ASTA 2014: Talks

Spineto, June 16 - 20

Leavitt path algebras of Cayley graphs arising from cyclic groups

Gene Abrams

Department of Mathematics, University of Colorado, Colorado Springs, Colorado 80918, U.S.A.
abrams@math.uccs.edu

Let n be any positive integer, and K any field. Let C_n denote the Cayley graph corresponding to the cyclic group $\mathbb{Z}/n\mathbb{Z}$ with respect to the generators $\{1, n-1\}$. In this talk we describe the Leavitt path algebra $L_K(C_n)$. Specifically, we show that there are exactly four isomorphism classes of such Leavitt path algebras, arising as the algebras corresponding to the graphs C_i $(3 \leq i \leq 6)$. The main tool utilized in our analysis is the Algebraic Kirchberg-Phillips Theorem.

Classical Lie theory from the point of view of monads

A. Ardizzoni*

J. Gomez-Torrecillas, C. Menini

Department of Mathematics "Giuseppe Peano", University of Turin, Via Carlo Alberto 10, I-10123 Torino, Italy
[alessandro.ardizzoni@unito.it]

We show that the functor from the category of bialgebras to the category of vector spaces sending a bialgebra to its subspace of primitive elements can be recovered by means of a construction involving the concept of monad. This talk is mainly based on the work [1].

[1] A. Ardizzoni, J. Gomez-Torrecillas and C. Menini, *Monadic Decompositions and Classical Lie Theory*, Appl. Categor. Struct., Online First.

On the dot product graph of a commutative ring

Ayman Badawi

Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates
abadawi@aus.edu

Let A be a commutative ring with nonzero identity, $1 \leq n < \infty$ be an integer, and $R = A \times A \times \cdots \times A$ (n times). The *total dot product graph* of R is the (undirected) graph $TD(R)$ with vertices $R^* = R \setminus \{(0, 0, ..., 0)\}$, and two distinct vertices x and y are adjacent if and only if $x \cdot y = 0 \in A$ (where $x \cdot y$ denote the normal dot product of x and y). Let $Z(R)$ denote the set of all zero-divisors of R. Then the *zero-divisor dot product graph* of R is the induced subgraph $ZD(R)$ of $TD(R)$ with vertices $Z(R)^* = Z(R) \setminus \{(0, 0, ..., 0)\}$. It follows that each edge (path) of the classical zero-divisor graph $\Gamma(R)$ is an edge (path) of $ZD(R)$. We observe that if $n = 1$, then $TD(R)$ is a disconnected graph and $ZD(R)$ is identical to the well-known zero-divisor graph of R in the sense of Beck-Anderson-Livingston, and hence it is connected. In this paper, we study both graphs $TD(R)$ and $ZD(R)$. For a commutative ring A and $n \geq 3$, we show that $TD(R)$ ($ZD(R)$) is connected with diameter two (at most three) and with girth three. Among other things, for $n \geq 2$, we show that $ZD(R)$ is identical to the zero-divisor graph of R if and only if either $n = 2$ and A is an integral domain or R is ring-isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$.

[1] Badawi, A., On the dot product graph of a commutative ring, to appear in *Comm. Algebra* (2015).
[2] Anderson, D. F., Badawi, A., On the zero-divisor graph of a ring, *Comm. Algebra* (36)(2008), 3073-3092.
[3] Beck, I., Coloring of commutative rings, *J. Algebra* **116**(1988), 208–226.
[4] Anderson, D. F., Livingston, P. S., The zero-divisor graph of a commutative ring, *J. Algebra* **217**(1999), 434–447.
[5] Anderson, D. F., Mulay, S. B., On the diameter and girth of a zero-divisor graph, *J. Pure Appl. Algebra* **210**(2007), 543–550.
[6] Lucas, T. G., The diameter of a zero-divisor graph, *J. Algebra* **301**(2006), 3533–3558.

On subnormal subgroups of $GL_n(D)$ satisfying a generalized group identity

Mai Hoang Bien

Leiden Universiteit, the Netherlands and Università degli Studi di Padova, Italy.
[maihoangbien012@yahoo.com]

Let D be a division ring with infinite center, n be a positive integer and $w(x_1, x_2, \cdots, x_m) = 1$ be a generalized group identity over the general linear group $GL_n(D)$. The aim of this small talk is to prove that every subnormal subgroup of $GL_n(D)$ which satisfies the generalized group identity $w(x_1, x_2, \cdots, x_m) = 1$ is central.

Commuting properties for the defect functor associated to a homomorphism

Simion Breaz*
Jan Žemlička

Department of Mathematics, Babeş-Bolyai University, Faculty of Mathematics and Computer Science, Str. Mihail Kogălniceanu 1, 400084 Cluj-Napoca, Romania
[bodo@math.ubbcluj.ro]

Department of Algebra, Charles University in Prague, Faculty of Mathematics and Physics Sokolovská 83, 186 75 Praha 8, Czech Republic
[zemlicka@karlin.mff.cuni.cz]

2000 Mathematics Subject Classification: 18C35, 16E30

If $\beta : L \to P$ is a homomorphism in an abelian category, we consider the functor $\mathrm{Def}_\beta = \mathrm{CokerHom}(\beta, -)$, called *the defect functor* associated to β. This notion is a common generalization for the following well-known functors: $\mathrm{Hom}(L, -)$ if $P = 0$, $\mathrm{Ext}^1(\mathrm{Coker}(\beta), -)$ if β is monic and P is projective, respectively the defect functor associated to a short exact sequence. We study commuting properties with respect to some kinds of direct limits (as direct unions and direct sums), and we apply the general results to characterize the modules M such that $\mathrm{Ext}^1(M, -)$ has the same commuting properties.

Module Structures in Rank One Restricted Enveloping Algebras

Stefan Catoiu

Department of Mathematics, DePaul University, Chicago, IL 60614, USA
[scatoiu@condor.depaul.edu]

We investigate both the one-sided and the adjoint module structure of the restricted enveloping algebra $u(sl_2)$ and its quantum analogue. Using this, one can describe the ideals of these algebras. Specifically, we look at the lattice and the monoid of ideals, and give presentations by generators for all these ideals.

