Application of Entropy Viscosity Methods for Shallow Water Flows

Manuel Quezada de Luna[1], Matthew Farthing[1], Jean-Luc Guermond[2], Christopher Kees[1], Casey Miller[3], Bojan Popov[2], and Timothy Weigand[3]

[1] *USACE Engineer Research and Development Center.*
[2] *Texas A&M University Department of Mathematics.*
[3] *University of North Carolina Department of Environmental Sciences and Engineering*

The two-dimensional, depth-averaged shallow water equations are a well-studied system of hyperbolic PDEs. They are used throughout science and engineering to model free-surface flows in systems where vertical scales are assumed to be small with respect to horizontal length scales [2]. Despite this widespread use, their numerical solution remains a challenge in many cases due to the presence of sharp fronts, non-trivial interactions with surface topography and bathymetry, as well as wetting and drying [5]. This is particularly true for continuous Galerkin finite element approximations.

Here, we will present an approximation for the shallow water equations based on recently developed entropy viscosity techniques [4, 1]. The approach fits naturally into a continuous Galerkin finite element framework, is second order, well balanced, and positivity preserving. In addition to computational results benchmarking its performance for several well-known test problems, we will discuss some of the broader challenges of bringing advanced numerical methods and computational tools into practice within government (and possibly other) large organizations. Finally, we will present some early steps to extend the entropy viscosity approach to new formulations of density dependent flow and transport [6] as well as generalized shallow water models that account rigorously for surface geometry in areas of steep slope or high curvature [3].

References

[1] J.-L. Azerad, P. Guermond and B. Popov, *Well-balanced second-order approximation of the shallow water equation with continuous finite elements*, SIAM Journal on Numerical Analysis, 55 (2017) 3203–3224.

[2] M. Brocchini and N. Dodd, *Nonlinear shallow water equation modeling for coastal engineering*, Journal of Waterway, Port, Coastal, and Ocean Engineering, 2, (2008) 104–120.

[3] I. Fent, I. Bachini, and M. Putti. *Modeling shallow water flows on general terrains*, Advances in Water Resources, In Review, (2017).

[4] J.L. Guermond and M. Nazarov. *maximum-principle preserving C^0 finite element method for scalar conservation equations*, Computer Methods in Applied Mechanics and Engineering, 272 (2014) 198–213.

[5] R. Hinkelmann, Q. Liang, V. Aizinger, and C. Dawson, *Editorial: Robust shallow water models*, Environmental Earth Sciences, 74 (2015) 7273–7274.

[6] T. Weigand, P. Schultz, D. Giffen, M. Farthing, A. Crockett, C. Kelley, W. Gray, and C. Miller. *Modeling non-dilute species transport using the thermodynamically constrained averaging theory*, Water Resources Research, In Review (2018).