Analysis and Control on Networks: trends and perspectives

Padova, March 9th – 11th, 2016
DEI - Aula Magna "Lepschy"

Book of Abstracts

Università degli Studi di Padova

Dipartimento Matematica

Organizing and Scientific Committee

Fabio Ancona (Università di Padova)	Laura Caravenna (Università di Padova)
Annalisa Cesaroni (Università di Padova)	Giuseppe M. Coclite (Università di Bari)
Claudio Marchi (Università di Padova)	Andrea Marson (Università di Padova)

Speakers

Boris Andreianov	(Université de Tours)
Georges Bastin	(Université Catholique de Louvain)
Alberto Bressan	(Penn State University)
Fabio Camilli	(Università di Roma La Sapienza)
Rinaldo M. Colombo	(Università di Brescia)
Emiliano Cristiani	(IAC, CNR)
Pierre Degond	(Imperial College, London)
Marco Di Francesco	(Università de L'Aquila)
Mauro Garavello	(Università di Milano Bicocca)
Olivier Glass	(Université Paris-Dauphine)
Paola Goatin	(INRIA Sophia Antipolis - Méditerranée)
Martin Gugat	(F.-A.-Universität Erlangen-Nrnberg)
Ke Han	(Imperial College, London)
Michael Herty	(RWTH Aachen University)
Serge Paul Hoogendoorn	(Delft University of Technology)
Cyril Imbert	(CNRS, Ecole Normal Supériore Paris)
Axel Klar	(Technische Universität Kaiserslautern)
Corrado Lattanzio	(Università de L'Aquila)
Pierangelo Marcati	(GSSI, Università de L'Aquila)
Amos Maritan	(Università di Padova)
Massimiliano Daniele Rosini	(Lublin University of Technology)
Andrea Tosin	(Politecnico di Torino)
Emmanuel Trélat	(Université Pierre et Marie Curie)

Well-posedness for some LWR models on a junction

Boris Andreianov
Université de Tours, France
boris.andreianov@lmpt.univ-tours.fr

The main goal of the talk is to demonstrate well-posedness, in the frame of vanishing viscosity limits, for the Cauchy problem on a traffic junction where m incoming and n outgoing roads meet. The traffic on each road is governed by an LWR model: for all $h \in \{1, \dots, m, m+1, \dots, m+n\}$

$$\partial_t \rho_h + \partial_x(\rho_h v_h) = 0, \;\; v_h = V_h(\rho_h), \;\; V_h : [0, R] \to [0, V_{max,h}].$$

Mass conservation is prescribed at the junction, but also additional admissibility conditions are implicitly contained in the requirement that the solution is a limit of vanishing viscosity solutions. One's hope is to write these implicit admissibility conditions under the form of suitable entropy inequalities, and then derive uniqueness of solutions satisfying the latter. Thus, the main question is: which form should take such entropy inequalities at the junction?

This problem was addressed in [3], where it was completely solved in the case where $n = m$ and all the $\{m + n\}$ roads are governed by the same closure law $v = V(\rho)$. This talk addresses the general case. We provide a suitable intrinsic characterization of vanishing viscosity limits at the junction based upon a complete description of the set of constant per branch stationary solutions, in the same spirit as in the works [1,2] devoted to the discontinuous-flux case $n = m = 1$. We prove uniqueness of admissible solutions; we exploit [3] in order to justify their existence and the link to the vanishing viscosity approximation.

It should be stressed that very different Riemann solvers for the junction were considered in the literature. We discuss extension of results to junction conditions other than the vanishing viscosity. Roughly speaking, such extension is possible only under monotonicity assumptions at the junction Riemann solver. In absence of natural regularization procedures, one can prove existence via finite volume approximation based on branches' and junction Godunov fluxes.

References

[1] B. Andreianov and C. Cancès, On interface transmission conditions for conservation laws with discontinuous flux of general shape, *J. Hyperbolic Differ. Equ.* **12** (2015) 343–384.

[2] B. Andreianov, K. H. Karlsen and N. H. Risebro, A theory of L^1-dissipative solvers for scalar conservation laws with discontinuous flux, *Arch. Ration. Mech. Anal.* **201** (2011), 27–86.

[3] G. M. Coclite and M. Garavello, Vanishing viscosity for traffic on networks, *SIAM J. Math. Anal.* **42** (2010), 1761–1783.