Algebras with self-duality and isotropic sub-modules of a self-dual module

Giovanni Cerulli Irelli

Department of Mathematics, Università Sapienza, Roma, Italy
[cerulli.math@googlemail.com]

We consider a finite dimensional algebra $A = kQ/I$ given by a quiver with relations, with the extra property that the quiver Q is endowed with an involutive anti-automorphism which leaves the ideal I invariant . Such an antiautomorphism induces an involutive self-duality on the category mod$A$ of finite dimensional A-modules. Motivated by problems in linear algebra, like classification problems of orthogonal-symplectic multiple flag varieties, I will present some results concerning self-dual representations. In particular, I will consider the subrepresentations which are isotropic and I will provide some geometric properties, in some very special cases. This is an ongoing project, partially in collaboration with F. Esposito and G. Carnovale and partially with E. Feigin and M. Reineke.

Coherence and generalized morphic property of triangular matrix rings

Jianlong Chen*

Qiongling Liu

Department of Mathematics, Southeast University, Nanjing 210096, P. R. China
[jlchen@seu.edu.cn]

Let R be a ring. R is left coherent if each of its finitely generated left ideals is finitely presented. R is called left generalized morphic if for every element a in R, $l(a) = Rb$ for some $b \in R$, where $l(a)$ denotes the left annihilator of a in R. We investigate the coherence and the generalized morphic property of the upper triangular matrix ring $T_n(R)$ $(n \geq 1)$. It is shown that R is left coherent if and only if $T_n(R)$ is left coherent for each $n \geq 1$ if and only if $T_n(R)$ is left coherent for some $n \geq 1$. And an equivalent condition is obtained for $T_n(R)$ to be left generalized morphic. Moreover, it is proved that R is left coherent and left Bézout if and only if $T_n(R)$ is left generalized morphic for each $n \geq 1$.

Duality pairs induced by Gorenstein projective modules with respect to semidualizing modules

Nanqing Ding

Department of Mathematics, Nanjing University, Nanjing 210093, China
[nqding@nju.edu.cn]

Let C be a semidualizing module over a commutative Noetherian ring R. We investigate duality pairs induced by C-Gorenstein projective modules. It is proven that R is Artinian if and only if $(\mathcal{GP}_C, \mathcal{GI}_C)$ is a duality pair if and only if $(\mathcal{GI}_C, \mathcal{GP}_C)$ is a duality pair and $M^+ \in \mathcal{GI}_C$ whenever $M \in \mathcal{GP}_C$, where $\mathcal{GP}_C$ $(\mathcal{GP}_C)$ is the class of C-Gorenstein projective (C-Gorenstein injective) R-modules. In particular, we give a necessary and sufficient condition for a commutative Artinian ring to be virtually Gorenstein. Moreover, we get that R is Artinian if and only if the class $\mathcal{GP}$ of Gorentein projective R-modules is preenveloping. As applications, some new criteria for a semidualizing module to be dualizing are given provided that R is a commutative Artinian ring. This talk is a report on joint work with Y. Geng and J. Hu.

Self-Dual Codes over Commutative and Non-Commutative Rings

Steven T. Dougherty

University of Scranton, Scranton PA, 18518 USA
[prof.steven.dougherty@gmail.com]

A linear code C of length n over a finite Frobenius ring R is a submodule of R^n. The ambient space is equipped with an inner-product and a dual code $C^\perp$ is defined with respect to that inner-product. A self-dual code is

a code C that satisfies $C = C^{\perp}$. Self-dual codes are important as algebraic structures themselves and are related to unimodular lattices, finite designs, and modular forms. We shall describe the structure of self-dual codes and give existence results in both the commutative and non-commutative cases.

Purity in categories of sheaves

Sergio Estrada

Departamento de Matemáticas, Universidad de Murcia, Campus del Espinardo 30100, Murcia, Spain
[sestrada@um.es]

The category of (quasicoherent) sheaves on a scheme is locally finitely presented under fairly general assumptions on the scheme. So the general Purity Theory on finitely accessible categories applies. But we can also define Purity locally on the stalks. And this definition is local in a geometric sense, and seems to be more well-behaved than the categorical one. In the talk we will discuss the two notions and the Relative Homological Algebra attained to the two exact structures.

Power Graphs of Rings

Mary Flagg

Department of Mathematics, Computer Science and Cooperative Engineering, University of St. Thomas, 3800 Montrose, Houston, TX, 77006, USA
[flaggm@stthom.edu]

Recently the connections between graph theory and ring theory have received significant attention in the literature. A number of different graphs have been defined on rings: the zero divisor graph, unit graph and total graph, among others. Given a ring R, one may also look at the additive group $(R, +)$ and the graphs defined on the group. One such graph is the directed power graph. Given a semigroup S with associative multiplication, the directed power graph $G(S) = (V, E)$ is defined by the vertex set V being the elements of S and $(x, y) \in E$ if and only if $x \neq y$ and $y = x^m$ for some positive integer m. For a ring R, one may define two directed power graphs: one on the additive abelian group $(R, +)$ and the other on the multiplicative semigroup $(R, *)$. I will investigate the connection between the multiplicative power graph of a ring and its algebraic properties. In particular, given an abelian group, A, with endomorphism ring $E = End(A)$, what is the connection between the graph theoretic properties of the power graph (zero divisor graph, etc.) of E and the algebraic properties of the underlying group A?

Gorenstein homological algebra relative to weakly Wakamatsu (co)tilting modules

Driss Bennis

Department of Mathematics, Laboratory of Analysis, Algebra and Decision Support, Faculty of Science, BP 1014, Mohammed V-Agdal University, Rabat, Morocco.

J. R. García Rozas*

Luis Oyonarte

Departamento de Matemáticas, Universidad de Almería, 04071 Almería, Spain.
[jrgrozas@ual.es]

In the last years (Gorenstein) homological dimensions relative to a semidualizing module C have been subject of several works as interesting extensions of (Gorenstein) homological dimensions.

In this talk we give a negative answer to the following natural question: Is the condition on C to be a semidualizing module necessary so that the relative homological dimensions preserve their properties? The investigation of this question leads to an extension to the noncommutative case of the concepts of G_C-projective module and dimension and $\mathcal{P}_C$-projective dimension (weakening the condition of C being semidualizing as well). We prove that indeed they share the principal properties of the classical ones and relate these two new dimensions. We show that if C is what we call a weakly Wakamatsu tilting module then both dimensions coincide on modules with finite $\mathcal{P}_C$-projective dimension. Dual results by using G_C-injective modules and weakly Wakamatsu cotilting modules are also established.

