Large scale behaviour of interacting diffusions: from stochastic control to functional inequalities

Book of abstracts

September, 18–20, 2024

Preface

The goal of the workshop is to discuss recent advances on the study of asymptotic properties of interacting particle systems, with a special emphasis on controlled systems. Topics of interest include but are not limited to propagation of chaos, infinite dimensional Hamilton-Jacobi equations and ergodicity. We also aim at fostering discussions on the interplay between control methods and other emerging techniques to establish functional inequalities.

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Abstracts list

A variational approach to the spatial Smoluchowski equation

20 September
9:30 - 10:20

Luisa Andreis

Politecnico di Milano

Since Smoluchowski introduced his coagulation equation in 1917, research has focused on understanding the solutions to this equation and related coagulation models. In 2000, Norris introduced the cluster coagulation model, extending Smoluchowski's framework to allow particles with properties beyond mass, such as shape or location. In this talk, we focus on particles with mass and spatial location in a Polish space S. We analyze the associated Markovian particle system, where particle pairs merge after random times, with their combined mass and new location determined by a kernel. Using statistical mechanics, we derive a (conditional) large-deviation principle for the particle distribution, identify the rate function, and characterize the minimizer(s). In certain cases, these minimizers solve the spatial version of Smoluchowski equation. This talk is based on a joint work with W. König, H. Langhammer and R.I.A. Patterson (WIAS Berlin).

HJB equations on the set of probability measures

19 September
9:30 - 10:20

Charles Bertucci

IPParis

I will motivate the need to study HJB equations on the set of probability measures on two examples: optimal control and large deviations of some mean field systems. Then, I will focus most of the talk on comparison principles for such type of equations, and the intrinsic difficulties which are associated. The answer I will present relies on the idea that the Wasserstein distance suffices to make the famous doubling of variables argument.

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20 September
12:30 - 13:20

Malliavin calculus for rough stochastic differential equations

Michele Coghi

Università degli studi di Trento

We study rough stochastic differential equations (RSDEs), as introduced in [Hocquet, Friz, Le, 21], as a mean to model McKean-Vlasov diffusions with a common (rough) noise. We show that RSDEs are Malliavin differentiable. We use this to prove existence of a density when the diffusion coefficients satisfy standard ellipticity assumptions. Moreover, when the coefficients are smooth and the diffusion coefficients satisfy a Hörmander condition, the density is shown to be smooth. The key ingredient is to develop a comprehensive theory of linear rough stochastic differential equations, which could be of independent interest.

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18 September
10:20 - 11:10

Stochastic dynamics and the Polchinski equation

Benoît Dagallier

New York University

Abstract: I will discuss a general framework to obtain asymptotic properties of statistical mechanics and field theory models. The basic, well known idea is to build a dynamics that samples from the model and control its long time behaviour. There are many ways to build such a dynamics, the Langevin dynamics being a typical example. In this talk I will introduce another, the Polchinski dynamics, based on renormalisation group ideas. It is also known under the name of stochastic localisation and can equivalently be viewed as a high-dimensional Hamilton-Jacobi equation. The time variable of the Polchinski dynamics represents a certain notion of scale in the model under consideration. I will motivate the construction of the dynamics in the context of statistical mechanics models and explain how it can be used to prove functional inequalities via a generalisation of Bakry and Emery's convexity-based argument. The talk is based on joint work with Roland Bauerschmidt and Thierry Bodineau and the review paper https://arxiv.org/abs/2307.07619 .

Stable Optimal Parameters for Deep and Wide Resnets Trained with Langevin Dynamics

19 September
10:20 - 11:10

Samuel Daudin

Université Paris Cité

Performing regression tasks with deep neural networks can be modeled as an optimal control problem for an ordinary differential equation. We investigate a relaxation of this problem where controls are taken to be probability measures over the parameter space and the cost involves an additional entropy penalization. We are particularly interested in the stability of the optimal parameters – where stability is understood in terms of unique solvability of a certain linearized system of PDEs. We show that, for "a lot" of initial data (in terms of the initial distribution of the features), there is actually a unique stable global minimizer in the control problem. Moreover we prove that the (continuous analog of the) gradient descent with backpropagation converges exponentially fast when initialized nearby a stable minimizer. This is a joint work with François Delarue (Nice).