Joint work with: Giuseppe Maria Coclite (*Università degli Studi di Bari, Italy*) and Carlotta Donadello (*Université de Franche-Comté, Besançon, France*)

Stability and boundary feedback control of networks of conservation and balance laws

Georges Bastin

Department of Mathematical Engineering,
ICTEAM, Université catholique de Louvain, Belgium

Georges.Bastin@uclouvain.be

The operation of many physical networks having an engineering relevance can be represented by hyperbolic systems of conservation or balance laws in one space dimension. Typical examples are hydraulic networks (for water supply, irrigation or navigation), road traffic networks, electrical line networks, gas transportation networks, networks of heat exchangers, communication networks, blood flow networks etc.

Such physical networks can generally be schematized by using a graph representation. The edges of the network represent the physical links (for instance the pipes, the canals, the roads, the electrical lines, etc ...) that are governed by hyperbolic systems of conservation or balance laws. Typically, the links carry some kind of flow and the network has only a few nodes where flows enter or leave the network. The other nodes of the network represent the physical junctions between the links. The mechanisms that occur at the junctions are described by 'junction models' under the form of algebraic or differential relations that determine the boundary conditions of the PDEs.

In this talk, we consider networks that can be monitored and controlled at the junctions. These control systems can be open loop unstable and subject to unmeasured boundary disturbances. We address the issue of feedback stabilization and disturbance rejection. Explicit necessary and sufficient conditions for robust stabilization are provided in the frequency domain for linear systems. Using a Lyapunov approach, sufficient stability conditions are given for quasi-linear systems. The theory is illustrated with application examples to electrical networks, hydraulic networks and road traffic networks.

References

[1] G. Bastin and J-M. Coron, *Stability and Boundary Stabilization of 1-D Hyperbolic Systems* , Birkhauser, (2016), in press.

[2] J-M. Coron, R. Vazquez, M. Krstic and G. Bastin, Local exponential H2 stabilization of 2x2 quasilinear hyperbolic systems using backstepping, *SIAM Journal on Control and Optimization*, **Volume 51(3)** (2013), pp. 2005-2035.

[3] G. Bastin, J-M. Coron and S-O Tamasoiu, Stability of linear density-flow hyperbolic systems under PI boundary control, *Automatica*, **Volume 53(1)** (2015), pp. 37-42.

[4] J-M. Coron, G. Bastin and B. d'Andrea-Novel, Dissipative boundary conditions for one-dimensional nonlinear hyperbolic systems, *SIAM Journal on Control and Optimization*, **Volume 47(1)** (2008), pp. 1460-1498.

Joint work with: Jean-Michel Coron (*Laboratoire Jacques-Louis Lions, Sorbonne Université, Université Pierre et Marie Curie Paris 06, France*)

Traffic flow on networks: recent results and open problems

Alberto Bressan

Center for Interdisciplinary Mathematics, Mathematics Department
Penn State University, USA

bressan@math.psu.edu

The talk will focus on conservation law models for traffic flow on a network of roads. Some recent progress on the modeling of intersections will be reviewed, together with problems of global optimization and users' equilibria. A number of related open problems will be discussed.

Parabolic models for chemotaxis on weighted networks

Fabio Camilli
Dipartimento di Scienze di Base e Applicate per l'Ingegneria,
"Sapienza" Università di Roma,
camilli@dmmm.uniroma1.it

We consider the Keller-Segel model for chemotaxis on networks, both in the doubly parabolic case and in the parabolic-elliptic one. Introducing appropriate transition conditions at vertices, we prove the existence of a time global and spatially continuous solution for each of the two systems. The main tool is the use of the explicit formula for the fundamental solution of the heat equation on a weighted graph and of the corresponding sharp estimates.

Joint work with: Lucilla Corrias (*Université d'Evry Val d'Essonne*)

Control Problems in Structured Population Models

Rinaldo M. Colombo
INdAM Unit, University of Brescia
rinaldo.colombo@unibs.it

Various structured population models lead to initial–boundary value problems for balance laws, often equipped with non-local boundary conditions. First, the present talks overviews recent results on the well posedness of such models. Different analytic environments are considered, i.e., measure spaces and sets of **BV** functions. In the latter case, examples are shown where a realistic modeling requires renewal equations to be set on graphs. Then, various control problems are stated and, where possible, an effective strategy to tackle them is proposed and pursued.

References

[1] Rinaldo M. Colombo, Mauro Garavello: *Control of Biological Resources on Graphs*. Preprint, 2015.

[2] Rinaldo M. Colombo, Mauro Garavello: *Stability and Optimization in Structured Population Models on Graphs*. Mathematical Biosciences and Engineering, 12, 2, 311-335, 2015.

[3] Rinaldo M. Colombo, Piotr Gwiazda, Magdalena Rosińska: *Optimization in Structure Population Models through the Escalator Boxcar Train*. Preprint, 2016.