[1] Y. Geng and N. Ding, *W-Gorenstein modules*, *J. Algebra* **325** (2011), 132–146.
[2] H. Holm and P. Jorgensen, *Semi-dualizing modules and related Gorenstein homological dimensions*, *J. Pure Appl. Algebra* **205** (2006), 423–445.
[3] D. White, *Gorenstein projective dimension with respect to a semidualizing module*, *J. Comm. Algebra* **2** (2010), 111–137.

Quantifier elimination for certain discretely ordered modules

Petr Glivický

Institute of Mathematics, Academy of Sciences of the Czech Republic, Žitná 25, Praha 1, 11567, Czech Republic
[glivicky@math.cas.cz]

We present a quantifier elimination result for certain class of discretely ordered modules, which is an analogue of the theorem of Baur and Monk [1], [2] for (unordered) modules.

More precisely: We say that a discretely ordered (commutative) integral domain D is a *doded* if

- for all pairs $q, r \in D$, $r > 0$, the regular (with respect to the ordering) Euclidean algorithm starting at (q, r) ends in finitely many steps,
- there is a "degree" function $deg : D \to \mathbb{N}$ with $rng(deg)$ an initial segment of $\mathbb{N}$ and $deg(q) \leq deg(r) \Leftrightarrow (\exists n \in \mathbb{N})|q| \leq n|r|$.

A discretely ordered module M (with the least positive element 1) over a discretely ordered domain D is called *D-integrally divisible* if for all $x \in M$, $0 < r \in D$ there are $y, z \in M$ with $0 \leq z < r1$ such that $x = ry + z$.

Theorem ([3])**:** *Let D be a doded and M a D-integrally divisible discretely ordered D-module. Then every formula is in M equivalent to a disjunction of positive primitive formulas. Moreover, M has quantifier elimination in the language extended by functions $q^{-1}(x) = \lfloor x/q \rfloor$ for all $0 < q \in D$.*

Further, we provide detail characterization of definable sets in the modules in question. We also outline applications of the above result in model theory of (Peano) arithmetic.

[1] Baur, W., Elimination of quantifiers for modules. *Israel Journal of Mathematics* **25** (1976), 64-70.

[2] Monk, L., Elementary-recursive decision procedures. *Ph.D. thesis*, University of California, Berkeley, 1975.

[3] Glivický, P., Study of arithmetical structures and theories with regard to representative and descriptive analysis. *Ph.D. thesis*, Charles University, Prague, 2013.

Some notes on Saorín's problem

Michal Hrbek*

Jan Trlifaj

Department of Algebra, Charles University, Czech Republic
[hrbmich@gmail.com]

Let T be a 1-tilting module and let $(\mathcal{T}, \mathcal{F}) = (\mathrm{Gen}(T), \mathrm{KerHom}(T, -))$ be the associated tilting torsion pair. If $(\mathcal{T}, \mathcal{F})$ is a classical tilting torsion pair (that is, T is equivalent to a finitely generated 1-tilting module), then $\mathcal{F}$ is closed under direct limits. The question asked by Manuel Saorín is whether the converse also holds. We provide several classes of rings for which the answer is positive. These classes include all artinian, commutative or hereditary noetherian, and commutative semiartinian rings, together with Prüfer domains.

Tate-Betti and Tate-Bass numbers

Alina Iacob *

Edgar Enochs, Sergio Estrada, Sinem Odabasi

Department of Mathematical Sciences, Georgia Southern University, 0203 Georgia Ave. Room 3008 Statesboro GA 30460, USA
[aiacob@georgiasouthern.edu]

We define Tate-Betti and Tate-Bass invariants for modules over a commutative noetherian local ring R. We prove the periodicity of these invariants provided that R is a hypersurface. In the case when R is a Gorenstein ring we show that a finitely generated R-module M and its Matlis dual have the same Tate-Betti and Tate-Bass numbers.

Commutator identities on group algebras

Tibor Juhász

Institute of Mathematics and Informatics, Eszterházy Károly College, H-3300 Eger, Leányka út 4, Hungary
[juhaszti@ektf.hu]

Let A be an associative algebra over the field K, and let S be a nonempty subset of A. Every element of S is considered to be a *Lie commutator of weight* 1 on S. A *Lie commutator of weight* $r > 1$ on S is an element $[x, y] = xy - yx$ of A, where x and y are Lie commutators of weight u and v on S and $u + v = r$. Let $K\langle x_1, \ldots, x_m\rangle$ be the polynomial ring in the non-commuting indeterminates $x_1, \ldots, x_m$ over the field K. A Lie commutator of weight r on the set of indeterminates $X = \{x_1, \ldots, x_m\}$ is said to be *multilinear Lie monomial of degree* r, if it is linear in each of its indeterminates. We will say that the subset S of A *satisfies a Lie commutator identity of degree* r, if there exists a nonzero multilinear Lie monomial f of degree r in $K\langle x_1, \ldots, x_m\rangle$ with $f(s_1, \ldots, s_m) = 0$ for all $s_1, \ldots, s_m \in S$.
A number of properties, such as Lie solvability and Lie nilpotence, can be characterized via specific Lie commutator identities. In this presentation we investigate the Lie derived lengths and Lie nilpotency indices of group algebras and their symmetric elements (with respect to the involution sending each group element to its inverse). Furthermore, we also show how these properties reflect the corresponding properties (derived length, nilpotency class) of the unit group.
This research was supported by the European Union and the State of Hungary, co-financed by the European Social Fund in the framework of TÁMOP 4.2.4. A/2-11-1-2012-0001 'National Excellence Program'.

Diophantine Sets of Representations

Sonia L'Innocente

School of Science and Technology, Mathematics Division, University of Camerino, Italy
[sonia.linnocente@unicam.it]

This is a joint work with Ivo Herzog [2].

