Integration on manifolds with mapped low-discrepancy points and greedy minimal k_s-energy points

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To integrate with the Quasi-Monte Carlo method (qMCM) on two-dimensional manifolds we consider two sets of points.

The first is the set of mapped low-discrepancy sequence by a *measure preserving map*. Low-discrepancy points are best choice to integrate functions through qMCM in the unit cube $[0,1]^d$ but to use them to integrate functions on a manifold we need to preserve their uniformity with respect to the Lebesgue measure.

The second is the *greedy minimal Riesz s-energy points* extracted from a suitable discretization of the manifold.

We chose greedy minimal energy points since thanks to the *Poppy-seed Bagel Theorem* [1] we know that the class of points with minimal Riesz s-energy, under suitable assumptions, are asymptotically uniformly distributed with respect to the normalized Hausdorff measure.

On the other hand, we do not know if the greedy extraction produce a set of points that are a good choice to integrate functions with the qMCM on manifolds.

Through theoretical considerations, by showing some properties of these points and by numerical experiments, we attempt to answer to these questions.

References

[1] D. P. Hardin and E. B. Saff *Minimal Riesz energy point configurations for rectifiable d-dimensional manifolds., Adv. Math., vol. 193, no. 1, pp. 174-204, 2005.*