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Relational doctrines and quotient completions

Quotients are everywhere in mathematics and several constructions have been developed to complete with quotients structures in which quotients are not natively available. There are many mathematical tools that one can adopt to study quotients. Among them Lawvere's doctrines [1, 2] provide a simple setting that generalise many different situations. Doctrines are functors $P\colon \mathbf{C}^{\mathbf{op}} \to \mathbf{Pos}$, based on a category $\mathbf{C}$ with finite products, that give an algebraic description of first order theories: objects and arrows of $\mathbf{C}$ model contexts and terms, products of $\mathbf{C}$ model context concatenation and P maps each object to a poset that models formulas on that object ordered by logical entailment.

In this talk we introduce the notion of *relational doctrine* as the functorial description of the *calculus of relations* [5]. Here one takes as primitive concepts (binary) relations instead of (unary) predicates, with basic operations, such as relational identities, composition and the converse of a relation.

Relational doctrines offer a natural setting where to deal with quotients. We present a universal construction that complete a relational doctrine with quotients (inspired by the elementary quotient completion of an elementary doctrine presented in [3, 4]) and we discuss on some relevant examples that the construction subsumes.

Since the calculus of relations is variable-free, we can drop finite products from the requirements that the base category of a relational doctrine must have. This allows to consider new examples such as the exact completion of a category with weak finite limits and categories of metric structures, such as metric spaces and non-expansive maps.

References:

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