Let k be a field of characteristic 0 and L the special linear Lie algebra $\mathrm{sl}(2, k)$. The Lie algebra L acts by derivations on the ring $k[x, y]$ of polynomials in two variables. This L-representation admits a direct sum decomposition of $k[x, y]$ into the subspaces $k[x, y]_n$ of homogeneous polynomials of total degree n. We will prove that if $\phi(v)$ is a positive-primitive formula in one free variable, and k is recursively presented, then the subset $\{n|\phi(k[x, y]_n) = 0\}$ of the natural numbers is recursive.
This result is part of the program, enunciated by L'I. and Macintyre [4], to extend the recursive presentation of k to one of the von Neumann k-algebra $U'(L)$ of definable scalars of the representation $k[x, y]$ introduced in [1]; and

to prove the decidability of the theory of $U'(L)$-modules. Furthermore, the arithmetic of the endomorphism ring of a particular infinite dimensional representation of $\mathrm{sl}(2,k)$ will be investigated [3].

1. I. Herzog, The pseudo-finite dimensional representations of $\mathrm{sl}(2,k)$. *Selecta Mathematica* **7** (2001), 241–290
2. I. Herzog, S. L'Innocente, Diophantine Sets of Representations, *Advances in Mathematics* **255** (2014), 338–351
3. I. Herzog, S. L'Innocente, The arithmetic of the theory of the Lie algebra $\mathrm{sl}(2,k)$, *In progress*
4. S. L'Innocente, A. Macintyre, Towards decidability of the theory of pseudo-finite dimensional representations of sl_2k; I, In: *A. Ehrenfeucht, V.W. Marek, M. Srebrny. Andrzej Mostowski and Foundational Studies.* IOS Press, 2007, 235–260.

Silting modules and ring epimorphisms over hereditary rings

Frederik Marks

Universität Stuttgart, Germany
[marks@mathematik.uni-stuttgart.de]

The new concept of silting modules ([2]) generalises tilting modules over an arbitrary ring as well as support τ-tilting modules over a finite dimensional algebra (see [1]). In this talk, we will discuss silting modules over hereditary rings and their interactions with ring epimorphisms. More precisely, we will show that homological ring epimorphisms correspond bijectively to certain "minimal" silting modules. This is joint and ongoing work with Lidia Angeleri Hügel and Jorge Vitória.

[1] Adachi, T., Iyama, O., Reiten, I., τ-tilting theory, to appear in *Comp. Math.*, preprint arXiv:1210.1036.
[2] Angeleri Hügel, L., Marks, F., Vitória, J., Silting modules, preprint arXiv: 1405.2531.

Some aspects of tilting

Francesco Mattiello*
Luisa Fiorot, Alberto Tonolo

Dipartimento di Matematica, Università degli Studi di Padova, via Trieste 63, 35121 Padova, Italy
[fiorot@math.unipd.it, mattiell@math.unipd.it, tonolo@math.unipd.it]

Let $\mathcal{G}$ be a Grothendieck category, let $D(\mathcal{G})$ be its unbounded derived category and denote by $(\mathcal{D}^{\leq 0}, \mathcal{D}^{\geq 0})$ the natural t-structure on $D(\mathcal{G})$. We present a classification theorem for the t-structures on $D(\mathcal{G})$ whose hearts $\mathcal{H}$ have n cohomologies, that is (up to a shift) $\mathcal{H} \subset \mathcal{D}^{\geq -n} \cap \mathcal{D}^{\leq 0}$, for some non-negative integer n.

The motivating example is that of the t-structure induced by a tilting object in $\mathcal{G}$.

On a Generalization of Goldie*-Lifting Modules

Elif Tugce Meric

Department of Mathematics, Celal Bayar University, 45140, Muradiye, Manisa, Turkey
[tugce.meric@cbu.edu.tr]

Our work is motivated by the problems which are given by [1]. We call M is a principally Goldie*-lifting module if for every proper cyclic submodule X of M, there is a direct summand D of M such that $X\beta^*D$. In this study, we focus on principally Goldie*-lifting modules as generalizations of lifting modules. It is investigated when direct summands and quotients of a principally Goldie*-lifting module inherit the property. We compare the classes of principally Goldie*-lifting, principally lifting and principally supplemented modules. Further, we obtain that principally Goldie*-lifting, principally lifting and principally $\oplus$-supplemented modules are equivalent on π-projective modules.

[1] Acar, U., Harmanci, A., Principally Supplemented Modules, *Albanian Journal of Math.*, **Vol 4**, **3** (2010), 79-88.
[2] Birkenmeier, G.F., Mutlu, F.T., Nebiyev, C., Sokmez, N., Tercan, A., Goldie*-supplemented Modules, *Glasgow Math. Journal*, **52(A)** (2010), 41-52.
[3] Kamal, M., Yousef A., Principally Lifting Modules, *Int. Electron. J.Algebra*, **2** (2007), 127-137.

[4] Kosan, T., Tutuncu Keskin D., H-supplemented Duo Modules, *Journal of Algebra and Its Applications*, **Vol. 6**, **No. 6** (2007), 965-971.

Salce's lemma in triangulated categories

George Ciprian Modoi*
Simion Breaz

Department of Mathematics, Babeş–Bolyai University, 1, Mihail Kogălniceanu, 400084, Cluj-Napoca, Romania
[cmodoi@math.ubbcluj.ro]

In [2] is developed a kind of approximation theory in abelian, or more general exact, categories, but replacing cotorsion and cotorsion–free classes with a pair of ideals satisfying a similar condition, that is to be orthogonal to each other with respect to Ext bifunctor. In this talk we do the same in triangulated categories, the rôle of the exact structure being played by a fixed proper class of triangles in the sense of [1].

[1] Beligianis, A., Relative Homological Algebra and Purity in Triangulated Categories, *J. Algebra*, **227** (2000), 268–361.
[2] Fu, X. H., Guil Asensio, P. A., Herzog, I., Torrecillas, B., Ideal Approximation Theory, *Adv. Math.* **244** (2013), 750–790.

Cotorsion pairs and Cartan-Eilenberg categories

Sinem Odabaşı

Departamento de Matemáticas, Facultad de Matemáticas, Campus de Espinardo, Universidad de Murcia, Spain
[sinem.odabasi1@um.es]

Cartan-Eilenberg categories were recently introduced by Guillén Santos, Navarro Aznar, Pascual and Roig, (see [1]). In this work we give a method of constructing Cartan-Eilenberg categories for abelian categories, based on cotorsion pairs. In particular we recover the left Cartan-Eilenberg structure for bounded below chain complexes of modules ([3]) and extend it to more general categories, including the category of (quasi-coherent)sheaves over a projective scheme.

