Invited talks

Large Cardinals beyond Choice

Joan Bagaria i Pigrau

ICREA, Barcelona

There are large cardinals whose existence implies the negation of the Axiom of Choice. One example is a Reinhardt cardinal, namely a cardinal that is the first ordinal moved by a non-trivial elementary embedding of the universe V of all sets into itself. Another, even stronger example, is that of a Berkeley cardinal, introduced by Woodin about 30 years ago and which is likely inconsistent with ZF, although so far no inconsistency has been found. In this talk we will present some recent results, in collaboration with Peter Koellner and W. Hugh Woodin, on the relative strength of the hierarchies of Reinhardt and Berkeley cardinals. We will also discuss briefly the relevance of the study of such cardinals for the foundations of set theory.

Entropy of actions of amenable groups

Antongiulio Fornasiero

Hebrew University of Jerusalem

Let G be an amenable group (or more generally, cancellative monoid). We describe the entropy of the action of G on various kind of "spaces". For actions on Abelian groups, we have the so-called algebraic entropy. We give some results for algebraic entropy: the addition theorem, a version of "Fubini", and the entropy of the Bernoulli shift.

κ-bounded exponential groups and exponential-logarithmic power series fields without log-atomic elements

Salma Kuhlmann

Universität Konstanz

A divisible ordered abelian group is an exponential group if its rank as an ordered set is isomorphic to its negative cone. Exponential groups appear as the value groups of ordered exponential fields, and were studied in [1]. In [2] we gave an explicit construction of exponential groups as Hahn groups of series with support bounded in cardinality by an uncountable regular cardinal κ. These κ-bounded Hahn groups are used in turn for the construction of the κ-bounded exponential logarithmic series fields, which are models of real exponentiation. These models are particularly interesting, since they are naturally similar to Conway–Gonshor's exp-log field of Surreal Numbers, and can therefore be exploited to investigate its properties. An exp-log series s is said to be log atomic if the nth-iterate of $\log(s)$ is a monomial for all $n \in \mathbb{N}$. Log-atomic (with respect to Gonshor's logarithm) surreal numbers exist and play a crucial role in defining derivations on the Surreal Numbers. In this talk I will present a modified construction of κ-bounded Hahn groups and exploit it to construct κ-bounded Hahn fields without log-atomic elements. This unexpected class of examples can be in turn used to investigate other possible logarithmic derivatives on the Surreal Numbers. This is ongoing joint work with A. Berarducci, V. Mantova and M. Matusinski.

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The complexity of the classification problem in ergodic theory

Martino Lupini

California Institute of Technology

Classical results in ergodic theory due to Dye and Ornstein–Weiss show that, for an arbitrary countable amenable group, any two free ergodic measure-preserving actions on the standard atomless probability space are orbit equivalent, i.e. their orbit equivalence relations are isomorphic. This motivates the question of what happens for nonamenable groups. Works of Ioana and Epstein showed that, for an arbitrary countable amenable group, the relation of orbit equivalence of free ergodic measure-preserving actions on the standard probability space has uncountably many classes. In joint work with Gardella, we strengthen these conclusions by showing that such a relation is in fact not Borel. This builds on previous work of Epstein, Tornquist, and Popa, and answers a question of Kechris.

Borel reducibility and its relatives

Luca Motto Ros

Università di Torino

In the last 30 years, Borel reducibility has proven to be an invaluable tool for tackling (and often solving, in a way or another) various classification problems arising in mathematics. However, there are situations in which such reducibility is not sufficiently strong to capture the essence of the problem or to give a satisfactory solution to it. In this talk we will discuss some strengthenings of the notion of Borel reducibility that have been proposed in the literature, compare them to one another, and mention some applications which motivated their introduction.

On relating type theories to (intuitionistic) set theories

Michael Rathjen

University of Leeds

Type theory, originally conceived as a bulwark against the paradoxes of naive set theory, has languished for a long time in the shadow of axiomatic set theory which became the mainstream foundation of mathematics in the 20th century. But type theories, especially dependent ones à la Martin-Löf, are looked upon favorably these days. The recent renaissance not only champions type theory as a central framework for constructive mathematics and as an important tool for achieving the goal of fully formalized mathematics (amenable to verification by computer-based proof assistants) but also finds deep and unexpected connections between type theory and other areas of mathematics.

One aspect, though, that makes type theories irksome is their overbearing syntax and rigidity. It is probably less known that they can often be related to set theories, albeit intuitionistic ones, and thereby rendered more accessible to those who favor breathing "set-theoretic" air, as it were (like the speaker). This connection was first unearthed by Peter Aczel in the previous century. Still, intuitionistic set theories can behave in rather unexpected ways and it is often surprising to find out which classical set theories they relate to. This and more recent developments will constitute the bulk of the talk.

Mathematics as a dynamic process: effects on the working mathematician

Giovanni Sambin

Università di Padova

Open minded meditation on Gödel's incompleteness, as opposed to Bourbaki's dogmatic denial of its significance, leads naturally to conceive mathematics as dynamic, partial, plural, that is, a conquered human achievement, instead of static, complete, unique, that is, a given absolute truth. In other words, it becomes possible to see mathematics as produced by a Darwinian process of evolution, as all other fields of science.

Unexpectedly, assuming deeply such a change of foundational attitude bring also many results and changes in the practice of mathematics. The talk will illustrate such novelties as: a foundation with two levels of abstraction, symmetry and duality in topology (and in general a deeper link between logic and topology), continuity as a commutative square, the mathematics of existential statements, embedding of pointwise into pointfree topology, conservativity of ideal aspects over real mathematics, algebraization of topology, ... So, with hindsight, one can see all signs of a new Kuhnian paradigm also for mathematics.

Recent Studies on Coordinatizing MV algebras

Philip Scott

University of Ottawa

In a paper with Mark Lawson (JPAA, 2017) we introduced a coordinatization program for MV-algebras. This work is in the spirit of von Neumann's Continuous Geometry, but uses recent developments in inverse semigroup theory. We develop a special class of Boolean inverse monoids, called AF inverse monoids, in analogy with standard (Bratteli) techniques for building AF C*-algebras. AF inverse monoids have the property that their lattices of principal ideals naturally form an MV-algebra. We say that an arbitrary MV-algebra can be co-ordinatized if it is isomorphic to an MV-algebra of principal ideals of some AF inverse monoid. Our main theorem is that every countable MV-algebra can be co-ordinatized. Our proofs are inspired by the recently evolving area of non-commutative Stone Duality, although moving beyond MV to the larger category of effect algebras. We survey some basic results, as well as recent advances and open problems inspired by work of F. Wehrung D. Mundici, W. Lu and on-going work with M. Lawson.

