CCC 2022 Tutorial

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1 An introduction to classical realizability

The theory of classical realizability was introduced by Krivine in the 90's to analyze the computational contents of classical proofs, following the connection between classical reasoning and control operators discovered by Griffin in 1990. More than an extension of Kleene's intuitionistic realizability, classical realizability is a complete reformulation of the principles of realizability, with strong connections with Cohen forcing.

In the first lecture, we shall first present the language of classical realizers (Krivine's λ_c calculus) and then see how to interpret each formula of second-order logic as a set of classical realizers, before proving that this interpretation is adequate w.r.t. the rules and axioms of classical second-order arithmetic.

2 Classical witness extraction and other problems

In this second lecture, we shall first see how classical realizability can be used to extract witnesses from classical realizers (or proofs) of existential formulas, depending on the logical complexity of the realized formula. In the second part of the lecture, we shall discuss the model-theoretic side of classical realizability (in PA2 or in ZF) and its connections with Cohen forcing, before concluding on some of its categorical aspects.