This research has been supported by the Consejería de Industria, Empresa e Innovación de la CARM by means of Fundación Séneca, the program of becas-contrato predoctorales de formación del personal investigador, 15440/FPI/10. It is also partially supported by MTM2010-20940-C02-02 with part of the sources coming from FEDER funds.

1. Guillén Santos, F., Navarro Aznar, V., Pascual, P. & Roig, A. (2010). A Cartan-Eilenberg approach to homotopical algebra. *J. Pure Applied. Alg., 214*, 140-164.
2. Hovey, M. (2002). Cotorsion pairs, model category structures, and representation theory. *Math. Z. 241(3)*, 553-592.
3. Pascual, P. (2012). Some remarks on Cartan-Eilenberg categories. *Collect. Math., 63*, No. 2, 203-216.

Fixed divisor of polynomial matrices

Giulio Peruginelli

Department of Mathematics, University of Padova, Via Trieste 63, 35121 Padova, Italy

Let R be a commutative ring. The fixed divisor of a polynomial $g(X)$ in $R[X]$ is defined as the ideal of R generated by the values of $g(X)$ over R. We generalize this classical notion by evaluating $g(X)$ over matrices over R. Let $M_n(R)$ be the R-algebra of $n \times n$ matrices over R. Given a prime ideal P of R and $g \in R[X]$ we look for the highest power of P such that $g(M)$ is in $M_n(P^k)$, for each $M \in M_n(R)$ (that is, all the entries of these polynomial matrices are in P^k). In order to determine the fixed divisor of $g(X)$ over matrices, we show that it is sufficient to consider companion matrices. Moreover, if R is a Dedekind domain with finite residue fields, we show that we can consider companion matrices of primary polynomials. Applications of these notion are related to integer-valued polynomials over matrices and polynomially dense subsets of matrices.

Representation embeddings preserve complexity

Mike Prest

School of Mathematics, Alan Turing Building, University of Manchester, M13 9PL, UK
mprest@manchester.ac.uk

We show that representation embeddings induce embeddings of lattices of pp formulas and hence are non-decreasing on dimensions, such as Krull-Gabriel dimension, uniserial dimension and width, which can be defined in terms of these lattices.

For a ring R, let pp_R^n denote the lattice of (equivalence classes of) pp formulas for R-modules, in n free variables. Equivalently, this is the lattice of finitely generated subfunctors of the nth power of the forgetful functor from mod-R to **Ab**.

Suppose that ${}_SB_R$ is a bimodule such that B_R is finitely presented. Choose a finite generating (n-)tuple for B_R. Then there is an induced map $\mathrm{pp}_S^1 \to \mathrm{pp}_R^n$ which is a homomorphism of lattices. If R, S are Krull-Schmidt and $- \otimes_S B_R$ is a representation embedding from mod-S to mod-R, then this induced map between lattices is an embedding.

It follows that if R and S are Krull-Schmidt rings and there is a representation embedding from Mod-S to Mod-R then, if the width of S is undefined, so is that of R.

In particular, if R is a finite-dimensional algebra of wild representation type then the width of the lattice of pp formulas for R-modules is undefined. So, if R is countable, then there will be a superdecomposable pure-injective R-module.

Eventually homological isomorphisms in module recollements

Chrysostomos Psaroudakis

Universität Stuttgart, Institut für Algebra und Zahlentheorie, Stuttgart, Germany
[Chrysostomos.Psaroudakis@mathematik.uni-stuttgart.de]

Let Λ be an Artin algebra and e an idempotent element. Our aim is to present a common context where we can compare the algebras Λ and $a\Lambda a$ with respect to Gorensteinness, singularity categories and the finite generation condition Fg for the Hochschild cohomology. For this, we introduce the notion of eventually homological isomorphisms between abelian categories. We will explain this new notion for recollements of module categories. In particular, we characterize when the exact functor $e\Lambda \otimes_\Lambda - : \mathsf{mod}\,\Lambda \to \mathsf{mod}\,e\Lambda e$ is an eventually homological isomorphism. Then, under some conditions on the idempotent element e, we show that Λ is Gorenstein if and only if $e\Lambda e$ is Gorenstein, the singularity categories of Λ and $e\Lambda e$ are equivalent and that Λ satisfies Fg if and only if the algebra

$e\Lambda e$ satisfies Fg. Furthermore we will provide applications and examples. This work is joint with Øystein Skartsæterhagen and Øyvind Solberg.

[1] Chrysostomos Psaroudakis, Øystein Skartsæterhagen and Øyvind Solberg, Gorenstein categories, singular equivalences and finite generation of cohomology rings in recollements, arXiv:1402.1588.

Irreducible representations of Leavitt path algebras over arbitrary graphs

Kulumani M. Rangaswamy

University of Colorado, Colorado Springs, USA
[krangasw@uccs.edu]

Let L be the Leavitt path algebra of a graph E over a field K. Irreducible representations of L induced by vertices and infinite paths are described. The cardinality of a single isomorphism class of irreducible representations is computed. Leavitt path algebras having at most countably many non-isomorphic irreducible representations are characterized.

Generalized injectivity and approximations

Serap Şahinkaya*

Jan Trlifaj

Department of Algebra, Charles University in Prague, Faculty of Mathematics and Physics Sokolovská 83, 186 75 Praha 8, Czech Republic
[srpsahinkaya@gmail.com]

The importance of injective modules in algebra comes from two facts: (i) their structure is well-known for many classes of rings, and (ii) each module has a (unique) injective envelope. In this work, we investigate approximation properties of some classic generalizations of injective modules, more precisely, the Ci- and quasi-injective modules. We prove that these classes provide for approximations only in exceptional cases (when all Ci modules are injective, or pure-injective).

Approximations and Mittag-Leffler modules

Jan Šaroch*

Lidia Angeleri Hügel, Jan Trlifaj

Department of Algebra, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, Praha 8, Czech Republic

[saroch@karlin.mff.cuni.cz]

We show several new applications of relative Mittag-Leffler modules within the scope of approximation theory of modules. Our results include:

1. the class of all flat Mittag-Leffler modules is precovering only over right perfect rings (and a generalization of this to 1-tilting case);
2. a particular instance of Enochs' conjecture that each covering class of modules is closed under direct limits;
3. generalization of the Countable Telescope Conjecture for module categories to non-hereditary cotorsion pairs.