Joint works with: Mauro Garavello, (*Università di Milano–Bicocca*); Piotr Gwiazda (*Polish Academy of Sciences*); Magdalena Rosińska (*University of Warsaw*).

Analysis and control of an (almost) intersection-free model for traffic flow on networks

Emiliano Cristiani
Istituto per le Applicazioni del Calcolo,
Consiglio Nazionale delle Ricerche, Rome, Italy
e.cristiani@iac.cnr.it

In this talk we first recall the main properties of the macroscopic (almost) intersection-free traffic flow model introduced in [1,2], and we show that this model corresponds to the microscopic Follow-the-Leader model on networks when the number of vehicles tends to infinity [4]. We also use the model as building block to describe and control the dynamics of a large number of drivers in the framework of mean field games. We assume that each driver aims at reaching a specific destination in minimal time and can choose his path runtime according to the traffic conditions, taking decisions at junctions [3].

References

[1] G. Bretti, M. Briani and E. Cristiani, An easy-to-use algorithm for simulating traffic flow on networks: Numerical experiments, *Discrete Contin. Dyn. Syst. Ser. S*, **7** (2014), pp. 379-394.

[2] M. Briani and E. Cristiani, An easy-to-use algorithm for simulating traffic flow on networks: Theoretical study, *Netw. Heterog. Media*, **9** (2014), pp. 519-552.

[3] E. Cristiani and F. S. Priuli, A destination-preserving model for simulating Wardrop equilibria in traffic flow on networks, *Netw. Heterog. Media*, **10** (2015), pp. 857-876.

[4] E. Cristiani and S. Sahu, On the micro-to-macro limit for first-order traffic flow models on networks, preprint (2015), to appear on *Netw. Heterog. Media*.

Joint work with: G. Bretti (*IAC-CNR*), M. Briani (*IAC-CNR*), F. S. Priuli (*IAC-CNR*), S. Sahu (*Durham University*).

Models of network formation

Pierre Degond

Department of Mathematics, Imperial College London, London SW7 2AZ, United Kingdom

pdegond@imperial.ac.uk

In this talk, we will present a modelling framework for the formation and evolution of networks and give two examples of applications: the first one is the formation of ant trails; the second one is inspired from vasculo or angiogenesis of blood capillaries. We believe this framework can apply to other types of networks in which the topology and topography of nodes and links is fuzzy and evolutive.

A deterministic particle approximation for nonlinear conservation laws

Marco Di Francesco
University of L'Aquila
marco.difrancesco@univaq.it

We present recent results on the deterministic particle approximation of nonlinear conservation laws. In [1], the unique entropy solution to

$$\rho_t + (\rho v(\rho))_x = 0$$

with a given initial datum in L^∞ and with strictly monotone v was rigorously approximated by the empirical measure of the follow-the-leader particle system

$$\dot{x}_i(t) = v\left(\frac{m}{x_{i+1}(t) - x_i(t)}\right) . \tag{1}$$

Said result is based on a discrete version of the classical Oleinik one-sided jump condition for L^∞ initial data and on a BV contraction estimate for BV initial data. The former requires some additional conditions on v, which reduces to strict concavity of the flux $\rho v(\rho)$ in case $v(\rho)$ is a power law. The convergence result also holds for the discrete density constructed from the particle system. The results in [1] have been recently extended to the Aw-Rascle-Zhang model for traffic flow in [2], where a similar BV contraction estimate has been proven, based on the interpretation of the system as a multi-population model. Finally, we shall present an extension of this technique to the Hughes model for pedestrians

$$\rho_t - (\rho v(\rho)\frac{\phi_x}{|\phi_x|})_x = 0\,, \quad |\phi_x| = c(\rho)\,, \tag{2}$$

on a bounded interval with Dirichlet boundary conditions. In [3] we prove the rigorous convergence of a suitable adaptation of the particle scheme (1) to the unique entropy solution to the IBV problem for (2).

References

[1] M. Di Francesco and M. D. Rosini, Rigorous derivation of nonlinear scalar conservation laws from follow-the-leader type models via many particle limit, *Archive for rational mechanics and analysis*, **217 (3)** (2015), pp. 831-871.