Contributed talks

Nonstandard methods in combinatorial number theory

Lorenzo Luperi Baglini

University of Vienna

Combinatorial number theory is currently in the spotlight of the mathematical community, thanks to the recent solutions of some long standing open problems. As problems in this area are very difficult to solve with classical techniques, in the last few years new methods based on nonstandard analysis have been introduced, as they can be used to reduce the complexity of the mathematical objects that one needs in a proof. In this talk we will focus on some of these nonstandard techniques, in particular in the context of the partition regularity of nonlinear Diophantine equations.

A predicate extension of real valued logic

Stefano Baratella

Università di Trento

We study a predicate extension of an unbounded real valued propositional logic that has been recently introduced by D. Zambella and by the author. The latter, in turn, can be regarded as an extension of both the abelian logic and of the propositional continuous logic. Among other results, we prove that our predicate extension satisfies the property of weak completeness (the equivalence between satisfiability and consistency) and, under an additional assumption on the set of premisses, the property of strong completeness (the equivalence between logical consequence and provability). Eventually we discuss some topological properties of the space of types in our logic.

Projective Fraïssé Limits of Partial Orders

Gianluca Basso

Université de Lausanne and Università di Torino

The Kechris–Pestov–Todorčević correspondence links Ramsey Theory, Fraïssé Theory and Topological Dynamics. In particular it states that the automorphisms group of the Fraïssé limit of a countable Fraïssé family F consisting of finite rigid structures is extremely amenable if and only if F has some Ramsey property. The previous result has been extended to the dual context of projective Fraïssé Theory by D. Bartošová and A. Kwiatkowska. In such a context we present our work on projective Fraïssé limits of partial orders and their quotients. This is joint work with R. Camerlo.

FROM BASIC LOGIC TO A QUANTUM LOGICAL APPROACH TO MATTE BLANCOS BI-LOGIC

Giulia Battilotti[1], Milos Borozan[2], and Rosapia Lauro Grotto[2]

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We consider a model in first order logic developed for quantum mechanics. It consists in formalizing assertions from the axiomatization of Quantum Mechanics [1], adopting suitable equations to define connectives, as in basic logic (reflection principle) [10, 8]. In the model first order domains are characterized as "infinite" when they correspond to pure states (prior to measurement) and "finite" when they correspond to mixed states after measurement. Such a characterization is extended to singletons as well. So one defines "infinite singletons", corresponding to pure states with respect to observables incompatible with the measured observable. This means that no closed term is available in the language in order to describe them. Infinite singletons are characterized intensionally, requiring that the universal and existential quantification coincide on them. In basic logic, this can be interpreted as the definition of a "symmetric" quantifier and eliminates negation (in the spin model of quantum mechanics, infinite singletons correspond to the eigenvalues of negation). We need to recall that the coexistence of closed and open terms was already pointed out by Freud in his monograph about aphasia: by explicitly referring to J. Stuart Mills Logik, he distinguished on this ground the so called word-representation (Wortvorstellung, i.e. the closed term) and thing-representation (Objektvorstellung or Sachvorstellung. i.e. the open term) [4]. We see that, in logic, considering infinite singletons gives a "symmetric" environment, corresponding to the "symmetric mode" that, following Matte Blanco, characterizes the primary process described by Freud [7]. We furtherly discuss how Matte Blancos "bivalent mode", that is the other mode of his Bi-logic, is derived from the collapse of the symmetric mode, which is characterized as "infinite" in Matte Blanco, to the finite. Modal logic represents an important additional tool for the further development of the model. A modality can be introduced in the quantum model itself [2], in order to define "the objective property of the particle with respect to every direction of the spin". This is not in contrast with the no-go given by the Kochen–Specker theorem of Quantum Mechanics, since the spin model is bivalent. The result is S4, characterized by Kurt Gödel as the modality which recovers the infinitary content of proofs. In foundations, our approach would allow to discuss the role of a pre-existing symmetric infinite with respect to the mathematical infinite. In psychoanalysis, we are interested in exploring a possible interpretation of the formal introduction of the modal system S4 in relation to two theoretical points: first, the shift from the First to the Second Topic description in Freudian Psychoanalysis, and second, the consideration of transitional dynamics and the role of external reality in the Object Relations approach. The model can furtherly explain other logical features of the human thinking, observed in psychology, beyond psychoanalysis, for example Becks cognitive distortions. Finally, we stress that, in the last times, the need for a formalization of Matte Blanco's theory has more and more emerged. We quote [6, 5] where a topological approach to bi-logic via ultra metric spaces is discussed.

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Surreal differential calculus and transeries

Alessandro Berarducci

Università di Pisa

Conway's field of surreal numbers contains both the real and the ordinal numbers, with the surreal sum and product of two ordinals coinciding with the Hessenberg sum and product. In the joint paper with Vincenzo Mantova "Surreal numbers, derivations and transseries" we proved that there is a meaningful way to take both the derivative and the integral (anti-derivative) of a surreal number, hence in particular of an ordinal number. The derivative of the ordinal ω is 1, the derivative of a real number is zero, and the derivative of the sum, product and exponentiation of surreal numbers obeys the expected rules. More difficult is to understand what should be the derivative of an ordinal power of ω, for instance the first epsilon-number, but this can be done in a way that reflects the formal properties of the derivation on a Hardy field (differentiable germs of non-oscillating real functions). In the joint preprint "Transseries as germs of surreal functions" we proved that many surreal numbers (properly containing a copy of the transseries) can indeed be interpreted as germs of differentiable functions on the surreals themselves. This can be done in such a way that the derivative acquires the usual analytic meaning as a limit. It is still open whether we can extend the result to the whole class of surreal numbers. I will report on these results.

The regularization of a logic

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Abstract

A variety $\mathcal{K}$ is called *irregular* whenever it satisfies an identity of the kind $f(x,y) \approx x$, where $f(x,y)$ is any term of the language in which x and y really occur. A variety is *regular*, when it is not irregular. Examples of irregular varieties abound in logic, since every variety with a lattice reduct is irregular as witnessed by the term $f(x,y) \coloneqq x \wedge (y \vee x)$. On the other hand, an identity $\varphi \approx \psi$ is said to be *regular* provided that exactly the same variables occur in φ and ψ. The algebraic study of regular varieties traces back to the pioneering work of Płonka [7], who introduced a class-operator $\mathcal{P}_l(\cdot)$ nowadays called *Płonka sums*, and used it to prove that any regular variety $\mathcal{K}$ can be represented as Płonka sums of a suitable irregular variety $\mathcal{V}$, in symbols $\mathcal{P}_l(\mathcal{V}) = \mathcal{K}$. In this case $\mathcal{K}$ is called the *regularization* of $\mathcal{V}$.

Over the years, regular varieties have been studied in depth from a purely algebraic perspective, see for example [1, 8, 5, 6, 4]. However, the recent discovery [2] that the regularization of the variety of Boolean algebras is the algebraic semantics of Paraconsistent Weak Kleene logic, i.e. a particular three-valued Kleene-like logic, showed that the notion of regularization could find interesting applications in logic as well. Building on this observation, in this contribution we develop a new notion of *logic-based* regularization, which on the one hand extends and subsumes the known algebraic one, and on the other hand explains on general grounds the relations between classical logic and Paraconsistent Weak Kleene logic discovered in [2]. Our investigation is carried on in the framework of abstract algebraic logic [3].

