

Bi-directional models of “Radically Synthetic” Differential Geometry

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The radically synthetic foundation for smooth geometry formulated in [Lawvere 2011] postulates a space T with the property that it has a unique point and, out of the internal monoid of endomorphisms of T , it extracts a submonoid R which, in many cases, is the (commutative) multiplication of a rig structure. The rig R is said to be ‘bi-directional’ if its subobject of invertible elements has two connected components. In this case, R may be equipped with a pre-order compatible with the rig structure, via the construction of a subrig of ‘non-negative quantities’. We adjust the construction of ‘well-adapted’ models of Synthetic Differential Geometry in order to build the first pre-cohesive toposes with a bi-directional R . (These seem to be also the first pre-cohesive models of SDG.) We also show that, in one of these models, the pre-order on R , derived radically synthetically from bi-directionality, coincides with that defined in the original model.