A coupling approach to the turnpike property in mean field games

19 September
11:40 - 12:30

Katharina Eichinger

IPParis

In this talk we prove the exponential turnpike property for a class of mean field games on R^d. The exponential turnpike property states that optimal trajectories stay exponentially close to a stationary state, called turnpike, if they are far from the initial and final time. Our technique is based on coupling by reflection adapted to controlled processes allowing us to treat controlled dynamics governed by an asymptotically convex potential. This enables us to prove existence and uniqueness of mean field game problems and their ergodic counterpart without monotonicity assumptions on the cost but rather a smallness condition on the dependence of the measure variable, and finally the exponential turnpike property. Based on joint work with Alekos Cecchin, Giovanni Conforti and Alain Durmus.

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20 September
11:40 - 12:30

Coarse correlated equilibria in mean field games

Markus Fischer

Università degli Studi di Padova

In the context of finite horizon mean field games with continuous time dynamics driven by additive Wiener noise, we introduce a notion of coarse correlated equilibrium in open-loop strategies. For non-cooperative many-player games, a coarse correlated equilibrium can be seen as a lottery on strategy profiles run according to a publicly known mechanism by a moderator who uses the (non-public) lottery outcomes to tell players in private which strategy to play. Players have to decide in advance whether to pre-commit to the mediator's recommendations or to play without seeing them. We justify our definition by showing that any coarse correlated solution of the mean field game induces approximate coarse correlated equilibria for the underlying N-player games. An existence result for coarse correlated mean field game solutions, not relying on the existence of classical solutions, will be given; an explicitly solvable example will be discussed as well. Joint work with Luciano Campi and Federico Cannerozzi (Università di Milano)

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19 September
15:20 - 16:10

Well-posedness for Hamilton-Jacobi equations for stochastic control problems: A new view on the classical approach using couplings

Richard Kraaij

TU Delft

Stochastic control problems for controlled Markov processes can be infinitesimally characterized using a second order Hamilton-Jacobi-Bellman (HJB) equation. The classical work of Crandall-Ishii-Lions (1992) establishes how to obtain uniqueness of viscosity solutions for controlled diffusion processes, and a collection of recnet works has pushed the estimates to include classes of spatially inhomogeneous controlled Lévy processes. We introduce a new perspective of the classical proof methods in terms of Markovian couplings, enabling the extendion of the applicability to a wider range of Lévy type processes and paving the pay for extensions to new contexts. Based on joint work (in progress) with Serena Della Corte (Delft), Fabian Fuchs (Bielefeld) and Max Nendel (Bielefeld).

Strong propagation of chaos for systems of interacting particles with nearly stable jumps.

19 September
14:30 - 15:20

Elisa Marini

Università degli Studi di Padova

We consider a system of N particles, described by SDEs driven by Poisson random measures, where the coefficients depend on the empirical measure of the system. Every particle jumps with a rate depending on its position. When this happens, all the other particles of the system receive a small random kick which is distributed according to a heavy tailed random variable belonging to the domain of attraction of an α-stable law and suitably scaled. We call these jumps collateral jumps. Moreover, in the bounded variation case, the jumping particle itself undergoes a macroscopic, main jump. Such systems appear in the modeling of large neural networks, such as the human brain. We introduce a suitable coupling to prove that our system has the conditional propagation of chaos property: as $N \to +\infty$, the finite particle system converges to an infinite exchangeable system which obeys a McKean-Vlasov SDE driven by an α-stable process, and particles in the limit system are independent, conditionally on the driving α-stable process.

Approximation of 2D Navier-Stokes equations with vorticity creation at the boundary by stochastic interacting particle systems

18 September
09:30 - 10:20

Mario Maurelli

Università di Pisa

We consider a suitable stochastic interacting particle system in a bounded domain with reflecting boundary, including creation of new particles on the boundary prescribed by a given source term. We show that such a particle system approximates 2D Navier-Stokes equations in vorticity form and impermeable boundary, the creation of particles modeling a prescribed vorticity creation at the boundary. Joint work with Francesco Grotto and Eliseo Luongo.