In order to introduce the notion of regularization of a given logic, we need to extend first the construction of Płonka sums to logical matrices. A *direct system* of logical matrices is a triple $X = \langle I, \{\mathbf{A}_i, F_i\}_{i\in I}, \varphi_{ij}\rangle$ where, I is a join-semilattice of indexes, $\langle \mathbf{A}_i, F_i\rangle_{i\in I}$ is a family of logical matrices, and $\varphi_{ij} : \mathbf{A}_i \to \mathbf{A}_j$ is a homomorphism such that $\varphi_{ij}(F_i) \subseteq F_j$ for every $i \leq j$. The Płonka sum over a direct system X is the logical matrix $\mathcal{P}_l(X) \coloneqq \langle \mathcal{P}_l(\mathbf{A}_i), \bigcup_{i\in I} F_i\rangle$. Now, recall that every logic $\vdash$ can be naturally associated with a class of logical matrices, usually called the *reduced models* of $\vdash$. With this technology at hand, we are ready to define the *regularization* $\vdash_r$ of a logic $\vdash$ as the logic determined by all Płonka sums of reduced models of $\vdash$. Remarkably, the regularization $\vdash_r$ could be equivalently defined syntactically, since it is not difficult to observe that

$$\Gamma \vdash_r \varphi \iff \text{there is } \Delta \subseteq \Gamma \text{ s.t. } \mathrm{var}(\Delta) \subseteq \mathrm{var}(\varphi) \text{ and } \Delta \vdash \varphi.$$

In other words, regular logics admit also a syntactical definition as logics of variable inclusion. From this starting point, we investigate logico-algebraic properties of $\vdash_r$ such as structural completeness, its location in the Leibniz and Frege hierarchy (both as a logic and as a Gentzen system), and the structure of its reduced models.

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Describing limits of bounded sequences of measurable functions via nonstandard analysis

Emanuele Bottazzi

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In functional analysis, there are different notions of limit for a bounded sequence of L^p functions. Besides the pointwise limit, that does not always exists, the behaviour of a bounded sequence of L^p functions can be described in terms of its weak (weak-$\star$ if $p = 1$ or $p = \infty$) limit, or by introducing a measure-valued notion of limit in the sense of Young measures. By using nonstandard analysis, we will show that for every bounded sequence $\{z_n\}_{n\in\mathbb{N}}$ of L^p functions there exists a function of a hyperfinite domain that simultaneously generalizes the weak, weak-$\star$ and Young measure limits of the sequence. Let ${}^*\mathbb{R}$ be a set of hyperreals of nonstandard analysis, let $\varepsilon \in {}^*\mathbb{R}$ be a positive infinitesimal, and let $\mathbb{X} = \{n\varepsilon : n \in {}^*\mathbb{Z}\}$. For all open $\Omega \subseteq \mathbb{R}^k$, let $\mathbb{G}(\Omega)$ be the vector space of ${}^*\mathbb{R}$-valued functions defined on $\Omega_{\mathbb{X}} = {}^*\Omega \cap \mathbb{X}^k$. We will prove that, for every bounded sequence $\{z_n\}_{n\in\mathbb{N}}$ in $L^p(\Omega)$, there exists a (non unique) function $u \in \mathbb{G}(\Omega)$ such that

- u represents the weak (weak-$\star$ if $p = 1$ or $p = \infty$) limit of the sequence $\{z_n\}_{n\in\mathbb{N}}$ in the sense that for all $g \in C^0(\Omega)$ it holds

$$^\circ\left(\varepsilon^k \sum_{x\in{}^*\Omega\cap\mathbb{X}^k} u(x)\,{}^*g(x)\right) = \lim_{n\to\infty}\int_\Omega z_n(x)g(x)dx;$$

- u represents the Young measure limit of the sequence $\{z_n\}_{n\in\mathbb{N}}$ in the sense that for all $f \in C^0(\mathbb{R})$ with $\lim_{|x|\to\infty} f(x) = 0$ and for all $g \in C^0(\Omega)$ it holds

$$^\circ\left(\varepsilon^k \sideset{}{^*}\sum_{x\in{}^*\Omega\cap\mathbb{X}^k} f(u(x))\,{}^*g(x)\right) = \lim_{n\to\infty}\int_\Omega f(z_n(x))g(x)dx.$$

Thus, functions of $\mathbb{G}(\Omega)$ can be used to simultaneously describe very different behaviours of bounded sequences of L^p functions; moreover, we believe that the study of functions in $\mathbb{G}(\Omega)$ will allow to define new notions of standard limits that could be successfully applied to the study of ill-posed problems from functional analysis.

The bi-embeddability relation for countable abelian groups

Filippo Calderoni

Università di Torino

We study the equivalence relation of bi-embeddability on standard Borel spaces of countable abelian groups in the framework of Borel reducibility. We show that, while the relation of bi-embeddability on torsion-free abelian groups is complete analytic, bi-embeddability on torsion groups is not. This is joint work with Simon Thomas.

Wadge hierarchies versus generalised Wadge hierarchies

Riccardo Camerlo

Polytechnic of Turin

The talk presents results, still in progress, about reducibility by continuous relations on second countable, T_0 topological spaces. The main result is that either this reducibility coincides with Wadge reducibility, or there is no topology whatsoever that turns it into Wadge reducibility. An attempt is made towards a description of the spaces that fit in each of the two cases.

Theories of presheaf type as a basic setting for topos-theoretic model theory

Olivia Caramello

Università degli Studi dell'Insubria (Como) and IHES

I will review the notion of classifying topos of a first-order (geometric) theory and explain the central role enjoyed by theories of presheaf type (i.e. classified by a presheaf topos) in the context of the topos-theoretic investigation of the model theory of geometric theories. After presenting a few main results and characterizations for theories of presheaf type, I will illustrate the generality of the point of view provided by this class of theories by discussing a topos-theoretic framework unifying and generalizing Fraïssé's construction in model theory and topological Galois theory and leading to an approach to the problem of the independence from ℓ of ℓ-adic cohomology.

Weak Yet Strong restrictions of Hindman's Finite Sums Theorem

Lorenzo Carlucci

University of Rome I "La Sapienza"

Hindman's Finite Sums Theorem is a celebrated result in Ramsey's Theory stating that any finite colouring of the positive integers admits an infinite set such that all non-empty finite sums of distinct elements from that set have the same colour.

Calibrating the strength of Hindman's Theorem is one of the main open problems in reverse mathematics since some thirty years since the seminal results of Blass Hirst and Simpson. The strength of Hindman's Theorem is between the $(\omega + 1)$-th Turing Jump and the Halting Set. In terms of reverse mathematics, the theorem is provable from ACA_0^+ and implies ACA_0, over RCA_0.

