Simulation of Shallow-Water Flows on General Terrain

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Shallow Water Equations are classically used as models of environmental fluid dynamics when the horizontal (longitudinal and lateral) components of the flow field are predominant with respect to the vertical components. This is the so called Shallow Water (SW) hypothesis. Application of SW equations for surface water flow are considered where general topographic features are encountered. The presence of a general terrain plays an important role, increasing the geometrical complexity of the fluid streamlines. It is then difficult to accurately identify the negligible velocity component under the SW hypothesis. We propose a derivation of the SW model obtained by integrating the Navier-Stokes (NS) equations along a direction locally normal to the bed topography [1]. This direction is used as an approximation of the "cross-flow" integration path, defined by the condition of being at each point orthogonal to the NS velocities. Along these paths it is possible to precisely state the hydrostatic pressure condition, thus enabling the actual reduction of the 3D NS equations to the 2D SW model. This yields a set of equations in intrinsic coordinates, characterized by non-autonomous fluxes and sources containing the metric induced by the bottom. The resulting SW equations are closely related to the model developed by Bouchout-Westdickemberg [2], and shares similar approximations and limitations in terms of the bottom geometry. For our model, we show second order of approximation of the NS equations with respect to an appropriate "geometric" aspect ratio parameter that includes information on local curvatures. Moreover, we prove that the system admits a conservative energy equation, in case of no dissipation, and it is well balanced (preserves the "lake-at-rest" condition).

We then derive a first-order Godunov type Finite Volume scheme defined intrinsically on the bed surface, and address difficulties and limitations of the proposed scheme, discussing possible future developments. A number of test cases performed over irregular bottom topography are used to show the effectiveness of the numerical approach and to verify the importance of considering the geometric features of the bed topography in the equations. The numerical results obtained show that it is important to take into full consideration the geometrical features of the terrain even for relatively mild and slowly varying curvatures.

References

[1] I. Fent, M. Putti, C. Gregoretti, S. Lanzoni, *Modeling Shallow Water Flows on General Terrains*, Adv. Water Res., (2017).

[2] F. Bouchut, M. Westdickenberg, *Gravity driven shallow water models for arbitrary topography*, Comm. Math. Sci., 2 (2004), 359–389.