Grassmannians over Rings

Phill Schultz

School of Mathematics and Statistics, The University of Western Australia, Nedlands, WA, Australia

phill.schultz@uwa.edu.au

The classical Grassmanian $\mathcal{G}(k, n)$ is the set of k–dimensional subspaces of the n–dimensional complex space $\mathbb{C}^n$, characterised by three properties:

1. $\mathcal{G}(k, n)$ can be embedded in a finite dimensional space $\mathbb{C}^N$;
2. $\mathcal{G}(k, n)$ has a decomposition into orbits of a group action;
3. Elements of $\mathcal{G}(k, n)$ are characterised up to isomorphism by their intersections with flags of subspaces of $\mathbb{C}^n$.

I show that for every ring R and positive integers $k \leq n$, there is a class of free modules satisfying the same three properties.

A semigroup-theoretical view of direct-sum decompositions and associated combinatorial problems

Daniel Smertnig*

Nicholas R. Baeth, Alfred Geroldinger, David J. Grynkiewicz

Institut für Mathematik und Wissenschaftliches Rechnen,
Karl-Franzens-Universität Graz, Heinrichstraße 36, 8010 Graz, Austria
[daniel.smertnig@uni-graz.at]

Let R be a ring and let $\mathcal{C}$ be a small class of R-modules. Denote by $\mathcal{V}(\mathcal{C})$ a set of representatives of isomorphism classes of $\mathcal{C}$. Then the direct sum operation induces the structure of a commutative semigroup on $\mathcal{V}(\mathcal{C})$ by means of $[M] + [N] = [M \oplus N]$. The semigroup $\mathcal{V}(\mathcal{C})$ carries all information about direct sum decompositions in $\mathcal{C}$, and hence the study of direct sum decompositions in $\mathcal{C}$ can be reduced to the study of the factorization theory of the semigroup $\mathcal{V}(\mathcal{C})$. This semigroup theoretical point of view has been prevalent since it was shown that $\mathcal{V}(\mathcal{C})$ is a Krull monoid whenever $\mathrm{End}_R(M)$ is semilocal for all $M \in \mathcal{C}$.

We pursue this approach in a number of cases where known module theoretic results allow a purely algebraic description of the semigroup $\mathcal{V}(\mathcal{C})$. Suppose that the monoid $\mathcal{V}(\mathcal{C})$ is Krull with a finitely generated class group (for example, when $\mathcal{C}$ is the class of finitely generated torsion-free modules and R is a one-dimensional reduced Noetherian local ring). In this case we study the arithmetic of $\mathcal{V}(\mathcal{C})$ using new methods from zero-sum theory. Furthermore, based on module-theoretic work of Lam, Levy, Robson, and others we study the algebraic and arithmetic structure of the monoid $\mathcal{V}(\mathcal{C})$ for certain classes of modules over Prüfer rings and hereditary Noetherian prime rings.

[1] Geroldinger, A., Baeth, N. R., Grynkiewicz, D. J., Smertnig, D., A semigroup-theoretical view of direct-sum decompositions and associated combinatorial problems, *J. Algebra Appl.*, to appear, arXiv:1404.7264.

Smashing localizations of rings of weak global dimension 1

Jan Stovicek

Department of Algebra, Charles University in Prague, Faculty of Mathematics and Physics Sokolovská 83, 186 75 Praha 8, Czech Republic
[stovicek@karlin.mff.cuni.cz]

I will present a joint work with Silvana Bazzoni (arXiv:1402.7294), where we use dg algebras to show that for rings of weak global dimension at most one, smashing localizations of $D(R)$ correspond bijectively to homological epimorphisms starting in R. If, moreover, R is a valuation domain, we have a classification of all smashing localizations in terms of disjoint collections of intervals in the Zariski spectrum of R. This allows us to decide the Telescope Conjecture not only for valuation domains, but also for other commutative rings of weak global dimension at most one.

Uniform Bounds of Artin-Rees type

Janet Striuli

Department of Math. and C.S., Fairfield University, Fairfield, CT 06824, U.S.A.
[jstriuli@fairfield.edu]

In many ways researchers have explored the uniform behavior that noetherian rings display. One important example of such uniform behavior is due to a theorem of Huneke: given a ring R which is, for example, essentially of finite type over a local noetherian ring, there exists an integer h, depending on an ideal J, that verifies the Artin-Rees Lemma $I^n \cap J \subseteq I^{n-h}J$ *uniformly* for all ideals $I \subseteq R$ and for all $n \geq h$.

In this talk we show that this property holds *uniformly* for all for all high syzygies. In particular, we show that for a local noetherian ring R of dimension d, there exists a uniform bound h such that $I^n F_i \cap Im\partial_{i+1} \subseteq I^{n-h} Im\partial_{i+1}$ for all $n \geq h$, for all ideals $I \subset R$, and for all free resolutions $(F_\bullet, \partial_\bullet)$ of d-syzygies modules.

This theorem answers a question of Eisenbud-Huneke and is joint work with Ian Aberbach and Aline Hosry.

Some model theory of modules over Bézout domains

Carlo Toffalori

School of Science and Technology, Division of Mathematics, University of Camerino, Via Madonna delle Carceri 9, Camerino, Italy
[carlo.toffalori@unicam.it]

I'll survey some joint work with Gena Puninski (Minsk) on the model theory of modules over a Bézout domain B. In particular we deal with the case

$B = D + XQ[X]$ where D is a principal ideal domain and Q is its field of fractions. We describe the Ziegler spectrum of these rings B and, using that, we prove the decidability of the theory of B-modules when D is sufficiently recursive.

[1] Puninski, G., Toffalori, C., Some model theory of modules over Bézout domains. The width, *J. Pure Applied Algebra*, to appear.
[2] Puninski, G., Toffalori, C., Decidability of modules over a Bézout domain $D + XQ[X]$ with D a principal ideal domain and Q its field of fractions, *J. Symbolic Logic*, to appear.