Recently Dzhafarov et al. proved that the lower bound on the full theorem is already true for its restriction to sums of at most 3 distinct terms. In joint work with Kołodziejczyk, Lepore and Zdanowski we showed that this is already true for its restriction to sums of one or two terms.

Note that no proof of the latter restrictions is known that does not also prove the full Finite Sums Theorem. Whether such a proof exists is indeed an open problem in combinatorics posed by Hindman, Leader and Strauss. Consequently, no better upper bound is known to hold for that restriction other than the ACA_0^+ upper bound on the full Finite Sums Theorem itself.

It is a natural question to ask whether natural restrictions of Hindman's Theorem exist that have non-trivial lower bounds as well as upper bounds better than the full theorem.

We introduced a number of such "weak yet strong" restrictions at various level of proof-theoretic and computability-theoretic strength. First we present an infinite family of restrictions of Hindman's Theorem that are equivalent to ACA_0. Second we introduce a restriction of Hindman's Theorem in which monochromaticity is required only for sums of adjacent elements and prove it to be between Ramsey's Theorem for pairs and the Increasing Polarized Ramsey's Theorem for pairs of Dzhafarov and Hirst.

We discuss the role of a sparsity condition on the solution set which we call the apartness condition.

Topological embeddability between functions

Raphaël Carroy

Universität Wien

A function f embeds topologically in a function g if there are two topological embeddings b and c such that $b \circ f = g \circ c$. This notion was considered by Solecki to find finite bases for non σ-continuous functions between Polish spaces. Since then, finite bases have been found for several other classes of functions, including a finite basis for non Baire class one functions (joint work with Benjamin Miller).

As a quasi-order, topological embeddability is however far from being always "simple". As a matter of fact, even on spaces of continuous functions it is very often analytic complete. I will pinpoint exactly when topological embeddability is an analytic complete quasi-order on continuous functions between Polish 0-dimensional spaces. This gives that on these spaces of functions, topological embeddability is either analytic complete or well-founded and without infinite antichains (joint work with Yann Pequignot and Zoltan Vidnyanszky).

Descriptive Set Theory and Automata

Filippo Cavallari

University of Turin and Université de Lausanne

In this talk I introduce a quite recent field of research, that is, the connection between Descriptive Set Theory and Automata Theory. The overlap between these two areas is that in Automata Theory the space which we work with is, in fact, the Cantor set 2^ω, that is a well-known uncountable Polish space. In Descriptive Set Theory subsets of the Cantor set can be stratified through topological hierarchies, like Borel Hierarchy and Wadge Hierarchy, while in Automata Theory these spaces can also be studied in terms of "regularity", that is the property of being recognised by an automaton. This double point of view leads to many interesting questions about the interplay between topological complexity and regularity. While we have a complete picture of these relationships in the case of automata on words, the case of automata on trees is still a *terra incognita*. In this talk I will present the state of art of the research on automata on trees and I will show some new results that are concerned with general regular tree languages that lie in low levels of Borel Hierarchy and Wadge Hierarchy.

Normalization by Evaluation in Linear Logic

Jules Chouquet[1], Giulio Guerrieri[2], Luc Pellissier[3], Lorenzo Tortora De Falco[4], and Lionel Vaux[5]

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In logic and programming language semantics, normalization-by-evaluation is a technique of computing the output/normal form of a program/proof P by appealing only to the denotational semantics of P. This semantic approach bypasses the traditional syntactic notion of computation as a reduction relation in a term rewrite system. We aim to implement the normalization-by-evaluation paradigm in the multiplicative-exponential fragment of linear logic without weakening, using the relational semantics as denotational model. In our work the notion of Taylor expansion of a linear logic proof-net plays a crucial role.

Informational semantics and depth-bounded natural deduction for classical logic

Marcelo D'Agostino

Università degli Studi di Milano

We present an informational view of classical propositional logic that stems from a kind of informational semantics whereby the meaning of a logical operator is specified solely in terms of the information that is actually possessed by an agent. In this view the inferential power of logical agents is naturally bounded by their limited capability of manipulating "virtual information", namely information that is not implicitly contained in the data. Although this informational semantics cannot be expressed by any finitely-valued matrix, it can be expressed by a non-deterministic 3-valued matrix that was first introduced by W.V.O. Quine in [4], to describe what he called the "primitive meaning of the logical operators", but ignored by the logical community. Following [3, 1, 2] we shall present this informational semantics and show how it naturally gives rise to an infinite hierarchy of tractable approximations to classical propositional logic. This hierarchy can be used to model the inferential power of resource-bounded agents and admits of a uniform proof-theoretical characterization that is half-way between a classical version of natural deduction and semantic tableaux. This will bring to light a general approach to logical deduction in classical logic that is quite different from the standard Gentzen-style approach (whether in the natural deduction or sequent calculus format), while preserving some of its desirable proof-theoretical properties.

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Ultraproducts and Spec($\hat{\mathbb{Z}}$)

Paola D'Aquino

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We give a description of the spectra of $\hat{\mathbb{Z}}$ and of the finite adeles using ultraproducts. In describing the prime ideals and the localizations of $\hat{\mathbb{Z}}$, ultrapowers of the group $\mathbb{Z}$ and ultraproducts of the rings of p-adic integers are used. This is joint work with A. Macintyre and M. Otero.

Infinitary logic and compact Hausdorff spaces

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Riesz Spaces are lattice-ordered linear spaces over the field of real numbers $\mathbb{R}$. They have had a predominant rôle in the development of functional analysis over ordered structures, due to the simple remark that most of the spaces of functions one can think of are indeed Riesz Spaces.

Not very known is the rôle that vector lattices play in logic. Given any positive element u of a Riesz Space V, the interval $[0, u]$ can be endowed with a stucture of Riesz MV-algebra. These structures have been defined in the setting of Łukasiewicz logic as expansion of MV-algebras – the *standard* semantics of the infinite valued Łukasiewicz logic – and in [1] is proved that Riesz MV-algebras are categorical equivalent to Riesz Spaces with a strong unit. Henceforth, vector lattices and logic are closely related.

Our aim is to exploit the connection between Riesz Spaces and MV-algebras to deepen the link between functional analysis and Łukasiewicz logic. Our first step will be the introduction of a notion of *limit of formulas*, which will be used to characterize the (uniform) norm convergence in Riesz Spaces [4, 5]. Then, we will see how such a notion is related to *order-convergence* in Riesz MV-algebras and how this connection sparkled the idea of adding an infinitary disjunction to the logic $\mathcal{RL}$ of Riesz MV-algebras, obtaining the logical system $\mathcal{IRL}$.

