Pluripotential Numerics

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Pluripotential Theory is the study of the complex Monge-Ampere operator and plurisubharmonic functions in several complex variables. In this work we introduce numerical methods for the approximation of the main (global) quantities in Pluripotential Theory as the *extremal plurisubharmonic function* V_E^* of a compact $\mathcal{L}$-regular set $E \subset \mathbb{C}^n$, its *transfinite diameter* $\delta(E)$, and the *pluripotential equilibrium measure* $\mu_E := (\mathrm{dd}^c V_E^*)^n$.

The methods rely on the computation of a *polynomial mesh* for E and numerical orthonormalization of a suitable basis of polynomials. We prove the convergence of the approximation of $\delta(E)$ and the uniform convergence of our approximation to V_E^* on all $\mathbb{C}^n$; the convergence of the proposed approximation to μ_E follows. Our algorithms are based on the properties of polynomial meshes and Bernstein Markov measures.

Numerical tests are presented for some simple cases with $E \subset \mathbb{R}^2$ to illustrate the performances of the proposed methods. .

References

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