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Coarse embeddings into a Hilbert space, coarse amenability, and expanders

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The concept of coarse embedding was introduced by Gromov in 1993. It plays a crucial role in the study of large-scale geometry of groups and the Novikov higher signature conjecture. Coarse amenability, also known as Guoliang Yu's propertyA, is a weak amenability-type condition that is satisfied by many known metric spaces. It implies the existence of a coarse embedding into a Hilbert space. In this expository talk, we discuss the interplay between infinite expander graphs, coarse amenability, and coarse embeddings. We present several 'monster' constructions, in the setting of metric spaces of bounded geometry, including a recent construction, jointly with Romain Tessera, of relative expander graphs which do not weakly contain any expander. This research was partially supported by my ERC grant ANALYTIC no. 259527.

Cotorsion pairs, model structures, and recollements

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Very recently, a recollement due to Krause was interpreted via model category methods by Becker. Motivated by this, we look in general at recollements of triangulated categories by way of model category theory. We will see that from the model category point of view a recollement is equivalent to 3 interrelated cotorsion pairs in some abelian category, or more generally, an exact category. We will illustrate this with a number of examples, some recovering known recollement situations, but others being new. For instance, we will describe an analog to the recollement of Krause that holds in any locally finitely presentable Grothendieck category, but which depends on our choice of generating set. Other nice examples follow from the authors joint work with Mark Hovey and Daniel Bravo on Gorenstein homological algebra.

Ideal Approximation Theory and Phantom Morphisms

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We develope a general theory of phantom morphisms in additive exact categories associated to subfunctors of the Ext functor. In particular, when the exact category has enough projective and injective modules, we prove that an ideal I in the category is special precovering if and only if there exists a subfunctor F of Ext with enough injective morphisms such that I is the ideal of phantom morphisms associated to the subfunctor F. We will specially stress the connection of our results with the classical notion of phantom morphisms in triangulated categories, as well as other new applications. We will also outline some existence theorems for phantom covers associated to ideals of morphisms.

Based on joint works with S. Estrada, X. Fu, I. Herzog, F. Ozbek and B. Torrecillas.

Construction of modules with a prescribed direct sum decomposition: the case of Artinian modules revisited

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Though artinian modules are one of the basic objects in Ring and Module Theory, significant results on their direct sum decompositions are relatively recent. In 1993 Camps and Dicks proved that artinian modules have a semilocal endomorphism

ring, and the first examples showing that the decomposition of an artinian module into indecomposable summands is not, in general, unique were given by Facchini, Herbera, Levy and Vamos in 1996.

Both results are closely related because one way to explain this lack of uniqueness is via the endomorphism ring of the artinian module: indecomposable finitely generated projective modules over a semilocal ring, in general, fail to satisfy the Krull-Schmidt theorem. This became completely clear when Wiegand in 2001 showed that any monoid that can be realized as the monoid of isomorphism classes of finitely generated projective modules over a semilocal ring can also be realized as the monoid of isomorphism classes of direct summands of an artinian (finitely generated) module. So the direct sum behavior of artinian modules is as good/bad as the direct sum behavior of finitely generated projective modules over a semilocal ring.

The question we address now is what happens with infinite direct sums of artinian modules? In a series of recent papers with P. Prihoda we have developed a theory to study infinitely generated projective modules, which is particularly successful in the case of noetherian semilocal ring. In this talk I will explain how this theory can be applied to construct infinite direct sums of (finitely generated) artinian modules which model the same behavior of projective modules over noetherian semilocal rings.

The mono-epi exact category of arrows.

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Let $(A;E)$ be an exact category. The category (Arr (A); Arr (E)) of arrows of A also has the structure of an exact structure and there is a full and faithful exact embedding of (A;E) into (Arr (A); Arr(E)) given by sending an object A to its identity morphism 1_A. The main result is that the smallest subfunctor of Ext in (Arr (A); Arr(E)) that contains the image of (A;E) is itself exact. This exact category is denoted by (Arr(A); ME) and is called the mono-epi exact category of arrows.

