Analysis and Control of Network Dynamics For Evolutionary Matrix Games

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An evolutionary approach to coordination of self-interested agents

Special issues on robotics
Science Magazine
October 2014 and November 2007

Advances in

- mobile sensor platforms
- intelligent autonomous robots

"selfishness": individuals maximize their own payoffs, might leading to a great cost to the group

Challenges

- local information vs. global team goal
- unknown, changing environment

An evolutionary approach to coordination of self-interested agents

An evolutionary approach to coordination of self-interested agents

Key difference from the existing control of complex systems

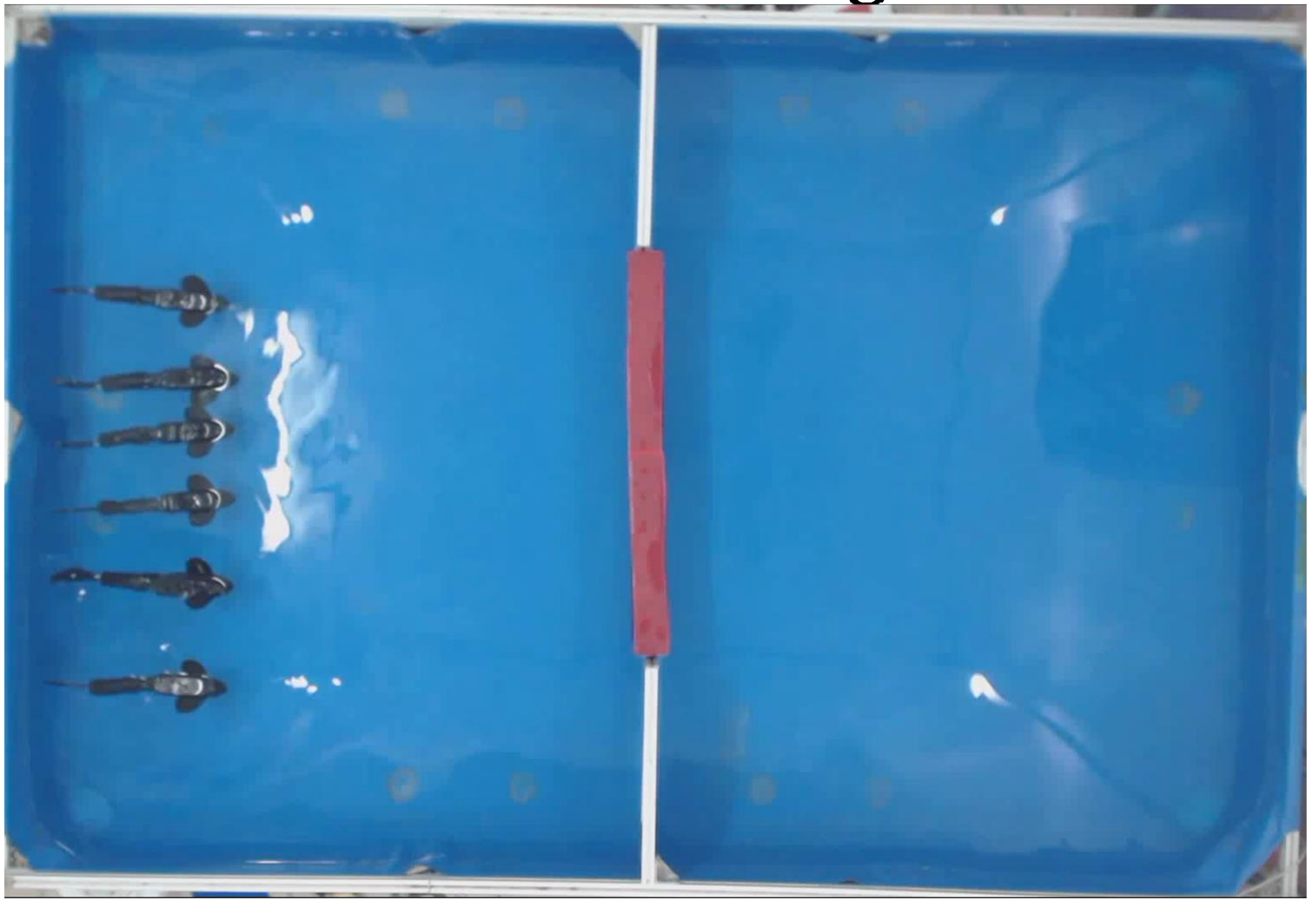
- distributed controller
- adaptive control evolves with changing environment

