Biologically inspired formulation of Optimal Transport Problems

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The Optimal Transport Problem (OTP) is an expanding area of mathematics that looks for the minimal-cost resource reallocation. Within this theory, the Congested Transport Problem (CTP) and the Branched Transport Problem (BTP) studies mass reallocation strategies where mass concentration along the transport paths is either penalized or encouraged [4]. Only recently, numerical methods for the solution of the OTP have been proposed.

In [1, 2] we proposed a model coupling a diffusion equation with ODE imposing a transient dynamics to the diffusion coefficient. These equations describe a infinite-dimensional extension of biologically inspired model introduced in [5], which can be efficiently solved numerically [2, 3]. Numerical evidence and partial mathematical results suggest that the long-time solution of this coupled system approaches the solution of the CTP, the classical OTP with Euclidean distance and, for certain parameter-choice, the singular and fractal-like structures typically emerging from BT problems.

We present the theoretical and numerical evidences supporting our claims, together with some applications of the model to real-world problems, such as the formation of the river basin network. Moreover, we present how simple modification of the model leads to the formation of loops in complex networks, another crucial but not completely understood characteristic of natural network.

References

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