Under suitable conditions on (A;E), The Ghost Lemma, Salce's Lemma and Wakamatsu's Lemma hold in the mono-epi exact category (Arr (A); ME) of arrows. I will explain the statements of these results and give some idea of their proofs.

The Ghost Lemma in (Arr(A); ME) will be applied to prove that if a ring R is semiprimary, then the nilpotency index of the phantom ideal is bounded by the nilpotency index of the Jacobson radical of R. For the special case of a finite group ring, I will present Benson's proof on the bound of the nilpotency index of the phantom ideal.

This is joint work with X.H. Fu.

Dominant dimension and gendo-symmetric algebras

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Classical Morita-Tachikawa correspondence provides a bijection between pairs (Λ artin algebra, M generator-cogenerator) and algebras Γ of dominant dimension at least two. When Λ is symmetric, Γ is called gendo-symmetric. After giving intrinsic characterisations and examples of gendo-symmetric algebras, properties of these algebras will be discussed. In particular, there exists a comultiplication, which can be used to determine the dominant dimension.

This is a report about joint work with Ming Fang (Chinese Academy of Sciences, Beijing).

Morphisms determined by objects and flat covers

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We describe a procedure for constructing morphisms in additive categories, combining Auslander's concept of a morphism determined by an object with the existence of flat covers. Also, we show how flat covers are turned into projective covers, and we interprete these constructions in terms of adjoint functors.

Pure projective torsion-free modules over some commutative rings

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The subject of the talk is inspired by the notion of a generalized lattice that appeared in the work of Butler, Campbell and Kovács about a decade ago. For a lattice-finite R-order Λ, where R is a Dedekind domain, generalized lattices were shown to be pure projective Λ-modules. Quite natural question is for which orders every generalized lattice is a direct sum of lattices. We study similar questions in the context of some commutative noetherian rings of Krull dimension 1. Our approach is different from the one used in representation theory, it is based on several results which have been used to study projective modules over (non-commutative) noetherian rings.

Hearts of t-structures which are Grothendieck or module categories

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Given a Grothendieck category $\mathcal{G}$, we study conditions under which a t-structure in the derived category $\mathcal{D}(\mathcal{G})$ has a heart which is a Grothendieck category or even a module category. We shall concentrate on the case of the so-called Happel-Reiten-Smal∅ t-structure associated to a torsion pair in $\mathcal{G}$ and in the case of an arbitrary compactly generated t-structure in $\mathcal{D}(R) = \mathcal{D}(R - Mod)$, where R is a commutative Noetherian ring.

The first main result to be presented states that if $\mathbf{t} = (\mathcal{T}, \mathcal{F})$ is a torsion pair in $\mathcal{G}$ such that the heart of the associated Happel-Reiten-Smal∅ t-structure is a Grothendieck category, then the torsionfree class $\mathcal{F}$ is closed under taking direct limits in $\mathcal{G}$, the converse being also true for commonly used torsion pairs.

For the case of a compactly generated t-structure in $\mathcal{D}(R)$, where R is a commutative Noetherian ring, in terms to be made precise in the talk, we will show that the associated heart is essentially always a Grothendieck category while it is very rarely a module category.

This is a report about joint work with Carlos Parra.

From Cluster algebras, via Semi-invariants to Maximal green sequences

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It was the notion of cluster algebras that brought together many notions from different parts of mathematics, quite often giving many different interpretations to the same objects or structures. In this talk I will concentrate on the path that I followed (there are very many other directions). The beginning of my interest was a need for categorical interpretation of the combinatorics of cluster algebras. One of the fundamental notions of cluster algebras, clusters, had a beautiful interpretation in cluster categories as cluster tilting objects, hence making clear connection to already well developed tilting theory of finitely generated modules. It is worth pointing out that recently, there is more interest in understanding the large tilting modules as well, in the context of cluster algebras and related topics. The next step in my interest came from looking at cluster complexes and viewing them as domains of semi-invariants. And the most recent is the inter- pretation of the same picture of the domains of semi-invariants as a tool for determining maximal green sequences again, a notion coming from mutations in cluster algebras, which is used in physics.