[2] M. Di Francesco, S. Fagioli, and M. D. Rosini, Many particle approximation for the Aw-Rascle-Zhang second order model for vehicular traffic, *Submitted preprint.*

[3] M. Di Francesco, S. Fagioli, M. D. Rosini, and G. Russo, Deterministic particle approximation of the Hughes model in one space dimension, *In preparation.*

Joint work with: Simone Fagioli (*University of L'Aquila*), Massimiliano D. Rosini (*Lublin University of Technology*), Giovanni Russo (*University of Catania*)

An optimal solution at junctions for traffic

Mauro Garavello
Università di Milano Bicocca
mauro.garavello@unimib.it

We consider a junction composed by m incoming roads and n outgoing ones. On each road we describe traffic by the Lighthill-Whitham-Richards model. We address our attention to an optimal control problem for the Cauchy problem at the junction. More precisely, given $T > 0$, we want to select a solution to the Cauchy problem which maximizes an integral junctional J, which depends on the solution over the whole time interval $(0, T)$. We present various results about this problem. In particular, optimal solutions can not be constructed by using Riemann solvers at the junction.

Joint work with: F. Ancona (Università di Padova), A. Cesaroni (Università di Padova), and G.M. Coclite (Università di Bari)

Some results on the boundary control of systems of conservation laws

Olivier Glass
CEREMADE, Université Paris-Dauphine
glass@ceremade.dauphine.fr

The theory of boundary control for system of conservation laws leave many open problems, especially in the context of entropy solutions, since the seminal works by Ancona-Coclite [1], Ancona-Marson [2] and Bressan-Coclite [3]. The general question is to "appropriately" drive the system by choosing the boundary conditions relevantly. As is well-known, the boundary conditions are themselves a difficult issue.

In this talk, I will discuss two partial results in the field. The first one establishes a controllability result for a very particular system: the Euler system for compressible flows, when one controls on both sides of the domain. Investigate further controllability properties at a node connecting two intervals would be a natural next step. The second result, in collaboration with Jean-Michel Coron (Paris), Sylvain Ervedoza (Toulouse), Shyam Sundar Ghoshal (L'Aquila) and Vincent Perrollaz (Tours) considers a problem of boundary feedback stabilization for 2×2 systems, as a continuation of what is known in the case of classical solutions. These types of boundary feedbacks can typically be used on connected intervals.

References

[1] Ancona F., Coclite G.M., On the attainable set for Temple class systems with boundary controls, SIAM J. Control Optim. 43 (2005), no. 6, pp. 2166–2190.

[2] Ancona F., Marson A., On the attainable set for scalar nonlinear conservation laws with boundary control, SIAM J. Control Optim. 36 (1998), no. 1, pp. 290–312.

[3] Bressan A., Coclite G.M., On the boundary control of systems of conservation laws, SIAM J. Control Optim. 41 (2002), no. 2, pp. 607–622.

Optimal control of networks of discretized PDEs: application to road traffic management

Paola Goatin
Inria Sophia Antipolis Méditerranée
paola.goatin@inria.fr

Macroscopic traffic flow models derived from fluid dynamics are very popular nowadays both for vehicular and pedestrian flows. They offer a sound mathematical basis relying on well posedness results for hyperbolic non-linear conservation laws, as well as fast and efficient numerical tools consisting of finite volume schemes. In particular, traffic dynamics on road networks can be described by conservation laws coupled with suitable boundary conditions at junctions. This approach allows for efficient coordinated ramp-metering and rerouting strategies for reducing traffic congestion.

References

[1] M.L. Delle Monache, J. Reilly, S. Samaranayake, W. Krichene, P. Goatin and A. Bayen, A PDE-ODE model for a junction with ramp buffer, *SIAM J. Appl. Math.*, **74(1)** (2014), 22-39.

[2] J. Reilly, W. Krichene, M.L. Delle Monache, S. Samaranayake, P. Goatin and A. Bayen, Adjoint-based optimization on a network of discretized scalar conservation law PDEs with applications to coordinated ramp metering, *J. Optim. Theory Appl.*, to appear.

[3] S. Samaranayake, J. Reilly, W. Krichene, M.L. Delle Monache, P. Goatin and A. Bayen, Discrete-time system optimal dynamic traffic assignment (SO-DTA) with partial control for horizontal queuing networks, in revision.

Joint work with: Alexandre Bayen (*University of California - Berkeley*), Maria Laura Delle Monache (*University Rutgers - Camden*), Walid Krichene (*University of California - Berkeley*), Jack Reilly (*Google*), Samitha Samaranayake (*Cornell University*).

Control on gas distribution networks

Martin Gugat
Friedrich-Alexander-Universität Erlangen-Nürnberg
martin.gugat@fau.de

We consider the isothermal Euler equations as a model for the gas flow through pipes. In a network, the flow through the junctions is governed by algebraic node conditions that are linear in terms of the Riemann invariants. We consider stationary states and traveling waves solutions on the networks. We present results on the existence of solutions on networks and also on the stabilization of the system locally around desired stationary states.

