Generalized block tuned preconditioners for SPD eigensolvers

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Given an $n \times n$ symmetric positive definite (SPD) matrix A and an SPD preconditioner P we propose a new class of generalized block tuned (GBT) preconditioners. These are defined as a p-rank correction of P with the property that arbitrary (positive) parameters $\gamma_1, \dots, \gamma_p$ are eigenvalues of the preconditioned matrix. We propose to employ these GBT preconditioners to accelerate iterative solution of linear systems like $(A - \theta I)\boldsymbol{s} = \boldsymbol{r}$ in the framework of iterative eigensolvers. We give theoretical evidence that a suitable, and effective, choice of the scalars γ_j is able to shift p eigenvalues of $P(A - \theta I)$ very close to one. Numerical experiments on various matrices of very large size [2] shows that the proposed preconditioner is able to yield an almost constant number of iterations, for different eigenpairs, irrespective on the relative separation between consecutive eigenvalues. We also give numerical evidence that the GBT preconditioner is as efficient as the spectral preconditioner [1], and far superior than the latter on matrices with highly clustered eigenvalues, in the acceleration of a number of iterative eigensolvers such as simplified Jacobi-Davidson [4] and the DACG method [3].

References

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