$\mathcal{IRL}$ has Dedekind σ-complete Riesz MV-algebras as models and its Lindenbaum-Tarski algebra is the Dedekind σ-completion of the Lindenbaum-Tarski algebra of $\mathcal{RL}$. Moreover, the models of the infinitary logic are *norm-complete* with respect to an appropriate norm. The latter result, together with Kakutani's duality, is instrumental in proving that the model of $\mathcal{IRL}$ are dual (as appropriate categories) to *basically disconnected* compact Hausdorff spaces and providing in this way an axiomatization for such spaces.

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A generalized Chang completeness theorem

Antonio Di Nola[1], Giacomo Lenzi[1], Vincenzo Marra[2], and Luca Spada[1]

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Let C be a product MV-algebra which is linearly ordered and torsion free. Let us call MV-C-algebras the MV-algebras equipped with a multiplication by elements of C. Let R be the ring such that $\Gamma(R, 1) = C$, where Γ is the Mundici functor. Let R^q be the field of quotients of R and let $C^q = \Gamma(R^q, 1)$. We show that the variety of MV-C-algebras is generated by C^q. This generalizes Chang completeness theorem which is the particular case when $C = \{0, 1\}$.

Left distributive algebras beyond I0

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The connection between large cardinals and left-distributive algebras is one of the most intriguing success stories of the theory of large cardinals. LD-algebras are algebras with one operator that satisfies the left-distributive law. At first sight, they have nothing to do with large cardinals, as they can be small, countable, even finite. Yet there is a connection: we say that I3 holds if there is an elementary embedding from V_λ to itself. It turns out that such embeddings form a free LD-algebra, therefore any result on such an algebra will propagate, thanks to the universal nature of free algebras, to all LD-algebras.

But I3 is not the strongest axiom, there is a hierarchy of other axioms above it. What kind of structure will generate the embeddings related to them? Isomorphic or completely different? A thorough study will show that to have a genuinely new structure one has to go far beyond I0, the strongest common large cardinal, into an experimental hierarchy of E^0_α axioms, and that weak independence properties can depend from the properness or non-properness of the embeddings involved.

Categorical quotient completions, local cartesian closure and choice principles

Jacopo Emmenegger and Erik Palmgren

Stockholms Universitet

The Constructive Elementary Theory of the Category of Sets (CETCS) is a modification of Lawvere's ETCS introduced by Palmgren in order to account for the structuralist character of Bishop's set theory. Abstracting on the setoid construction in Martin-Löf type theory, models of this theory can be seen as the exact completion of the category of their choice objects and can then be characterised in terms of properties of the latter.

Most of these properties can be obtained from known results in the theory of exact completions, however this is not the case for local cartesian closure since, to apply Carboni and Rosolini's characterisation, a stronger choice principle must be internally valid. For this reason we have introduced a property of the choice objects, inspired from an axiom of Aczel's CZF, that ensures the local cartesian closure of the model in the most general case.

The whole treatment strongly relies on the internal logic of the categories involved, and if time allows we will also discuss connections with the more general quotient completion of elementary doctrines by Maietti and Rosolini.

Towards an Implementation in LambdaProlog of the Two Level Minimalist Foundation

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In 2005 M. Maietti and G. Sambin [1] argued about the necessity of building a foundation for constructive mathematics to be taken as a common core among relevant existing foundations in axiomatic set theory, such as Aczel-Myhill's CZF theory, or in category theory, such as the internal theory of a topos, or in type theory, such as Martin-Löf's type theory and Coquands Calculus of Inductive Constructions. Moreover, they asked the foundation to satisfy the "proofs-as-programs" paradigm, namely the existence of a realizability model where to extract programs from proofs. Finally, the authors wanted the theory to be appealing to standard mathematicians and therefore they wanted extensionality in the theory, e.g. to reason with quotient types and to avoid the intricacies of dependent types in intensional theories.

In the same paper they noticed that theories satisfying extensional properties, like extensionality of functions, can not satisfy the proofs-as-programs requirement. Therefore they concluded that in a proofs-as-programs theory one can only represent extensional concepts by modelling them via intensional ones in a suitable way, as already partially shown in some categorical contexts.

Finally, they ended up proposing a constructive foundation for mathematics equipped with two levels: an intensional level that acts as a programming language and is the actual proofs-as-programs theory; and an extensional level that acts as the set theory where to formalize mathematical proofs. Then, the constructivity of the whole foundation relies on the fact that the extensional level must be implemented over the intensional level, but not only this. Indeed, following Sambin's forget-restore principle, they also required that extensional concepts must be abstractions of intensional ones as result of forgetting irrelevant computational information. Such information is then restored when extensional concepts are translated back at the intensional level.

In 2009 M. Maietti [2] presented the syntax and judgements of the two levels, together with a proof that a suitable completion of the intentional level provides a model of the extensional one. The proof is constructive and based on a sequence of categorical constructions, the most important being the construction of a quotient model and a notion of canonical isomorphism between dependent setoids.

In the talk we will present an ongoing project to implement the following software components:

1. a type checker for the intensional level

2. a reformulation of the extensional level that allows to store syntactically proof objects that are later used to provide the information to be restored when going from the extensional to the intensional level

3. a type checker for the obtained extensional level

4. a translator from well-typed extensional terms to well-typed intensional terms

Combining the translator with a proof extraction component it will be possible to extract programs from proofs written in the extensional level.

We have chosen LambdaProlog [3] (and its recent implementation ELPI [4], by the second author) as the programming language to write the two type checkers and the translator. The benefits are that LambdaProlog takes care of the intricacies of dealing with binders and alpha-conversion and moreover a LambdaProlog implementation of a syntax-directed judgement is just made of simple clauses that are almost literal translations of the judgemental rules. This allows humans (and logicians in particular) to simply inspect the code to spot possible errors.

At the time of the submission of the abstract the two type-checkers are completed and we are working on the extension of the calculus and on the translator.

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On coherence, strict coherence and logic

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The aim of our investigation is to contribute to the logical foundations of probability theory. In particular, we will present a characterization of strict coherence in terms of finitely additive and normalized measures which further satisfy Carnap's regularity condition. We will also point out the fundamental role played by MV-algebras in the logical foundations of uncertain reasoning and we will introduce interesting refinements of de Finettis coherence revealed by the logico-algebraic and geometric methods we adopted to prove our main result. Finally, we will hint at ongoing research aimed at investigating deeper links between uncertainty measures and logical valuations.

L.E.J. Brouwer and A. Heyting on Foundational Labels: Their Creation and Use

Miriam Franchella

Università di Milano

The use of the three labels (logicism, formalism, intuitionism) to denote the three foundational schools of the early twentieth century is now firmly part of the literature. Still, they were not introduced by the founding fathers each for his own school. Furthermore, neither their number nor their adoption has been stable over the twentieth century. In this talk we will see the role and attitude that intuitionists of the first era (Brouwer and Heyting) have had in the production and use of foundational labels and I will advance the thesis that not only the creation but also the use of labels, far from being a mere gesture of academic reference to literature, can be a sign of the cultural operation each scholar wants to do.

