New advances on Hawkes processes for a better risk quantification

- WORKSHOP -

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Torre Archimede

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Room 2BC30

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Book of Abstracts

1. Yousra CHERKAOUI TANGI (CREST and Milliman, Paris, FR)

Title: Calibration Techniques for Marked Hawkes Processes with External Excitation in Cyber Attack Frequency Modeling

Abstract: Modeling the frequency and propagation of cyber attacks is essential in the insurance industry, as understanding the dynamics of contagion enables precise estimation of risk exposure and the development of effective mitigation strategies. Hawkes processes have been widely adopted in the literature for capturing the spread of such attacks due to their ability to model self-exciting events. However, standard linear Hawkes models typically
assume a constant baseline intensity and deterministic impact of each attack, overlooking the inherent variability observed in real-world scenarios. To address this limitation, we extend the Hawkes process framework in two directions: first, by adding external excitation to account for the arrival of cyber vulnerabilities that may trigger cyber attacks and second, by incorporating stochastic, unobservable marks to account for the random impact of each event. The marks indicate the level of contagiousness associated with each attack since not all cyber attacks have the same potential for contagiousness.
In this study, we develop and calibrate a marked Hawkes process tailored for modeling cyber attack frequency within the insurance context. We first present a general model and derive expectation calculations for two distinct kernel types: exponential and erlang. Focusing on the calibration of this process, we explore different estimation methodologies under different observation scenarios and nature of the marks, whether stochastic or deterministic. We compare two maximum likelihood estimation calibration methods: a deterministic approach based on the Hawkes likelihood, where the impact of the marks is replaced with their expectation, and a Monte Carlo based method. Through simulations across low, medium, and high marks variance scenarios, we demonstrate that while the deterministic method performs well under low variance conditions, Monte Carlo based methods offer superior accuracy in high variance settings. Our findings highlight the importance of selecting appropriate calibration techniques based on the underlying variability of contagion impacts.

2. Bernardo D'AURIA (University of Padova, Padova, IT)

Control for cyber-risk mitigation

Title: A stochastic model in Cybersecurity to mitigate the impact of cascading Cyberattacks

Abstract: Cyberattacks are an ever-growing threat in modern organizations, characterized by their unpredictability, clustering tendencies, and significant potential for cascading impacts. This talk explores the application of Hawkes processes, a self-exciting stochastic model traditionally used in fields such as seismology and finance, to the domain of cybersecurity as a model for the intensity of cyberattacks, such as to capture key dynamics such as escalation and clustering effects. Assuming that attacks become detectable when their intensity surpasses a certain threshold, we formulate

and solve an optimization problem aimed at minimizing cumulative losses. This approach aims to provide a structured framework for understanding and mitigating the impact of cyber threats.

3. Antoine HERANVAL (INRAE BioSP, Avignon, FR)

Title: Analyzing Contagious Cyber Incidents and Extreme Climate Events with Point Process Models
Abstract: In the first part of this presentation, we introduce a method to classify sectors based on their vulnerability to contagious cyber incidents. By integrating Multivariate Hawkes processes with the Classification and Regression Trees (CART) algorithm, we capture the temporal dynamics and interconnections of these events. Using the likelihood of the Hawkes process as a splitting criterion, the approach emphasizes incident frequency rather than severity. Results are validated through simulations and real-world cyber incident data, offering practical insights for risk management. The second part examines dependencies between different types of extreme climate events and the occurrence of compound events. Each event (e.g., temperature, precipitation, wind speed) is characterized by its time of occurrence and attributes such as magnitude, duration, or spatial extent. Using real-world climate datasets, we analyze statistical features like intensity functions and correlation measures to study how events interact across scales. We also propose models on spatial graphs to represent these dependencies, enabling the simulation and analysis of compound event across regions.

4. Benjamin MASSAT (Institut de Mathématique de Toulouse, Toulouse, FR)

Title: Normal approximation of Functionals of Point Processes: Application to Hawkes Processes.

Abstract: One central representation of point processes relies on the Poisson imbedding framework. Leveraging this representation, Stein's method combined with Malliavin calculus is employed to derive explicit upper bounds for the Wasserstein distance between functional point processes and Gaussian distributions. These general bounds are particularly insightful when applied to non-linear Hawkes processes, providing convergence rates under significantly relaxed assumptions compared to existing results in the literature. The general theoretical framework will be presented, with a focus on its application to Hawkes processes, showcasing both the mathematical tools and the improvements achieved in the convergence analysis.

5. Thomas PEYRAT (CREST and Exiom, Paris, FR)

Title: A compound Hawkes process with dependencies for insurance applications

Abstract: In insurance, one of the processes commonly used to model risk is the compound Poisson process. Assuming independence between the counting process (Poisson Process) and the claims (independent and identically distributed random variables), the calculation of the first two cumulants is immediate. However, these assumptions limit the scope of application of this process to certain risks. In order to remove some of these assumptions, we will present a similar model in which the counting process is a Hawkes process whose intensity is impacted by claims. We will also briefly present a methodology for calculating the first two cumulants of this process.

6. Anthony, RÉVEILLAC (Institut de Mathématique de Toulouse, Toulouse, FR)

Title: Poisson imbedding meets the Malliavin calculus: representations of point processes and application to Insurance

Abstract: In this talk we present a representation of point processes using the so-called Poisson imbedding and the Malliavin calculus. We discuss the use of these representations to provide several models in insurance modelling exhibiting clustering effects.