

POD-Galerkin Reduced Basis Methods for Parametric Problems in Computational Fluid Dynamics: State of the Art and Perspectives

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In this talk, we provide the state of the art of Reduced Order Methods (ROM) for parametric Partial Differential Equations (PDEs), and we focus on some perspectives in their current trends and developments, with a special interest in parametric problems arising in offline-online Computational Fluid Dynamics (CFD). Systems modelled by PDEs are depending by several complex parameters in need of being reduced, even before the computational phase in a pre-processing step, in order to reduce parameter space. Efficient parametrizations (random inputs, geometry, physics) are very important to be able to properly address an offline-online decoupling of the computational procedures and to allow competitive computational performances. Current ROM developments in CFD include: a better use of stable high fidelity methods, considering also spectral element method, to enhance the quality of the reduced model too; more efficient sampling techniques to reduce the number of the basis functions, retained as snapshots, as well as the dimension of online systems; the improvements of the certification of accuracy based on residual based error bounds and of the stability factors, as well as the the guarantee of the stability of the approximation with proper space enrichments. For nonlinear systems, also the investigation on bifurcations of parametric solutions are crucial and they may be obtained thanks to a reduced eigenvalue analysis of the linearised operator. All the previous aspects are very important in CFD problems to be able to focus in real time on complex parametric industrial and biomedical flow problems, or even in a control flow setting, and to couple viscous flows -velocity, pressure, as well as thermal field - with a structural field or a porous medium, thus requiring also an efficient reduced parametric treatment of interfaces between different physics. Model flow problems will focus on few benchmark cases in a time-dependent framework, as well as on simple fluid-structure interaction problems. Further examples of applications will be delivered concerning shape optimisation applied to industrial problems.

References

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