Semigroups of Modules

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The idea of making isomorphism classes of modules into an algebraic object is an old one. Witness the ideal class group of a Dekekind domain. For additive examples, using the direct sum as the operation, one has K-theory and Grothendieck groups. Even if the objects one is studying satisfy direct-sum cancellation, however, one loses a lot of information by formally adjoining inverses (negatives) to form an abelian group. Therefore we won't do that; we'll just stick with semigroups.

Given a ring R and a class $\mathcal{C}$ of R-modules closed under direct sums and direct summands, we consider the class $V(\mathcal{C})$ of isomorphism classes $[M]$ of modules M in $\mathcal{C}$. Assuming that $V(\mathcal{C})$ is a set, we make it into a semigroup by defining $[M] \oplus [N] = [M \oplus N]$. This way of thinking has led to many new discoveries. For example it has produced a negative answer to Krull's question: Do Artinian modules satisfy the Krull-Remak-Schmidt property (KRS)? Also, the semigroup approach allows us to say, in a very precise way, exactly how badly KRS can fail for finitely generated modules over a (commutative, Noetherian) local ring. A remarkable fact is that the same semigroups, namely, finitely generated reduced Krull semigroups, arise in many different contexts and have helped unify research in disparate areas: module theory over local rings, decompositions of torsion-free abelian groups of finite rank, factorization theory in integral domains, Boij-Söderberg theory, combinatorics, and simplicial topology. I will mention these connections briefly, but the main focus will be on the monoid $V(\mathcal{C})$, where $\mathcal{C}$ is the class of finitely generated torsion-free modules over a one-dimensional local domain.

Recollements and finitistic dimensions of algebras

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The finitistic dimension of a ring R, denoted by fin.dim(R), is by definition the supremum of projective dimensions of those left R-modules M which have a finite projective resolution by finitely generated projective modules. The finitistic dimension conjecture states that any Artin algebra should have finite finitistic dimension. This is a longstanding question (Bass, 1960) and has still not been settled. There are at least seven other unsolved main conjectures in homological representation theory of algebras, which are closely related to this conjecture.

In this talk, we shall study this conjecture from a very general view-point of recollements of derived module categories. Given such a recollement of derived categories of rings, we establish several upper bounds and relationships among the finitistic dimensions of rings involved. Here, we define homological widths (or cowidths) for bounded complexes of projective (or injective) modules. They are always natural numbers.

Theorem Let R_1, R_2 and R_3 be rings. Suppose that there exists a recollement among the derived module categories $\mathscr{D}(R_3)$, $\mathscr{D}(R_2)$ and $\mathscr{D}(R_1)$ of R_3, R_2 and R_1 :

$$\begin{tikzcd} \mathscr{D}(R_1) \arrow[r, "i_*"] & \mathscr{D}(R_2) \arrow[l, bend right, "i^*"'] \arrow[l, bend left, "i^!"] \arrow[r, "j^!"] & \mathscr{D}(R_3) \arrow[l, bend right, "j_!"'] \arrow[l, bend left, "j_*"] \end{tikzcd}$$

Then the following statements hold true:

(1) Suppose that $j_!$ restricts to a functor $\mathscr{D}^b(R_3) \to \mathscr{D}^b(R_2)$ of bounded derived module categories. Then $\text{fin.dim}(R_3) \leq \text{fin.dim}(R_2) + cw\big(j^!(\text{Hom}_{\mathbb{Z}}(R_2, \mathbb{Q}/\mathbb{Z}))\big)$, where $cw(-)$ is the homological cowidth map.

(2) Suppose that $i_*(R_1)$ is a compact object in $\mathscr{D}(R_2)$. Then

(a) $\text{fin.dim}(R_1) \leq \text{fin.dim}(R_2) + w(i^*(R_2))$.

(b) $\text{fin.dim}(R_2) \leq \text{fin.dim}(R_1) + \text{fin.dim}(R_3) + w\big(i_*(R_1)\big) + w\big(j_!(R_3)\big) + 1$, where $w(-)$ is the homological width map.

This result extends Happel's reduction techniques for finitistic dimenson conjecture to more general situations, generalizes some recent results in the literature, and can be applied to many other situations. The contents of the talk are taken from a joint work ”Recollements of derived categories III: finitistic dimensions” with H. X. Chen.

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