Brief presentation, followed by Q & A session with the audience on the forthcoming book:

Positive Topology.
A new practice in constructive mathematics

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Structures in Foundations of Mathematics

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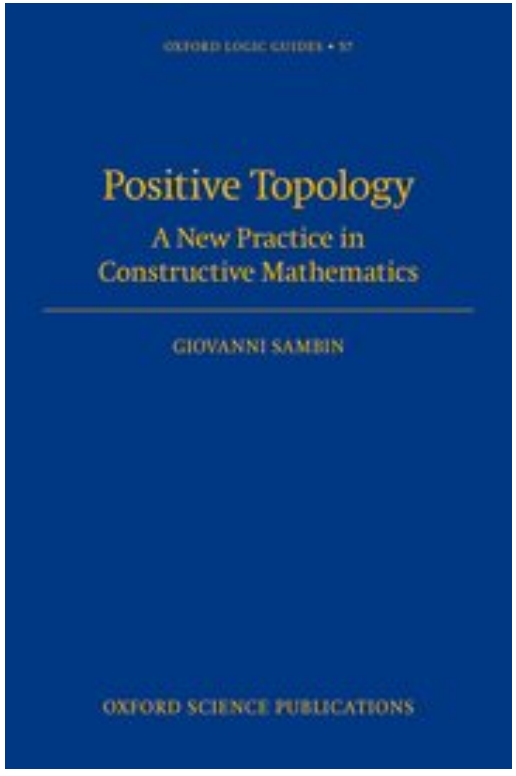

https://global.oup.com/academic/product/positive-topology-9780199232888

Vision

Mathematics is made by humans

It is a tool for survival. A cultural product, like music, economic systems, social rules (e.g on genders), view of the environment, etc. It is a natural and dynamic process.

This vision yields a safer and richer foundation, and lets many mathematical innovations to emerge.

From given objective truth to careful management of acquired information

No `LEM` to preserve positive existence $\exists \neq \neg\forall\neg$,

No `PSA` to preserve inductive generation set $\neq$ collection

No `AC` (and `AC!`) to preserve effectivity of operations function $\neq$ operation

Minimalist Foundation: a way to respect as much information as possible. Benefits: control (proof assistants), consistency (without formal systems), computational content (applications)

Can we do mathematics?

Certainly. To the contrary, mathematics is better than before (and one can feel well within it).

Mathematical novelties

Topology is crucial to gather and express all possible information, both computation (real mathematics) and spatial intuition (ideal mathematics)

Structures underlying topology
Duality int / cl , symmetry *points*/*observables* in a basic pair $\mathcal{X} = (S, \Vdash, S)$.
Topology = module of convergence over applied logic.
Concrete spaces = basic pairs + S is an index-set for a base.

Pointfree topology is a must
Points rarely form a set. Computational content is on the side of observables.
Pointfree topology is a way to maximize it.

- Positive topology $\mathcal{S} = (S, \lhd, \ltimes)$.
- Cover $\lhd$ to define formal opens, positivity relation $\ltimes$ to define formal closed.
- Usually $\lhd$ is generated by induction, and $\ltimes$ by coinduction.

Grothendieck expectation proved embedding of categories: **CSpa** $\hookrightarrow$ **PTop**

Ideal aspects as ideal spaces over a positive topology
Ideal space $\mathcal{I}p(\mathcal{S}) \equiv (IPt(\mathcal{S}), \ni, \mathcal{S})$ over a positive topology $\mathcal{S}$.
Local (hopefully) conservative idealisation to support intuition (as for continuous vision). The categories **ISpa** and **PTop** are **isomorphic**

The natural dynamic paradigm All this could become a new paradigm, in the sense of Kuhn, or perhaps the completion of the classical paradigm started with set theory. Anyway, a lot of future work.