Space and parameter reduction for the groundwater flow equation based on the stochastic moment equations

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Groundwater flow at steady state in a two-dimensional confined aquifer $\Omega \subset \mathbb{R}^2$ is modelled by the following diffusion equation:

$$\begin{cases} \nabla \cdot (T(\mathbf{x})\nabla h(\mathbf{x})) + f(\mathbf{x}) = 0 & \mathbf{x} \in \Omega \\ h(\mathbf{x}) = H(\mathbf{x}) & \mathbf{x} \in \Gamma_D \\ -\mathbf{q}(\mathbf{x}) \cdot \mathbf{n}(\mathbf{x}) = Q(\mathbf{x}) & \mathbf{x} \in \Gamma_N \end{cases} \tag{1}$$

where h is the hydraulic head [L], f is the forcing term [T^{-1}], H represents the fixed hydraulic head at the Dirichlet boundary Γ_D [L], Q represents the assigned flux at the Neumann boundary Γ_N [L T^{-1}], $\mathbf{q}(\mathbf{x}) = -T(\mathbf{x})\nabla h(\mathbf{x})$ is Darcy's flux [L T^{-1}], and $\mathbf{n}$ is the outgoing normal on the boundary. T is described as a second order random field having log-normal distribution ($Y = log(T)$, $Y \sim N(\mu_Y, \mathbf{C}_Y)$). Consequently, the head probability distribution (PD) is typically computed through a large number (N) of Monte Carlo (MC) simulations, solving (1) for different samples of Y.

Reduced order models (ROMs) [1], reduce the system dimension by projecting the solution on a suitable small space of dimension m ($m << n$, where n is the dimension on the full system model - FSM) thus obtaining fast MC simulations. The snapshot technique [2] computes the projection vectors as eigenvectors of the correlation matrix associated with a certain number (N_s, $N_s << N$) of FSM solutions. Here, we propose to replace this empirical computation of the head mean and covariance by the second-order solutions of the stochastic Moment Equations (MEs, [3]) associated with (1). MEs are deterministic equations whose unknowns are the moments of the head PD. Their numerical solution is characterized by better accuracy with respect the solution obtained relying solely on a limited number of MC realizations, and can be directly employed in the definition of the ROM matrices. To further decrease the computational cost of ROMs, the transmissivity field T is approximated by its truncated Karhunen-Loève (KL) representation, enabling the offline computation of the ROM matrices and significantly reducing the online costs.

The different ROMs are compared for a two-dimensional numerical exemplary setting depicting a flow in a rectangular domain. Our results suggest that the proposed ROM obtained with the aid of the second-orderhead mean and covariance computed by the MEs provides accurate estimations of the head pdf, with important online CPU savings, e.g., reducing the MC cost for 10,000 realizations from 80 s (FSM) to 15 s (ROM).

References

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