References

[1] M. Gugat, F.M. Hante, M. Hirsch-Dick and G. Leugering, Stationary states in gas networks, *Networks and Heterogeneous Media* **10**, (2015), pp. 295–320.

The Mathematical Foundation of Dynamic User Equilibria

Ke Han
Center for Transport Studies, Imperial College London
k.han@imperial.ac.uk

Dynamic traffic assignment (DTA) is the modeling of time-varying network traffic that is consistent with established traffic flow theory and travel choice behavior. Depending on the assignment principle, DTA problems can be broadly categorized as *system optimal* and *user equilibrium.* This presentation focuses on the latter, i.e. *dynamic user equilibrium* (DUE), which corresponds to a Nash-like game, and seeks network flow patters such that experienced travel cost, including delay and arrival penalties, is identical for those route and departure time choices selected by travelers between a given origin-destination pair. DUE is a widely pursued topic in transportation planning and engineering due to its capability to capture drivers' departure time and route choices in a wide range of contexts (e.g. network design, traffic signal control, congestion charging, route navigation).

In this presentation, formulation of the DUE model as a variational inequality and a differential variational inequality will be presented, followed by an existence result in a continuous-time setting. The network performance sub-model within DUE, known as *dynamic network loading*, will be described in conjunction with the Lighthill-Whitham-Richards model. Computational DUE problems will be analyzed in detail with several solution algorithms and their convergence conditions presented, as well as numerical results on several networks illustrated. Finally, we will mention a few extensions of DUE related to travel demand elasticity and bounded user rationality, as well as its applications to congestion pricing and signal optimization, followed by a few open mathematical problems.

References

[1] Bressan, A., Han, K., Optima and equilibria for a model of traffic flow, *SIAM Journal on Mathematical Analysis*, **43 (5)** (2011), pp. 2384-2417.

[2] Han, K., Szeto, W.Y., Friesz, T.L., Formulation, existence, and computation of boundedly rational dynamic user equilibrium with fixed or endogenous user tolerance, *Transportation Research Part B*, **79** (2015), pp. 16-49.

[3] Wardrop, J., Some theoretical aspects of road traffic research, in *ICE Proceedings: Part II, Engineering Divisions*, 1, (1952), pp. 325-362.

[4] Peeta, S., Ziliaskopoulos, A., Foundations of dynamic traffic assignment: The past, the present and the future, *Networks and Spatial Economics*, **1** (2001), pp. 233-265.

Joint work with: Alberto Bressan(*Penn State*), Terry Friesz (*Penn State*), Benedetto Piccoli (*Rutgers University*).

Control of traffic flow and supply chain networks

Michael Herty
RWTH Aachen University
`herty@mathc.rwth-aachen.de`

We are interested in the control of flows on networks. The dynamics on the arcs is typically governed by (systems of) hyperbolic conservation or balance laws. The coupling at nodal points of networks may involve a control variable. The time dependent control of such a variable is of importance not only from a theoretical point of view but also from the application. We present recent results on flow problems for traffic flow and high volume production lines. We discuss derivation and implementation of suitable controls

Pedestrian Dynamics and Networks: Theory, Modelling and Applications

Serge P. Hoogendoorn

Delft University of Technology, The Netherlands

S.P.Hoogendoorn@tudelft.nl

The need for accurate pedestrian flow models is huge: whether we want to predict if unsafe situations may occur during a mass event, or if we want to determine expected levels-of-service for a novel transfer station design, we need falsified theory and validated models. That said, the processes we are trying to model are highly complex, among other things due to the many interactions that underly the dynamics. These interactions entail both the operational level (walking, waiting), but also the tactical (route choice, activity scheduling) and strategic level (knowledge acquisition, memory decay), and lead to various interesting yet poorly understood phenomena (self-organisation, spontaneous phase transitions). This talk deals with recent developments in the field op pedestrian dynamics, in particular focussing on modelling of pedestrian operations in large scale infrastructures such as railway stations. We will present novel macroscopic modelling approaches for the combined operational and tactical level. We will discuss their derivation as well as the properties of the model. In looking for numerical solution approaches, we show the relation with (traditional) discrete network modelling. Having presented the models, we will briefly discuss different application in monitoring and state estimation, design assessment and optimisation in case of emergencies.