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18 September
11:40 - 12:30

Local convergence rates for Wasserstein gradient flows

Pierre Monmarché
Sorbonne Université

For the granular media equation, or other Wasserstein gradient flows associated to some free energy, the exponential convergence towards a unique global minimizer is known to follow from a suitable non-linear log-Sobolev inequality. However this inequality cannot hold when the free energy admits non-global local minimizers, as in the granular media case in a double-well potential with attractive interaction below the critical temperature. We will discuss how local inequalities can still be established in this context to obtain local convergence rates for initial conditions in a Wasserstein ball centered at local minimizers. From a practical point of view, this implies that the free energy of a particle system which approximates the flow decays quickly below the level of the local minimum, up to some error term. The same analysis works in the kinetic case (i.e. the Vlasov-Fokker-Planck equation). This is a joint work with Julien Reygner.

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19 September
16:40 - 17:30

Propagation of Chaos for the Kac walk

Carlo Orrieri
Università di Pavia

In the talk we discuss a quantitative version of chaos propagation for the classical Kac walk. We firstly improve the rate of convergence at time zero on the Kac sphere and subsequently show its propagation in time. At the center of this approach are refined version of local central limit theorems and Wild series. This is a joint project with Emanuele Dolera (UniPv)

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18 September
14:30 - 15:20

Gradient Flows for Statistical Computation - Trends and Trajectories

Sam Power
University of Bristol

Sam Power (University of Bristol) When conducting statistical estimation and inference, it is relatively commonplace that the computational burden takes the form of an optimisation task. This has long been recognised for tasks of parameter estimation, though there is an increasing recognition that other tasks of interest can be fruitfully interpreted as optimisation tasks over a space of probability measures, or even over some hybrid space involving both parameters and measures. In this talk, I will survey some trends in this area, touching on algorithm analysis and synthesis, identification of suitable problem structures, and highlighting opportunities for future contributions in this fast-growing area.

Dynamical Gibbs principle and associated stochastic control problems

18 September
16:40 - 17:30

Julien Reygner
CERMICS

In statistical physics, the Gibbs principle describes the asymptotic marginal distribution of a particle when the whole system is conditioned on a large deviation of its empirical measure. It lies at the basis of the equivalence between the canonical and microcanonical ensemble. We consider a similar question for the case of diffusion processes with constraints at all times, and show that the effect of conditioning results in an additional drift which encodes a mean-field like interaction. We also discuss the interpretation of this statement in terms of a stochastic control problem, with a constraint in distribution. This is a joint work with Louis-Pierre Chaintron and Giovanni Conforti.

Global contractivity for Langevin dynamics with distribution-dependent forces via couplings

18 September
15:20 - 16:10

Katharina Schuh
Technische Universität Wien

We analyse the long-time behaviour of both the classical second-order Langevin dynamics and the nonlinear second-order Langevin dynamics of McKean-Vlasov type. After giving a short recap on coupling methods we introduce a coupling approach to establish global contraction in an L_1 Wasserstein distance with an explicit dimension-free rate for pairwise weak interactions. For external forces corresponding to aκ-strongly convex potential, a contraction rate of order $O(\sqrt{k})$ is obtained in certain cases. But, the contraction result is not restricted to these external forces. It rather includes multi-well potentials and non-gradient-type external forces as well as non-gradient-type repulsive and attractive interaction forces. The proof is based on a novel distance function which combines two contraction results for large and small distances and uses a coupling construction adjusted to the distance. By applying a componentwise adaptation of the coupling we can prove uniform in time propagation of chaos bounds for the corresponding mean-field particle system.

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20 September
10:20 - 11:10

Weak convergence of the empirical measure for the Keller-Segel model in both subcritical and critical cases.

Yoan Tardy

IPParis

We show the weak convergence, up to extraction of a subsequence, of the empirical measure for the Keller-Segel system of particles in both subcritical and critical cases, for general initial conditions. This particle system consists of N planar Brownian motions interacting through a Coulombian attractive force, which is quite singular. In the subcritical case, a stronger result has been established by Bresch-Jabin-Wang at the price of two simplifications: the whole space $\mathbb{R}^2$ is replaced by a torus and the initial condition is assumed to be regular. In the subcritical case, our proof is fairly straightforward: we use a *two particles* moment argument, which shows that particles do not aggregate in finite time, uniformly in the number of particles. The critical case requires more work.
