Descriptive Complexity of qcb_0-Spaces

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In this talk, we will briefly review some recent work (made essentially within the last decade) on generalizing and effectivizing classical descriptive set theory (DST) which deals with hierarchies of sets, functions and equivalence relations on Polish spaces (completions of separable metrizable spaces).

Since several fields of mathematics and theoretical computer science deal with non-Hausdorff spaces (e.g., Zariski topologies on the prime ideals of commutative rings with 1 in algebraic geometry and computer algebra or domains in denotational semantics of programming languages), there was an intensive work on generalizing the classical DST from the traditional setting of separable metrizable spaces to the more general category of T_0-quotients of countably based spaces (qcb_0-spaces). The qcb_0-spaces include all countably based T_0-spaces (cb_0-spaces), but also many non-countably based spaces (like the Kleene-Kreisel continuous functionals that have applications to computable analysis and mathematical logic). Extending DST to non-Hausdorff cb_0-spaces is non-trivial since even the classical definition of Borel hierarchy needs a minor but important revision. Even less trivial task is to extend DST to non-countably based spaces, since at least two natural options for the classical hierarchies appear.

Theoretical computer science and computable analysis especially need an effective version of DST, as a foundation for computation theory in topological structures. A useful work in this direction was done within the classical computability theory for concrete zero-dimensional spaces like the discrete space ω and the Baire space $\mathcal{N}$. There was also some work on effective DST for Polish spaces and effective domains. In the last years, many people attempted to make next steps towards the "right" coherent version of effective DST beyond the effective Polish spaces. The task is highly non-trivial since there often appear different natural effectivizations of classical notions of DST. The search for the "right" effective versions of general enough classes of spaces resulted in proliferation of different notions of which it is not easy to choose the most useful ones. This leads to the sad situation that the terminology in this field is becoming chaotic: equivalent notions often obtain different names, and one and the same name is sometimes used to denote non-equivalent objects.

Since the (effective) DST is already very extensive, keeping this review of reasonable size forced me to be very selective. In particular, we do not mention many known facts of the classical DST. For the same reason, our discussion will mostly be a listing of some directions of current research, notions and formulations of some results.