A number that has an elementary trace function and no elementary sum approximation

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A function is *elementary* if it can be computed in time $O(2^n) \cup O\big(2^{2^n}\big) \cup O\big(2^{2^{2^n}}\big) \cup \cdots$, or equivalently, if it is in the third level $\mathcal{E}^3$ of the Grzegorczyk hierarchy. A strictly increasing function $A\colon \mathbb{N} \to \mathbb{N}$ is the *sum approximation* (*from below*) of an irrational number α if $\alpha = a + \sum_{i=0}^{\infty} \frac{1}{2^{A(i)+1}}$ for some integer a. A function $T\colon \mathbb{Q} \to \mathbb{Q}$ is a *trace function* for α if $|T(q) - \alpha| < |q - \alpha|$ for all $q \in \mathbb{Q}$. We show that there exists an irrational number with an elementary trace function and without an elementary sum approximation, as was conjectured by Kristiansen [Kri17, Section 5].

This is joint work with Akitoshi Kawamura.

References

[Kri17] Lars Kristiansen. On subrecursive representability of irrational numbers. *Computability*, 6(3):249–276, 2017.

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