Precohesive Toposes over arbitrary Base Toposes

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We first recall what it means for Grothendieck fibrations to have internal sums and being locally small.

Based on these notions we discuss famous results by J.-L. Moens and M. Jibladze from the 1980s characterizing finite limit preserving functors between elementary toposes as fibered toposes with internal sums. Geometric morphisms will be characterized as fibered toposes which have internal sums and are, moreover, locally small. The additional requirement of boundedness for geometric morphisms will be characterized in terms of having small generating families. Accordingly, bounded geometric morphisms will turn out as corresponding to fibered Grothendieck toposes over some base topos $\mathcal{S}$.

Finally, we will show that Lawvere's notion of *stably precohesive topos* over a base topos $\mathcal{S}$ can be characterized in terms of the additional properties *molecular*, *hyperconnected* and *local* for which we provide characterizations in terms of requirements for the corresponding fibered toposes.

Molecular (*aka* locally connected) and local will turn out as the requirements that Δ / Γ has a fibered left / right adjoint Π / ∇ and "hyperconnected" will turn out as the requirment that subobjects of $\Delta(I)$ are in 1-1-correspondence with subobjects of I in the base topos.

Finally, we will dicuss an open problem brought up by Lawvere and Menni in their 2015 TAC paper, namely whether one obtains a genuinely weaker notion of *precohesive geometric morphism* when replacing the requirement of being molecular by the requirement that the inverse image part of the geometric morphism just preserves ordinary exponentials.

References

[Str22]   T. Streicher. *Fibered Categories á la Jean Bénabou* a text available on the internet under `arXiv:1801.02927v18`.