

CCC 2022 Tutorial

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1. Borel combinatorics and reverse mathematics The area of Borel combinatorics takes classical combinatorial theorems, such as “every acyclic graph has a two-coloring”, and asks whether a Borel version of the theorem holds. Informally, if the given acyclic graph has a nice description, does it also have a two-coloring with a nice description? Usually the Borel version of the theorem fails, and two kinds of argument mostly suffice to demonstrate the failure: measure arguments and category arguments. Often both kinds of arguments work, but sometimes only one kind is known to work, and it is natural to wonder whether the other can also do it. Reverse mathematics provides a way to formalize this kind of question. In this talk, we will introduce Borel combinatorics and reverse mathematics, and indicate how informal questions about the former can be expressed in the latter.

2. Measure and category of Borel sets in reverse mathematics Within the reverse mathematics framework, we analyze the statements “Every Borel set has the property of Baire” and “Every Borel set is measurable” to show that category arguments and measure arguments can be distinguished by the framework, and are strictly less powerful than arguments which recurse directly on the structure of a Borel set. This framework can then be applied to query the necessity of measure and category methods in various theorems about Borel sets, with some partial results obtained towards the questions from the first talk. The results presented are joint with subsets of Astor, Dzhafarov, Flood, Montalbán, Solomon, Towsner and Weisshaar in various combinations.