Computational Complexity of Projection Operators

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We study the computational complexity of projection operators in Euclidean space by investigating their degrees in the Weihrauch lattice. As an example of application, we deal with the classical Whitney Extension Theorem for differentiable functions.

Ziegler Spectra of Serial Rings

Lorna Gregory[1] and Gena Puninski[2]

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The (right) Ziegler spectrum, Zg_R, of a ring R is a topological space attached to the (right) module category of R. This space encodes model theoretic (and algebraic) information about the module category.

A (right) module is uniserial is it submodules are totally ordered by inclusion and it is serial if it can be written as a finite direct sum of uniserial modules. A ring is serial if it is serial as both a left and a right module over itself.

In this talk I will describe a proof that the Ziegler spectra of all serial rings are sober, that is every irreducible closed subset of Zg_R is the closure of a point. I will then go on to explain how to use this proof to get a general technique for computing Ziegler spectra of uniserial rings after identifying topologically indistinguishable points.

Factorisation Theorems for Generalised Power Series

Sonia L'Innocente[1] and Vincenzo Mantova[2]

[1] University of Camerino
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Fields of generalised power series, with coefficients in a field and exponents in a divisible ordered abelian group, are a fundamental tool in the study of valued and ordered fields and asymptotic expansions. The subring of the series with non-positive exponents appear naturally when discussing exponentiation, as done in transseries, or integer parts. A notable example is the ring of omnific integers inside the field of Conway's surreal numbers. In general, the elements of such subrings do not have factorisations into irreducibles. In the context of omnific integers, Conway conjectured in 1976 that certain series are irreducible (proved by Berarducci in 2000), and that any two factorisations of a given series share a common refinement. Here we prove a full factorisation theorem for the ring of series with non-positive real exponents: every series is shown to be a product of irreducible series with infinite support and a factor with finite support which is unique up to constants. From this, we shall deduce a general factorisation theorem for series with exponents in an arbitrary divisible ordered abelian group, including omnific integers as a special case. To obtain the result, we prove that a new ordinal-valued function, based on Berarducci's order value, is a valuation on the ring of generalised power series with real exponents, and formulate some structure results on the associated RV monoid.

Social welfare relattions and descriptive set theory

Giorgio Laguzzi

University of Freiburg

Utility streams for infinite horizon have been widely investigated in economical theory in the last decades. From the set-theoretical point of view, infinite utility streams are nothing more than infinite sequences on certain topological spaces, and so they can be analyzed and studied with the usual method coming from forcing and descriptive set theory. In particular, Zame [3] and Lauwers [2] showed that the existence of Paretian social welfare relations satisfying intergenerational equity imply the existence of non-constructible objects, such as non-Ramsey and non-measurable sets.

In this talk I prove some connection also with another well-known regularity property, i.e., the Baire property, and I use Shelah's amalgamation ([1] for a very detailed and complete introduction) in order to show that the two above implications does not reverse. Moreover I also investigate other types of egalitarian pre-orders, such as pre-orders satisfying Pigou-Dalton's principle and Hammond's principle.

I thank Adrian Mathias to suggest me the reading of [2] and more generally to show me this connection between set theory and theoretical economics.

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Problems about the Jónsson distributivity spectrum

Paolo Lipparini

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It follows from results by B. Jónsson that if a variety $\mathcal{V}$ is congruence distributive, then, for every m, there is some k such that the congruence identity

$$\alpha\,(\beta \circ \gamma \circ \beta \cdots) \subseteq \alpha\beta \circ \alpha\gamma \circ \alpha\beta \cdots$$

holds in $\mathcal{V}$, with m occurrences of $\circ$ on the left and k occurrences of $\circ$ on the right.

Let $J_{\mathcal{V}}(m)$ denote the smallest value of k for which the above formula holds. We study the problem of which functions can be represented as $J_{\mathcal{V}}$, for $\mathcal{V}$ a congruence distributive variety. The set of such functions is closed under pointwise maximum. Moreover, we show that the value of $J_{\mathcal{V}}(m)$ puts some restrictive bounds on the values of $J_{\mathcal{V}}(m')$, for $m' > m$.

Generalized connectives of linear logic

Roberto Maieli

"Roma TRE" University

We present the so called generalized connectives of multiplicative linear logic (MLL). These general connectives were formerly introduced by J.-Y. Girard [2] but most of the results known after then are due to V. Danos and L. Regnier [1]. This talk elaborates on these seminal works and brings several innovations. A multiplicative generalized (or n-ary) connective can be defined by two pointwise orthogonal sets of partitions, P and Q, over the same domain $\{1, \ldots, n\}$. Actually, we can use partitions according two different points of view (i.e., two syntaxes), sequential and parallel, preserving the same notion of orthogonality ($P \perp Q$). Anyway, general connectives are more expressive in the parallel syntax since this allows to represent correct proofs, namely proof-nets, containing generalized connectives (n-ary links) that cannot be defined (decomposed) by means of the basic (binary) multiplicative ones, $\parr$ and $\otimes$. Dislike the standard proof-nets, these "more liberal" proof-nets do not correspond (sequentialize) to any sequential proof (if we exclude the trivial axioms). In this talk, we characterize an "elementary" class of non-decomposable connectives: the class of entangled connectives. Actually, entangled connectives are the "smallest" generalized multiplicative connectives (w.r.t. the number of partitions or, equivalently, w.r.t. the number of rules or switchings), if we exclude, of course, the basic ones. Surprisingly, non-decomposable generalized connectives witness an asymmetry between proof-nets and sequent proofs since the former ones allow to express a kind of parallelism (concurrency) that the latter ones cannot do.

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The isomorphism relation of classifiable shallow theories

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In their 2014 article, S. Friedman, T. Hyttinen and V. Kulikov convincingly linked Classification Theory and Generalized Descriptive Set Theory by showing that countable classifiable shallow first-order theories are exactly the ones with a Borel isomoprhism relation between sufficiently large models. In this paper we retrace their steps toward this result and answer positively an open question from the article regarding the existence of a correspondence between the depth of a theory and the Borel degree of its isomorphism relation. Specifically, we prove a descriptive set-theoretic analogue of Shelah's Main Gap Theorem: the isomorphism relation of a countable theory is either not Borel or has very small Borel rank, and the exact upper bound depends on the depth of the theory.

On K. Popper's Decomposition of Logical Notions

Enrico Moriconi

Università di Pisa

Popper's logical enquiries of the late 40s are experiencing a renewed interest, and many scholars are focusing this subject along different perspectives. Here, we intend to stress that Popper's intention was to reverse Tarski's order of priority — in his 1936 paper "On the notion of logically following" — taking the notion of "derivability", or "deducibility", or "logical consequence", as primitive, and trying on this basis to show that those signs are logical, or formative, which can be defined with the help of that primitive concept. This approach considers logic a metalinguistic enterprise, which can be applied to any language in which we can identify statements (Popper, 1947, p. 202). Devising logical notions, consequently, becomes a matter strictly linked to discussing principles of rational discussion. This is the origin of (e.g.) the detection of different kinds of negations, and of the study of their possible coexistence, on the one hand, and, on the other hand, of the capability to envisage "new" logics, as the then "rediscovered" and called dual-intuitionism.