Sociology: social dilemma in modern society

Biology: understanding cooperating behavior in social animals

An evolutionary approach to coordination of self-interested agents

An evolutionary approach to coordination of self-interested agents



Carry out the task repeatedly; adjust strategies each time

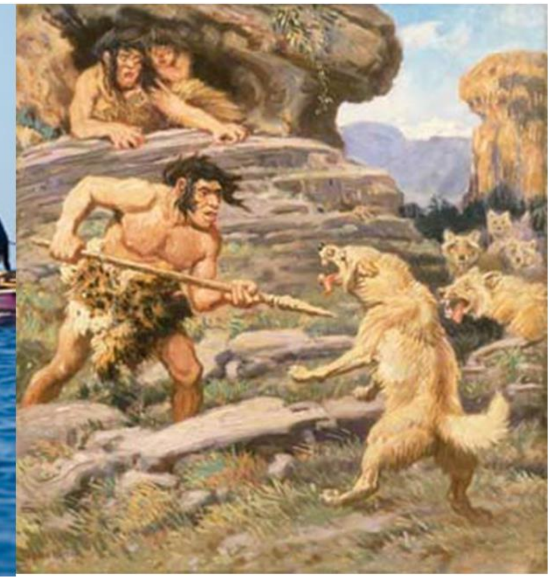
- each time the task is taken as a group game
- new insight into how cooperation emerge as an evolutionary outcome

Outline

- Paradox of cooperation
- Evolutionary matrix games
- Continuous-time replicator dynamics
- Discrete-time dynamics driven by switching probabilities
- Controlling evolutionary games on networks

The paradox of cooperation

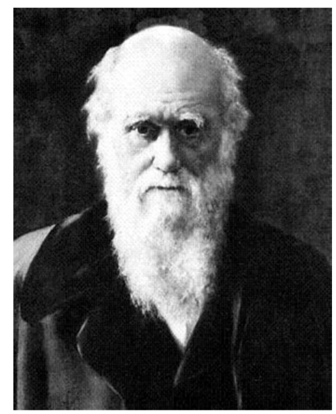




Natural selection is based on competition. How can it lead to cooperation?

Cooperation is often costly for the individual, while benefits are distributed over a collective

Cooperation (altruism) is an evolutionary puzzle!



Charles Darwin
(1809-1882)

Mechanism for evolution of cooperation is a central topic

THE QUESTIONS

The Top 25
Essays by our news staff on 25 big questions facing science over the next quarter-century.

- What Is the Universe Made Of?
- What is the Biological Basis of Consciousness?
- Why Do Humans Have So Few Genes?
- To What Extent Are Genetic Variation and Personal Health Linked?
- Can the Laws of Physics Be Unified?
- How Much Can Human Life Span Be Extended?
- What Controls Organ Regeneration?
- How Can a Skin Cell Become a Nerve Cell?
- How Does a Single Somatic Cell Become a Whole Plant?
- How Does Earth's Interior Work?
- Are We Alone in the Universe?
- How and Where Did Life on Earth Arise?
- What Determines Species Diversity?
- What Genetic Changes Made Us Uniquely Human?
- How Are Memories Stored and Retrieved?
- How Did Cooperative Behavior Evolve?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In a special collection of articles published beginning 1 July 2005, *Science* Magazine and its online companion sites celebrate the journal's 125th anniversary with a look forward -- at the most compelling puzzles and questions facing scientists today. A special, free news feature in *Science* explores 125 big questions that face scientific inquiry over the next quarter-century; accompanying the feature are several online extras including a reader's forum on the big questions. The Signal Transduction Knowledge Environment highlights some classic *Science* papers that have influenced the study of cell signaling. The Science of Aging Knowledge Environment looks at several important questions confronting researchers on aging. And *Science*'s Next Wave introduces us to four young scientists building their careers grappling with some of the very questions that *Science* has identified.