Cotilting modules over commutative noetherian rings

Jan Trlifaj

Univerzita Karlova, Matematicko-fyzikální fakulta, Katedra algebry, Sokolovská 83, 186 75 Praha 8, Česká Republika
[trlifaj@karlin.mff.cuni.cz]

Tilting and cotilting classes have recently been classified over all commutative noetherian rings R in [1]. It has also been shown that each cotilting class in Mod-R is induced by a minimal cotilting R-module, and a construction of these minimal R-modules has been presented in [2]. Moreover, colocalization has been used to relate cotilting R-modules to compatible systems of cotilting $R_{\mathfrak{m}}$-modules where $\mathfrak{m}$ runs over maxSpec(R). However, the corresponding results for tilting modules are still elusive.

1. L. Angeleri Hügel, D. Pospíšil, J. Šťovíček, J. Trlifaj, Tilting, cotilting, and spectra of commutative noetherian rings, to appear in *Trans. Amer. Math. Soc.*, arXiv:1203.0907.
2. J.Šťovíček, J.Trlifaj, D.Herbera, Cotilting modules over commutative noetherian rings, *J. Pure Appl. Algebra* **218**(2014), 1696–1711.
3. J. Trlifaj, S. Şahinkaya, Colocalization and cotilting for commutative noetherian rings, *J. Algebra* **408**(2014), 28–41.

Rings for which indecomposables are simple

Peter Vamos

University of Exeter
[P.Vamos@exeter.ac.uk]

The talk will investigate the structure of these rings. The commutative case is simple (= trivial) but the non-commutative case is more complicated.

A new basis of the sub Hopf algebra of the mod 2 Steenrod algebra

Tane Vergili*

Ismet Karaca

Department of Mathematics, Ege University, 35100 Bornova Izmir, Turkey
[tane.vergili@ege.edu.tr ismet.karaca@ege.edu.tr]

The mod 2 Steenrod algebra, $\mathcal{A}$, is the complete algebra of the stable cohomology operations and its dual is isomorphic to $\mathbb{Z}_2[\xi_1, \xi_2, ...]$ with $|\xi_k| = 2^k - 1$. The dual to the monomial basis is a basis for the Steenrod algebra known as the Milnor basis, denoted by $Sq(r_1, r_2, ...)$. The finite sub Hopf algebra $\mathcal{A}_n$ of $\mathcal{A}$ has an additive basis which are dual to $\xi_1^{r_1}...\xi_{n+1}^{r_{n+1}}$ in the range $0 \leq r_i \leq 2^{n+2-i} - 1$. Many researchers deal with describing a basis of $\mathcal{A}$ that can be restricted to basis of $\mathcal{A}_n$. Wood defines Y and Z basis that fits for $\mathcal{A}_n$ by giving an order to the Steenrod squares of type $Sq^{2^a(2^b-1)}$ which are known as atomic squares, and he leaves a problem whether it might have another ordering on them which gives a new basis for $\mathcal{A}_n$. Starting from this point of view, we investigate a new basis for the sub Hopf algebra of Steenrod algebra by changing the linear ordering on the atomic Steenrod squares.

[1] Milnor J., The Steenrod algebra and it's dual, *Annals of Math.* **67** (1958), 150–171.
[2] Steenrod N, E., and Epstein D. B. A., *Cohomology operations.* Annals of Math Studies Princeton University Press 50, Princeton, NJ, 1962.
[3] Wood R. M. W., Problems in the steenrod algebra, *Bull London Math. Soc.* **30** (1988), 449–517.
[4] Wood R. M. W., A note on bases and relations in the Steenrod algebra, *Bull. London Math. Soc.* **27** (1995), 380–386.

Surjunctivity and direct finiteness of group representations

Simone Virili

Departament de Matemàtiques, Universitat Autònoma de Barcelona, Edifici C - 08193 Bellaterra (Barcelona), Spain
[simone@mat.uab.cat]

Consider the following conjectures:
(Linear) Surjunctivity Conjecture. A map is *surjunctive* if it is non-injective or surjective. Given a set A, endow A^G with the product of the discrete topologies. The *Surjunctivity Conjecture* (first stated by Gottschalk in 1973) states that any continuous and G-equivariant map $\phi : A^G \to A^G$ is surjunctive provided A is finite. Similarly, given a field $\mathbb{K}$ and supposing that A is a $\mathbb{K}$-vector space, the *L-Surjunctivity Conjecture* (stated by implicitly Gromov and explicitly by Ceccherini-Silberstein and Coornaert) states that any continuous, linear and G-equivariant map $\phi : A^G \to A^G$ is surjunctive, provided A is finite dimensional.
Stable Finiteness Conjecture. A ring R is *directly finite* if $xy = 1$ implies $yx = 1$ for all $x, y \in R$. Furthermore, R is *stably finite* if $\mathrm{Mat}_k(R)$ is directly finite for all $k \in \mathbb{N}_+$. The *Stable Finiteness Conjecture* (stated by Kaplansky in 1969) states that the group ring $\mathbb{K}[G]$ is stably finite for any field $\mathbb{K}$. Notice that, $\mathrm{Mat}_k(\mathbb{K}[G]) \cong \mathrm{End}_{\mathbb{K}[G]}(\mathbb{K}[G]^k)$, so an equivalent way to state the Stable Finiteness Conjecture is to say that any endomorphism of a free right (or left) $\mathbb{K}[G]$-module of finite rank is either injective or non-surjective.

Some cases of the above conjectures are known to have positive solution but they are still open in general. After explaining some connections among the above conjectures, I will describe two strategies to solve them for amenable and sofic groups respectively. Furthermore, when studying stable finiteness, we will consider not just group rings $\mathbb{K}[G]$ but crossed products $R * G$, where R is a Noetherian ring.

In particular, in the amenable, the solution will follow by the existence of suitable real-valued invariants on the category $R * G - \mathrm{Mod}$ called *algebraic entropies*. In the sofic case, our approach relies on some general lattice theoretic results that can be applied both to the lattices of submodules (giving applications to the Stable Finiteness Conjecture) and to lattices of closed subspaces (giving applications to the L-Surjunctivity Conjecture).

Silting modules and silting complexes

Jorge Vitória

Università degli Studi di Verona
[jorge.vitoria@univr.it]

Support τ-tilting theory for finite dimensional algebras extends classical tilting theory, allowing to complete the parametrisation of certain structures in the module category and in its derived category ([1]). The new concept of silting modules ([2]) provides an adequate setup for such parametrisations over arbitrary rings, while keeping some of the features of (possibly large) tilting modules. In this talk, we will define silting modules and discuss their relations with silting complexes and (co-)t-structures in the derived category. This is joint work with Lidia Angeleri Hügel and Frederik Marks.

