Recent advancements in preconditioning techniques for large size linear systems on High Performance Computers

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The numerical simulations of real world engineering problems create models with several millions or even billions of degrees of freedom. Most of these simulations are centered on the solution of systems of non-linear equations, that, once linearized, become a sequence of linear systems, whose solution is often the most time-demanding task. Thus, in order to increase the capability of modeling larger cases, it is of paramount importance to exploit the resources of High Performance Computing architectures. In this framework, the development of new algorithms to accelerate the solution of linear systems, i.e. both algorithms and preconditioners for many-core architectures, is a really active research field. Our main focus is algebraic preconditioning and, among the various options, we elect to develop approximate inverses for symmetric and positive definite (SPD) linear systems [1], both as stand-alone preconditioner or smoother for AMG techniques. This choice is mainly supported by the almost perfect parallelism that intrinsically characterizes these algorithms. As basic kernel the Factorized Sparse Approximate Inverse (FSAI) developed in its adaptive form by Janna and Ferronato [2] is selected. Recent developments are i) a robust multilevel approach for SPD problems based on FSAI preconditioning, which eliminates the chance of algorithmic breakdowns independently of the preconditioner sparsity [3] and ii) a novel AMG approach featuring the adaptive FSAI method as a flexible smoother as well as new approaches to adaptively compute the prolongation operator. In this latter work, a new technique to build the prolongation is also presented [4]. In this talk, we describe these two new ideas and show their effectiveness in real world examples.

References

[1] L. Y. Kolotilina, A. Y. Yeremin, *Factorized sparse approximate inverse preconditionings I. Theory*, SIAM Journal on Matrix Analysis and Applications, 14(1) (1993) 45-58.

[2] C. Janna, M. Ferronato, *Adaptive pattern research for block FSAI preconditioning*, SIAM Journal on Scientific Computing, 33(6) (2011) 3357-3380.

[3] A. Franceschini, V. A. Paludetto Magri, M. Ferronato, C. Janna, *A Robust Multilevel Approximate Inverse Preconditioner for Symmetric Positive Definite Matrices*, SIAM Journal on Matrix Analysis and Applications, 39(1) (2018) 123-147.

[4] V. A. Paludetto Magri, A. Franceschini, C. Janna, *A Novel AMG Approach based on Adaptive Smoothing and Prolongation for Ill-conditioned Systems*, submitted to SIAM Journal on Scientific Computing on January, 2018.