

First-order completeness of implicative models

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Implicative algebras are a generalization of complete Heyting algebras intended to factorize the model-theoretic constructions underlying forcing and realizability, both in intuitionistic and classical logic. Each implicative algebra induces a *Set*-based tripos (in the sense of Hyland, Johnstone and Pitts) using a general construction that encompasses all triposes, since all *Set*-based triposes can be obtained from the latter construction (up to isomorphism).

In this talk, we will show that implicative algebras are also well-suited for studying models of first-order logic. For that, we shall introduce and study the notion of implicative model for 1st-order classical theories, as a natural generalization of Boolean-valued models. We will then prove that each 1st-order classical theory T is captured by some implicative model M , in the sense that a closed formula φ holds in M if and only if φ is derivable in T . We shall see that this result of strong completeness gives new insights about the usual completeness theorem for first-order logic, before presenting some perspectives in the study of first-order logic.