

# Rigorous Function Calculi-V

Pieter Collins  
Department of Advanced Computing Sciences  
Maastricht University

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## Abstract

Almost all problems in applied mathematics, including the analysis of dynamical systems, deal with spaces of real-valued functions on Euclidean domains in their formulation and solution. There are many packages available providing rigorous calculi for Euclidean functions, including Ariadne [1], AERN [2] and iRRAM [3], which are all part of the CID project [4]. In this talk, I will describe the some application of rigorous function calculus, focussing on ARIADNE, which has a semantics based on computable analysis [5].

I will first introduce some core problems, namely algebraic equations, differential equations, and constraint satisfaction problems, and discuss their use for the analysis of hybrid systems in ARIADNE [6, 7]. I will then discuss progress on more complicated classes of system, including differential inclusions [8], partial differential equations, and stochastic differential equations. Finally, I shall give some recent developments on applying function calculus to problems in artificial intelligence, and perspectives on further applications.

Throughout, I will mention the capabilities of packages providing a rigorous function calculus, including COSY Infinity [9], the CAPD Library [10], and FlowStar [11] as well as the previously-mentioned tools.

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