Greedy kernel-based function approximation by VKOGA: analysis and applications

Bernard Haasdonk[1] and Gabriele Santin[1]

[1] *Institute of Applied Analysis and Numerical Simulation, University of Stuttgart,*
haasdonk,gabriele.santin@mathematik.uni-stuttgart.de

Approximation of functions in Reproducing Kernel Hilbert Spaces (RKHS) is very attractive, as those spaces naturally cover traditional spline or certain Sobolev-spaces. At the same time, generic data-based approximation algorithms can be formulated and analyzed. In particular meshless (random/scattered) data can be used for generating the approximants. This makes the algorithms less prone to the curse-of-dimensionality from a computational viewpoint and very widely applicable. In the current presentation, we consider the vectorial kernel orthogonal greedy algorithm (VKOGA) [1], which is motivated by aiming at extreme sparseness of an approximate kernel expansion. The resulting approximants then allow very rapid evaluation and are potentially suited for real-time or multi-query settings [2]. From a theoretical viewpoint, convergence rates have been obtained for different versions of the VKOGA [1, 3]. We applied VKOGA in various settings and present results for function approximation in multiscale modelling [4], uncertainty quantification [5] as well as forecasting for implicit ODE integration [6].

References

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