[1] Adachi, T., Iyama, O., Reiten, I., τ-tilting theory, to appear in *Comp. Math.*, preprint arXiv:1210.1036.

[2] Angeleri Hügel, L., Marks, F., Vitória, J., Silting modules, preprint arXiv: 1405.2531.

An iterative meta-example constructed using power series

Sylvia Wiegand*

William Heinzer, Christel Rotthaus

Department of Mathematics, University of Nebraska, Lincoln, NE 68588-0130, USA

[swiegand@math.unl.edu]

Let x and y be indeterminates over a field k, let $R = k[x,y]_{(x,y)}$ and let R^* be the (x)-adic completion $k[y]_{(y)}[[x]]$ of R. We first apply a simple form of a basic construction that we have developed to adjoin an element σ of $xk[[x]]$ that is transcendental over $k(x)$; for example with k the rational numbers, take $\sigma = e^x - 1$. For this, set $A := k(x,y,\sigma) \cap k[y]_{(y)}[[x]]$. Then $A = C[y]_{(x,y)}$, where $C := k(x,\sigma) \cap k[[x]]$, a DVR. Thus the ring A is Noetherian and a regular domain; moreover A is a nested union of localized polynomial rings in three variables that is naturally associated to A.

We iterate the construction using $\tau \in yk[[y]]$ transcendental over $k(y)$. The resulting ring $A' := k(x,y,\sigma,\tau) \cap k[[x,y]]$ is a two-dimensional regular local domain with maximal ideal $(x,y)A'$ and completion $\widehat{A'} = k[[x,y]]$. There is a nested union B' of localized polynomial rings in four variables contained in and naturally associated to A'. Depending upon the choices of σ and τ, sometimes $B' = A'$ and sometimes B' is properly contained in A'.

We give some insights, results and examples concerning whether $B' = A'$ and whether B' is Noetherian.

Generalized Lie Derivations on Lie Ideals

Nihan Baydar Yarbil

Department of Mathematics, Ege University, 35100 Bornova Izmir, Turkey
[nihan.baydar.yarbil@ege.edu.tr]

The talk will focus on a recent progress in a joint work with N. Argaç. Let R be a ring. An additive map d from R into itself is called a derivation of R if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$ and is said to be a Lie derivation if $d\big([x,y]\big) = \big[d(x), y\big] + \big[x, d(y)\big]$ for all $x, y \in R$. By a generalized derivation of R we mean an additive map F from R into itself such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$ where d is the so called associated derivation of R . An additive map F from R into itself is called a generalized Lie derivation (in the sense of Nakajima) if $F\big([x,y]\big) = \big[F(x), y\big] + \big[x, d(y)\big]$ for all $x, y \in R$ where d is a Lie derivation . It is well known that every derivation is a Lie derivation, but when it comes to generalized derivations it is a little specific as we will discuss.

[1] A. Nakajima, On Generalized Higher Derivations, *Turk. J. Math.* **(24)** (2000), 295-311.
[2] T.K. Lee, Generalized derivations of left faithful rings, *Comm. in Alg.* **27(8)** (1999), 4057-4073.
[3] B. Hvala, Generalized derivations in rings, *Comm. in Alg.* **26(4)** (1998), 1147-1166.
[4] B. Hvala, Generalized Lie derivations in prime rings, *Taiwanese J.Math.* **(11)** (2007), 1425-1430.

Relative and Tate homology with respect to semidualizing modules

Xiaoxiang Zhang*

Zhenxing Di, Zhongkui Liu, Jianlong Chen

Department of Mathematics, Southeast University, Nanjing 211189, P. R. China
[z990303@seu.edu.cn]

In 2002, Avramov and Martsinkovsky [1] introduced the relative cohomology functors $\mathrm{Ext}^n_{\mathcal{G}}(M,-)$ and the Tate cohomology functors $\widehat{\mathrm{Ext}}^n_R(M,-)$. The relative cohomology functors $\mathrm{Ext}^n_{\mathcal{G}}(M,-)$ treat modules of G-dimension zero as projectives and induce a relative dimension which refines the classical projective dimension. The Tate cohomology functors $\widehat{\mathrm{Ext}}^n_R(M,-)$ are defined based on a complete resolution $\mathbf{T} \to \mathbf{P} \to M$. An interesting and deep result in [1] is the connections among $\mathrm{Ext}^n_{\mathcal{G}}$, Ext^n_R and $\widehat{\mathrm{Ext}}^n_R$.

In 2010, Sather-Wagstaff et al. [4] generalized the work [1] to arbitrary abelian categories. As a specific situation, they investigated Tate cohomology of modules over a commutative noetherian ring with respect to semidualizing modules. Recently, several relative Tor functors with respect to semidualizing modules were introduced and studied by Salimi et al. [3].

Inspired by [1] and [4], we define a kind of Tate homology of modules admitting Tate $\mathcal{F}_C$-resolutions over a commutative coherent ring R. We first characterize the class of modules admitting a Tate $\mathcal{F}_C$-resolution. Then an Avramov-Martsinkovsky type exact sequence is constructed to connect such Tate homology functors and relative homology functors. Finally, motivated by the idea of [4], we establish a balance result for such Tate homology over a Cohen-Macaulay ring with a dualizing module by using a result provided in [2].

[1] Avramov, L. L., Martsinkovsky, A., Absolute, relative, and Tate cohomology of modules of finite Gorenstein dimension, *Proc. Lond. Math. Soc.* **85** (2002), 393–440.

[2] Enochs, E. E., Estrada, S. E., Iacob, A. C., Balance with unbounded complexes, *Bull. London Mtath. Soc.* **44** (2012), 439–442.

[3] Salimi, M., Sather-Wagstaff, S., Tavasoli, E., Yassemi, S., Relative Tor functors with respect to a semidualizing module, *Algebr. Represent. Theory* **17** (2014), 103–120.

[4] Sather-Wagstaff, S., Sharif, T., White, D., Comparison of relative cohomology theories with respect to semidualizing modules, *Math. Z.* **264** (2010), 571–600.

Index