VEXPA: a Validated EXPonential Analysis method

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The solution of several physical problems involves the measurement of signals which fall exponentially with time. From these measurements, the task is to determine the complex exponential terms that are composing the signal, often using a variant of Prony's method [1, 2, 3]. It is known that these methods are very susceptible to perturbation of the samples. Moreover, in the case of noisy data, there is no direct way to evaluate the accuracy of the result. We propose a technique that can run on top of every variant of Prony's method and provides additional parameters that validate the output of the analysis.

The developed method is based on a few observations. At first, we analyze how it is possible to improve Prony's stability. This is achieved by decimation of the samples and the possibility to fix the aliasing issue. Second, we exploit a connection between Prony's method and Padé approximation [4, 5] that gives us information about the effect of the noise in Prony's method. At last, thanks to the decimation of the dataset, we are able to perform several independent analyses that are then passed to a cluster detection algorithm. The result of the cluster analysis serves to validate the output.

These observations lead to a new procedure that validates the result of the parametric Prony-based method to which it is added. So, the resulting technique divides the original problem into independent subproblems of smaller dimension. These subproblems can be solved in parallel and have a lower computational cost compared to the original one. In addition, the proposed technique provides a way to detect and remove outliers from the data.

References

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