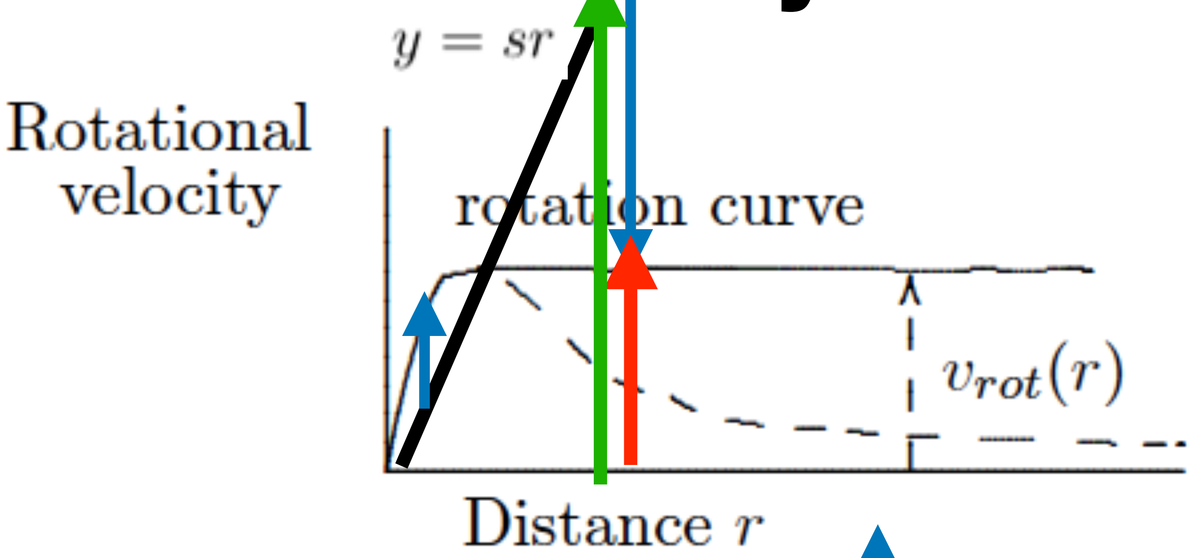
Final message

Galaxy rotation problem



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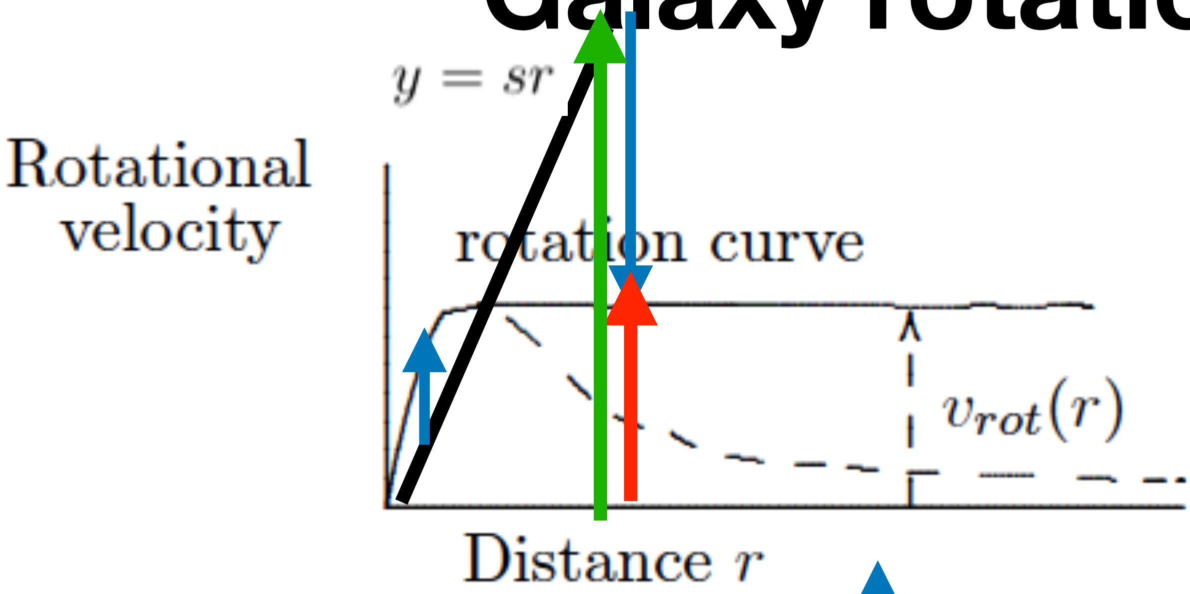
The area of celestial mechanics has a lot to offer to concerns from astronomy.

Galaxy rotation problem



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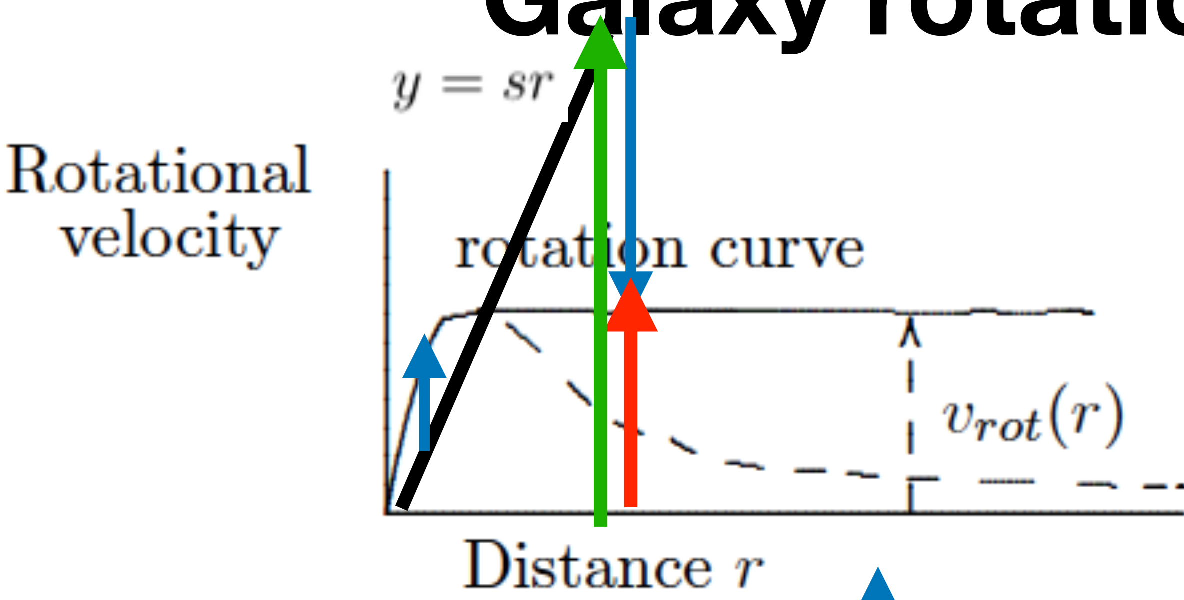

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Thanks for your attention!