Degenerate parabolic equations on networks

Cyril Imbert

CNRS & ÉNS (Paris)

Cyril.Imbert@ens.fr

In this paper, we study degenerate parabolic equations posed on *junctions*. These networks are made of one vertex and a finite number of infinite edges. Degenerate parabolic equations include Hamilton-Jacobi equations whose study on junctions (and networks) recently attracted a lot of attention. We explain here that the approach proposed by the first author and Monneau (2014) can be further developed to handle second order terms (disappearing at the junction) and generalized junction conditions such as the generalized Kirchoff ones. We prove that generalized junction conditions reduce to flux-limited ones, which are of control-type. We also prove a comparison principle for so-called L-relaxed solutions by modifying in an appropriate way the construction of the vertex test function (Imbert, Monneau – 2014).

Joint work with: Vinh Duc Nguyen (*Université Paris-Est*)

Kinetic and related macroscopic models for chemotaxis on networks

Axel Klar
TU Kaiserslautern
klar@mathematik.uni-kl.de

We discuss kinetic and associated macroscopic models for chemotaxis on a network. Coupling conditions at the nodes of the network for the kinetic problem are presented and used to derive coupling conditions for the macroscopic approximations. The results of the different models are compared and relations to a Keller-Segel model on networks are discussed. For a numerical approximation of the governing equations asymptotic preserving relaxation schemes are extended to directed graphs. Kinetic and macroscopic equations are investigated numerically and their solutions are compared for tripod and more general networks. Coupling of network problem with 2D equations is discussed as well.

References

[1] R. Borsche, A. Klar, and T.N.H. Pham, ar, T.N.H. Pham, Kinetic and related macroscopic models for chemotaxis on networks, to appear in *M3AS*, (2016)

[2] R. Borsche, S. Göttlich, A. Klar, and P. Schillen, The Keller-Segel model on networks, *M3AS*, (2014), pp. 221-247

[3] R. Borsche, A. Klar, S. Kühn, A. Meurer, Coupling traffic flow networks to pedestrian and crowd motion, *M3AS*, (2014), pp. 359-380

[4] R. Borsche, A. Klar, Flooding in urban drainage systems: coupling hyperbolic conservation laws for sewer systems and surface flow, *Int. J. Num. Meth. Fluids*, **76** (2014), pp. 789-810

Joint work with: Raul Borsche (*TU Kaiserslautern*), Simone Göttlich (*U Mannheim*), T. N. Ha Pham (*TU Kaiserslautern*), P. Schillen (*U Mannheim*)

Multiscale models for traffic flow dynamics

Corrado Lattanzio
Università deli Studi dell'Aquila
corrado@univaq.it

The aim of this talk is to present multiscale models in the description of traffic flow dynamics in presence of special (slower and larger) vehicles. The situation we are interested in is that of *moving bottlenecks*: the presence of the special vehicles influences significantly the whole traffic in terms of a nonnegligible capacity dropping of the total flow. The resulting models consist in ODEs for the trajectories of (microscopic) slower vehicles fully coupled with a conservation law for the (macroscopic) car density. The coupling takes into account both the microscopic vehicles as moving bottlenecks in the PDE for the regular car flow, and the effects of the latter on the ODEs.

Numerical simulations are provided, also in the case of a bus route in a urban network of roads.

References

[1] G. Bretti, E. Cristiani, C. Lattanzio, A. Maurizi, and B. Piccoli, A coupled PDE–ODE tracking algorithm for moving bottlenecks in traffic flow (forthcoming)

[2] I. Gasser, C. Lattanzio, and A. Maurizi, Vehicular traffic flow dynamics on a bus route, *Multiscale Model. Simul.*, **11** (2013), pp. 925-942

[3] C. Lattanzio, A. Maurizi, and B. Piccoli, Moving bottlenecks in car traffic flow: a PDE-ODE coupled model, *SIAM J. Math. Anal.*, **43** (2011), pp. 50-67

[4] C. Lattanzio, A. Maurizi, and B. Piccoli, Modeling and simulation of vehicular traffic flow with moving bottlenecks, in MASCOT09 Proceedings, vol. 15 of *IMACS Series in Computational and Applied Mathematics*, F. Pistella and R. M. Spitaleri editors (2010), pp. 181-190

Results in collaboration with: Gabriella Bretti and Emiliano Cristiani (*IAC–CNR*), Ingenuin Gasser (*Hamburg Univ.*), Amelio Maurizi, Benedetto Piccoli (*Rugters Univ., Camden*)

Quantum hydrodynamics analysis of Quantum synchronization over quantum Networks

Pierangelo Marcati
GSSI & Università deli Studi dell'Aquila
pierangelo.marcati@gssi.infn.it

We recall various models regarding classical synchronization [3] and we show how to construct the quantum corresponding Lohe model [4],[5]. We show how to use the QHD approach developed in [1],[2] to describe the problem. A proof regarding the 2-node case (QuBit) is given.