A proof-theoretic approach to formal epistemology

Sara Negri

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Ever since antiquity, attempts have been made to characterize knowledge through belief augmented by additional properties such as truth and justification. These characterizations have been challenged by Gettier counterexamples and their variants. A modern proposal, what is known as defeasibility theory, characterizes knowledge through stability under revisions of beliefs on the basis of true or arbitrary information. A formal investigation of such a proposal calls for the methods of dynamic epistemic logic: well developed semantic approaches to dynamic epistemic logic have been given through plausibility models (Baltag and Smets [1], Pacuit [4]) but a corresponding proof theory is still in its beginning. We shall recast plausibility models in terms of the more general neighbourhood models and develop on their basis complete proof systems, following a methodology introduced in Negri [3] and developed for conditional doxastic notions in Girlando et al. [2]. An inferential treatment of various epistemic and doxastic notions such as safe belief and strong belief will give a new way to study their relationships; among these, the characterizing equation, knowledge=belief stable under arbitrary revision, will be grounded through formal sequent calculus derivations.

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Polish Topologies for Graph Products of Groups

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We give several strong necessary conditions on the admissibility of a Polish group topology for an arbitrary graph product of groups $G(\Gamma, G_a)$, and use them to give a characterization modulo a finite set of nodes from Γ (under CH). As a corollary, we give a complete characterization under the assumption that all the factor groups G_a are countable (without CH).

Temporal irreflexivity in terms of a propositional constant

Matteo Pascucci

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This presentation illustrates an alternative method to force irreflexivity in temporal frames (i.e. relational frames used to interpret Priorean temporal logic). The method at issue is based on a non-standard semantics for propositional constants (in which they are not associated with fixed sets of instants) and on the notion of strict range of a system.

Stream Abstract Machines: Parallel And Non-Deterministic Execution

Marco Pedicini[1] and Mario Piazza[2]

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We study an approach to abstract machines and optimal reduction via streams, infinite sequences of elements. We first define a sequential abstract machine capable of performing Directed Virtual Reduction (DVR) and then we extend it to its parallel version, whose equivalence is explained through the properties of DVR itself. The result is a formal definition of the lambda-calculus interpreter called Parallel Environment for Lambda Calculus Reduction (PELCR), a software for lambda-calculus reduction based on the Geometry of Interaction. In particular, we describe PELCR as a stream-processing abstract machine, which in principle can also be applied to infinite streams. Finally, we present a demo implementation of our machine and we discuss the possibility of a non-deterministic execution.

Typoids in Martin-Löf's intensional type theory

Iosif Petrakis

Ludwig-Maximilians-Universität Munich

In this work we introduce the category of typoids, we prove some fundamental properties of univalent typoids, and we show how to encode certain higher inductive types within the theory of typoids.

Extension by conservation: from transfinite to finite proof methods in abstract algebra

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An additional axiom is conservative if reduction to the special case characterised by the axiom is possible. Semantically, this is typically put as an extension theorem, and proved by Zorn's lemma. The prime example perhaps is Krull's: every reduced commutative ring can be embedded into a product of integral domains, or alternatively for proving a definite clause about reduced rings one may assume that the ring be a domain. More often than not, however, a syntactical argument is possible at least for definite clauses, as shown before on individual instances, e.g., in locale theory, dynamical algebra, formal topology and proof analysis. We report on recent results in this area obtained by and with Davide Rinaldi and Daniel Wessel.

Measuring the Complexity of Reductions between Equivalence Relations

Luca San Mauro

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Joint work with Ekaterina Fokina and Dino Rossegger.

Computable reducibility is a tool to compare the complexity of various equivalence relations over the natural numbers. Based on this notion, we introduce degree spectra of reducibility and bi-reducibility. These spectra allow us to capture how much information is needed to compute possible reductions between given equivalence relations. We prove that any upward closed collection of Turing degrees with a finite basis can be realised as a reducibility spectrum, or as a bi-reducibility spectrum, and that reducibility and bi-reducibility spectra of Δ_2^0 equivalence relations can coincide, for any natural number n, with the union of n cones of Turing degrees.

The Paradox of No Properties in Ramified Logic

Giorgio Sbardolini

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This paper is a proof of concept for Ramified Intensional Logic. Russell's famous strategy to account for the paradoxes was ramification. It will be shown how to develop a version of Russell's insight into a simple framework, and it will be compared to rival accounts of the paradoxes. We will discuss what assumptions we are making when adopting a Ramified Logic, and show that some of Russell's own (and perhaps dubious) assumptions aren't indispensable. I will argue that Ramified Logics are available tools for analysis, and that they allow us to retain a classical language and a rich metaphysics.

Consider the following puzzle: Is there something with no properties? Yes, according to some contemporary metaphysicians (e.g. Ted Sider) who call such things bare particulars, and possibly also according to classical metaphysics (e.g. Aristotle's 'prime matter', or Locke's 'substratum'). However, suppose there is something with no properties. Let it be 'a'. But having no properties is a property (. . . isn't it?). So a has a property. Contradiction. Call this proof the Paradox of No Properties.

We can make the proof more rigorous by invoking a Property Comprehension Principle. This is a statement that establishes the existence of something (a property) given that a certain condition is satisfied. The question is, however, what to make of the proof we gave. It is perhaps a reductio of the claim that there's something with no properties—but that forces us to reject a lot of contemporary and classical metaphysics. It is perhaps a sound proof of a true contradiction—but that too forces us to reject a lot of contemporary metaphysics, by acceptance of a dialetheist ontology. I don't think that logic is metaphysically innocent, but I also don't think that existence or non-existence come cheap. In this paper I present an account of the Paradox in Ramified Intensional Logic. This will allow us to say that the proposition that a has no properties, and the proposition that a has a property (i.e. that of having no properties) don't contradict each other.

Ramified Logic are inspired by Russell's original insight into the logical paradoxes. We divide the type of propositions into levels, and thus the type of properties accordingly as functions from objects to propositions. I further simplify Russell's system by tying the increase of level only to the syntax of quantifiers. I argue further that acceptance of this type-theoretic restriction on quantification is independent on ontological restrictions imposed by any particular Comprehension Principle (although Russell may have thought so). The resulting logic system yields a flexible and consistent account of the paradoxes, on which negation and the other connectives maintain their classical definition, quantification has its standard interpretation, and on which the metaphysics is rich. Because of these theoretical virtues, I recommend acceptance of the framework for work in the metaphysics of intensional objects (properties, propositions). I address revenge problems by way of conclusion.

