Polynomial and RBF approximation on Lissajous nodes in MPI applications

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Magnetic Particle Imaging (MPI) is a medical imaging technology, currently under development, which is based on the detection of a tracer consisting of superparamagnetic iron oxide (SPIO) nanoparticles through the superimposition of different magnetic fields [4]. The acquisition of the signal is performed following a generated field-free point (FFP) along a suitable sampling trajectory. Among other possible choices, considering Lissajous curves as trajectories provides a series of advantages [5]. For a class of these curves, particular sampling points can be considered. We call such points Lissajous nodes.

In this talk, we first consider Lissajous nodes in order to obtain a polynomial reconstruction of the density of the tracer [3]. The polynomial interpolants on the Lissajous nodes can be expressed as multivariate Chebyshev series, which can be expressed also as Fourier series by a change of coordinates. In applications, many objects are represented by discontinuous functions. In this case, it is well-known that the presence of leap points in the function gives rise to the so called *Gibbs phenomenon*, which causes distortions and oscillations in the image reconstruction. This phenomenon can be treated using classic spectral filters. In addition, in [2] we consider adaptive spectral filters, which take into account the spatial positions of the discontinuities in the image.

In the last part of the talk, we consider a Radial Basis Function (RBF) interpolation approach on Lissajous nodes and we compare it to the polynomial reconstruction method. In this context, we consider Variably Scaled Kernels (VSKs), that turn out to be particularly suitable for the reconstruction of functions having steep gradients or discontinuities [1].

References

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