References

[1] P. Antonelli, P. Marcati, On the finite energy weak solutions to a system in Quantum Fluid Dynamics, *Comm. Math. Phys.* 287, (2009) no. 2, pp. 657-686.

[2] P. Antonelli, P. Marcati, The Quantum Hydrodynamics system in two space dimensions, *Arch. Ration. Mech. Anal.* 203, (2012) no. 2, pp. 499-527.

[3] S.-Y. Ha, T, Ha and J.-H. Kim, On the complete synchronization of the Kuramoto phase model, *J. Phys. D* 239, (2010) no. 17, pp. 1692-1700.

[4] M.A. Lohe, Non-Abelian Kuramoto model and synchronization, *J. Phys. A* 42, (2009) no. 39, 395101, 25 pp.

[5] M.A. Lohe, Quantum synchronization over quantum networks, *J. Phys. A* 43, (2010) no. 46, 465301, 20 pp.

Joint work with: Paolo Antonelli (*GSSI, L'Aquila*), Seung-Yeal Ha (*Seoul National University*), Dohyun Kim (*Seoul National University*)

Emergence of structural and dynamical properties of ecological mutualistic networks

Amos Maritan
Dipartimento di Fisica e Astronomia, Università di Padova, CNISM and INFN, via Marzolo 8, I-35131 Padova, Italy
maritan@pd.infn.it

Mutualistic networks are formed when the interactions between two classes of species are mutually beneficial and they are important examples of cooperation shaped by evolution. The topological properties of the ecological interaction networks have been the subject of sparkling research and they indicate non-random pattern of community organization. Indeed, ecologists have collected extensive data on species interactions showing that, independently of species composition and latitude, mutualistic networks (such as plant-pollinator systems) have nested architectures: specialist species, with only few mutualistic links, tend to interact with a proper subset of the many mutualistic partners of any of the generalist species. Despite sustained efforts to explain observed network structure on the basis of community-level stability or persistence, such correlative studies have reached minimal consensus. It will be show how nested interaction networks emerge as a consequence of an op! timization-variational principle. Nested networks also attenuates the impact of the propagation of perturbations on species abundance through localization of the principal eigenvector of the linearized dynamics.

References

[1] S. Suweis, J. Grilli, J.R. Banavar and A. Maritan, Effect of localization on the stability of mutualistic ecological networks, *Nat. Comm.*, **6** (2015), DOI: 10.1038/ncomms10179

[2] S. Suweis, F. Simini, J.R. Banavar and A. Maritan, Emergence of structural and dynamical properties of ecological mutualistic networks, *Nature*, **500** (2015) 449-452.

Joint work with: Samir Suweis, Jayanth Banavar, Jacopo Grilli and Filippo Simini

Conservation laws with point constraints on the flow and their applications

Massimiliano Daniele Rosini
Uniwersytet Marii Curie-Skłodowskiej
`mrosini@umcs.lublin.pl`

In this talk we present the recent results achieved for conservation laws with variable unilateral point constraints on the flow. Vehicular traffic flows through a toll gate and pedestrian flows through an exit door are the main motivating applications behind these equations.

More precisely, we first consider the Cauchy problem for a scalar conservation law with a *non-local* point constraint proposed in [2]. In particular, as underlined in [4], we also show how the regularity of the constraint operator impacts the well-posedness of the problem. We then present the *ad hoc* simulations performed in [5] to qualitatively validate the model in the framework of crowd dynamics, by proving its ability to reproduce typical phenomena at the bottlenecks, such as the faster-is-slower effect and the Braess' paradox.

The final part of the talk deals with the Cauchy problem for the second order model for vehicular traffic introduced in [1,7] with a point constraint, see [3,6].

References

[1] A. Aw and M. Rascle, Resurrection of "second order" models of traffic flow, *SIAM J. Appl. Math.* **60** (2000), pp. 916-938

[2] B. Andreianov, C. Donadello and M. D. Rosini, Crowd dynamics and conservation laws with nonlocal constraints and capacity drop, *Mathematical Models and Methods in Applied Sciences* **24** (2014), pp. 2685-2722

[3] B. Andreianov, C. Donadello and M. D. Rosini, A second order model for vehicular traffics with local point constraints on the flow, to appear on *Mathematical Models and Methods in Applied Sciences*

[4] B. Andreianov, C. Donadello, U. Razafison and M. D. Rosini, Riemann problems with non-local point constraints and capacity drop, *Mathematical Biosciences and Engineering* **12** (2015), pp. 259-278