Denominator respecting maps

Luca Spada

Università di Salerno

The talk addresses the following problem. Let I be any set, one can define a "denominator" function on $[0,1]^I$, by sending each point in $[0,1]^I \setminus \mathbb{Q}^I$ to 0 and otherwise to the least common multiple of the denominators of the coordinates, written in reduced form, (the `lcm` being 0, in case the set of denominators is unbounded). Suppose that the points of a compact Hausdorff space X are labeled with natural numbers by a function $d \colon X \to \mathbb{N}$.

When does there exist an embedding of X into $[0,1]^I$, for some set I, that preserves d?

By "preserving d" here we mean that points labeled by d with a natural number n go into points with denominator equal to n. The logic import of this problem might not be self-evident, please see the longer abstract for motivations.

Playing with equivalent forms of CH

Silvia Steila

University of Bern

There are several mathematical statements which are equivalent to the Continuum Hypothesis (CH). For instance a famous equivalence by Sierpiński is: CH holds if and only if there are two subsets A and B of the real plane, whose union covers the real plane and such that any vertical section of A and any horizontal section of B are countable. In this talk we are going to analyse some variants of such statements and relate the corresponding versions to opportune relations between cardinal invariants. (On-going work with Alessandro Andretta and Raphaël Carroy).

Generic solutions of exponential polynomials over the complex field

Giuseppina Terzo

Università degli Studi della Campania "L. Vanvitelli"

Zilber in [2] proved that the class of exponential algebraically closed fields of characteristic 0, satisfying a natural $\mathcal{L}_{\omega_1\omega}(Q)$ sentence, expressing strong exponential closure, countable closure and cyclicity of the kernel of exponentiation, has a unique model in every uncountable cardinality. He conjectured that the complex exponential field is the unique such model of cardinality $2^{\aleph_0}$. Marker in [1] investigated a simple case of the strong exponential closure axiom, assuming Schanuel's Conjecture. Following this line of research, we examine the next natural cases of the strong exponential closure axiom for the complex exponential field. Assuming Schanuel's Conjecture, we prove that certain exponential polynomials have always a solution in the complex field which is generic. Moreover, we give information on the transcendence degree of the set of solutions.

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Decidability of the theory of modules over Bzout domains with infinite residue fields

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This is a joint work with Lorna Gregory, Sonia L'Innocente and Gena Puninski. We deal with decidability of first order theories of modules over Pruefer, in particular Bézout, domains with infinite residue fields. We assume that all these domains are effectively given, in particular countable, in order to ensure that the decision problems of their modules makes sense.

Recall that a domain is Pruefer if all its localizations at maximal ideals are valuation domains, and is Bézout if every finitely generated ideal is principal (which allows to define a notion of greatest common divisor satisfying the Bézout identities). Bézout domains are also Pruefer.

The model theory of modules over Bézout domains, with some hints at Pruefer domains, has been recently studied by Gena Puninski and the author. They also proved with Sonia L'Innocente the decidability of the theory of modules over the ring of algebraic integers, and obtained a similar result over Bézout domains built from principal ideal domains via the so called $D + M$ construction.

On the other hand Lorna Gregory proved that the theory of modules over an effectively given valuation domains is decidable if and only if there is an algorithm which decides the prime radical relation, namely, for every $a, b \in V$, decides whether a is in the radical of b, equivalently whether the prime ideals of V including b also contain a.

We develop a similar analysis over Pruefer domain, in terms of a radical relation extending that of valuation domains. Our main result states that, if B is an effectively given Bézout domain, then B-modules have a decidable theory if and only if there is an algorithm which, given $a, b, c, d \in B$, decides whether, for all prime ideals P, Q of B with $P + Q$ a proper subset of B, if P contains b then it includes also a or, if Q contains d, then it includes also c.

Generalizations to Pruefer domains are also proved and discussed.

As said we focus on domains B whose residue fields with respect to maximal ideals are infinite. This condition applies to several interesting algebraic examples, like the ring of algebraic integers, and ensures that any elementary invariant of B is either 1 or infinity — which sometimes simplifies the analysis.

We Need LOv abstract machine for call-by-need calculus, and its complexity

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The work presented is part of a new research branch initiated by B. Accattoli concerning the study of time cost models for λ-calculus, working in particu- lar on the reasonable ones, that is, polynomially related to the one of Turing machines. This is achieved by means of abstract machines, that are implementation schemas for fixed evaluation strategies that are a compromise between theory and practice: they are concrete enough to provide a notion of machine, and abstract enough to avoid the many intricacies of actual implementations. Abstract machines can be used to prove that the number of β-steps is a reasonable time cost model, i.e. a metric for time complexity. The study can then be reversed, exploring how to use this metric to study the relative complexity of abstract machines, that is, the complexity of the overhead of the machine with respect to the number of β-steps. Such a study leads to a new quantitative theory of abstract machines, where machines can be compared and the value of different design choices can be measured. In the talk I will deal with the open call-by-need calculus: this evaluation strategy exploits the β axiom of call-by-name and the substitution on demand of call-by-value, allowing open terms. This strategy is based on a λ-calculus redefined by means of Linear Substitution Calculus and is more performing based than the two mentioned. A new AM for open call-by-need, called We NEED LOv, is presented and its reasonableness is proved. This achievement for the machine is obtained through a labelling of explicit substitutions and other fea- tures, whose combination in a single machine had never been presented before.

On some varieties generated by generalized rotations of residuated lattices

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The aim of this contribution is to show that several notable subvarieties of $\mathbb{RL}$ can be characterized as those classes whose directly indecomposable elements are rotations of residuated lattices. In particular we define a generalized notion of rotation in order to obtain a directly indecomposable bounded residuated lattice whose regular elements form a finite MV-chain. The varieties generated will have a retraction in their MV-skeleton. Moreover, we obtain categorical equivalences with respect to categories whose objects are triples made of an n-potent MV-algebra, a distributive residuated lattice, and an operator assigning to each MV-element a filter of the lattice.

Trace spaces of Counterexamples to Naimark's Problem

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In a paper from 1951, Naimark asked whether any C*-algebra with only one irreducible representation up to unitary equivalence is isomorphic to the algebra of compact operators (on any Hilbert space). No progresses where made on this long-standing open problem until, in the 2004 paper "Consistency of a counterexample to Naimark's problem", Akemann and Weaver produced, assuming Jensen's diamond principle, a wide family of such counterexamples. We will briefly present their construction, and study how to modify it in order to get specific properties on the trace spaces of such counterexamples.

Useful Axioms

Matteo Viale

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We outline that forcing axioms, the axiom of choice, Baire's category theorem, Large cardinals. Los theorem for ultrapowers, Shoenfield's generic absoluteness are all manifestations of the same type of non constructive principle and can all be suitably expressed in the language of partial orders.