[5] B. Andreianov, C. Donadello, U. Razafison and M. D. Rosini, Qualitative behaviour and numerical approximation of solutions to conservation laws with non-local point constraints on the flux and modeling of crowd dynamics at the bottlenecks, to appear on *ESAIM: Mathematical Modelling and Numerical Analysis*

[6] B. Andreianov, C. Donadello, U. Razafison, J. Y. Rolland and M. D. Rosini, Solutions of the Aw-Rascle-Zhang system with point constraints, to appear on *Networks and Heterogeneous Media*

[7] H. M. Zhang, A non-equilibrium traffic model devoid of gas-like behavior, *Transportation Res., Part B* **36** (2002), pp. 275-290

Joint work with: Boris Andreianov (*Université de Tours*), Carlotta Donadello (*Université de Franche-Comté*), Ulrich Razafison (*Université de Franche-Comté*), Julien Yves Rolland (*Université de Franche-Comté*).

A Boltzmann-type kinetic approach to traffic flow on road networks

Andrea Tosin
Department of Mathematical Sciences "G. L. Lagrange", Politecnico di Torino
andrea.tosin@polito.it

In this talk I will present a kinetic approach to the modelling of vehicular traffic on road networks. Sticking to the idea that the macroscopic traffic flow is ultimately generated by microscopic dynamics involving locally few vehicles, the approach consists in linking the changes of speed of the vehicles to binary interactions among them. Such microscopic interactions are then described probabilistically by implementing them in a Boltzmann-type collisional operator. The microscopic point of view partly retained by the kinetic approach further allows one to devise conditions at the junctions of the network (such as e.g., flow splitting and priority rules) inspired by a close look at the individual behaviour of the drivers (e.g., choice of the speed at which to cross the junction).

References

[1] L. Fermo and A. Tosin, A fully-discrete-state kinetic theory approach to traffic flow on road networks, *Math. Models Methods Appl. Sci.*, **25** (2015), pp. 423-461

[2] M. Herty and S. Moutari, A macro-kinetic hybrid model for traffic flow on road networks, *Comput. Methods Appl. Math.*, **9** (2009), pp. 238-252

Joint work with: Luisa Fermo (*Department of Mathematics and Computer Science, University of Cagliari*)

Sparse control and stabilization to consensus of collective behavior models

Emmanuel Trélat

Université Pierre et Marie Curie (Paris 6), Laboratoire Jacques-Louis Lions and Institut Universitaire de France

emmanuel.trelat@upmc.fr

In the last years there have been many works on models of self-alignment and consensus dynamics, with the objective of modelling and explaining self-organization. However, the evidence that in practice self-organization does not necessarily occur leads to the natural question of whether it is possible to externally influence the dynamics in order to promote the emergence of some desired patterns. Once this question is posed, one is also faced with the issue of defining the best way of obtaining the result, seeking for the most "economical" way to achieve a certain outcome.

In recent works with M. Caponigro, M. Fornasier, B. Piccoli, and F. Rossi, we address the issue of finding the sparsest control strategy in order to lead us optimally towards a given outcome, in this case the achievement of a state where the group will be able by self-organization to reach an alignment consensus.

As a consequence we provide a mathematical justification to the general principle according to which "sparse is better": in order to achieve group consensus, a policy maker not allowed to predict future developments should decide to control with stronger action the fewest possible leaders rather than trying to act on more agents with minor strength. Our main model is the Cucker-Smale model, both in finite dimension and in infinite dimension (kinetic Cucker-Smale model, obtained by mean-field limit). We establish local and global sparse controllability properties to consensus.

References

[1] M. Caponigro, M. Fornasier, B. Piccoli, E. Trélat, Sparse stabilization and optimal control of the Cucker-Smale model, *Math. Cont. Related Fields*, **Vol. 3 no. 4** (2013), pp. 447-466

[2] M. Caponigro, M. Fornasier, B. Piccoli, E. Trélat, Sparse stabilization and control of alignment models, *Math. Models Methods Applied Sciences*, **Vol. 25 no. 3** (2015), pp. 521-564

[3] B. Piccoli, F. Rossi, E. Trélat, Control to flocking of the kinetic Cucker-Smale model, *SIAM J. Math. Anal.*, **Vol. 47 no. 6** (2015), pp. 4685-4719

[4] B. Piccoli, F. Rossi, E. Trélat, Control of the 1D continuous version of the Cucker-Smale model, in *American Control Conference (ACC)* (2015), pp. 1264-1269

Joint works with: Marco Caponigro (*CNAM Paris*), M. Fornasier (*Münich*), B. Piccoli (*Rutgers Camden*), F. Rossi (*Marseille*).