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How Did Cooperative Behavior Evolve?

REVIEW

Five Rules for the Evolution of Cooperation

Martin A. Nowak

Cooperation is needed for evolution to construct new levels of organization. Genomes, cells, multicellular organisms, social insects, and human society are all based on cooperation. Cooperation means that selfish replicators forgo some of their reproductive potential to help one another. But natural selection implies competition and therefore opposes cooperation unless a specific mechanism is at work. Here I discuss five mechanisms for the evolution of cooperation: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection. For each mechanism, a simple rule is derived that specifies whether natural selection can lead to cooperation.

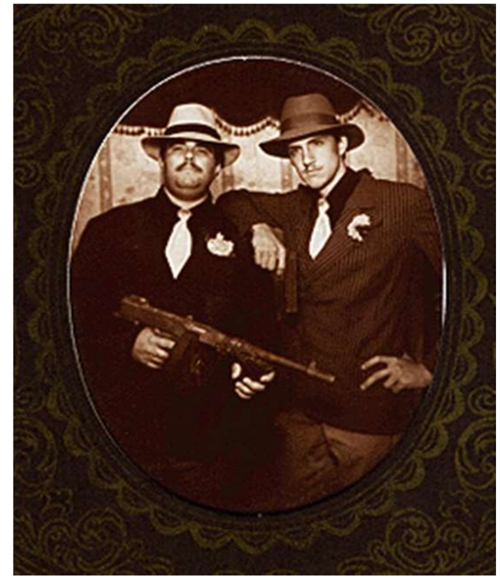
M. A. Nowak, *Science*, V314, 1560-1563, 2006

- Kin selection
- Direct reciprocity (“tit-for-tat”)
- Indirect reciprocity
- Network reciprocity
- Group selection

rich theory – limited predictive power…

Free-rider problem: the Prisoner's dilemma

	cooperate	defect
cooperate	$b-c$	$-c$
defect	b	0



$b > c > 0$

Mutual cooperation more profitable than mutual defection

But: under all circumstances to defect is the dominating strategy

LETTERS TO NATURE

Evolutionary games and spatial chaos

Martin A. Nowak & Robert M. May

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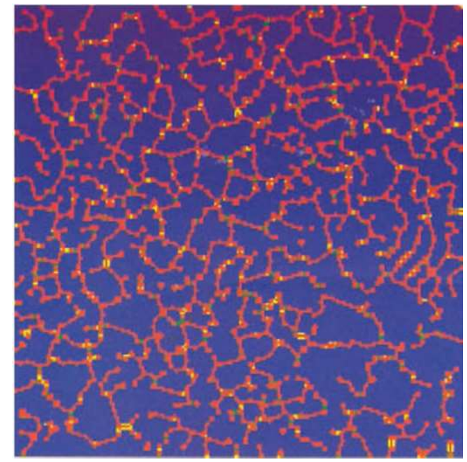
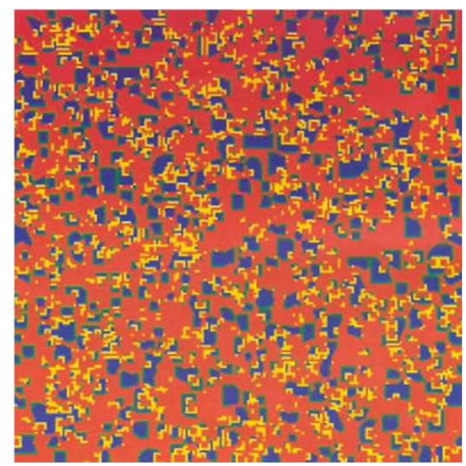
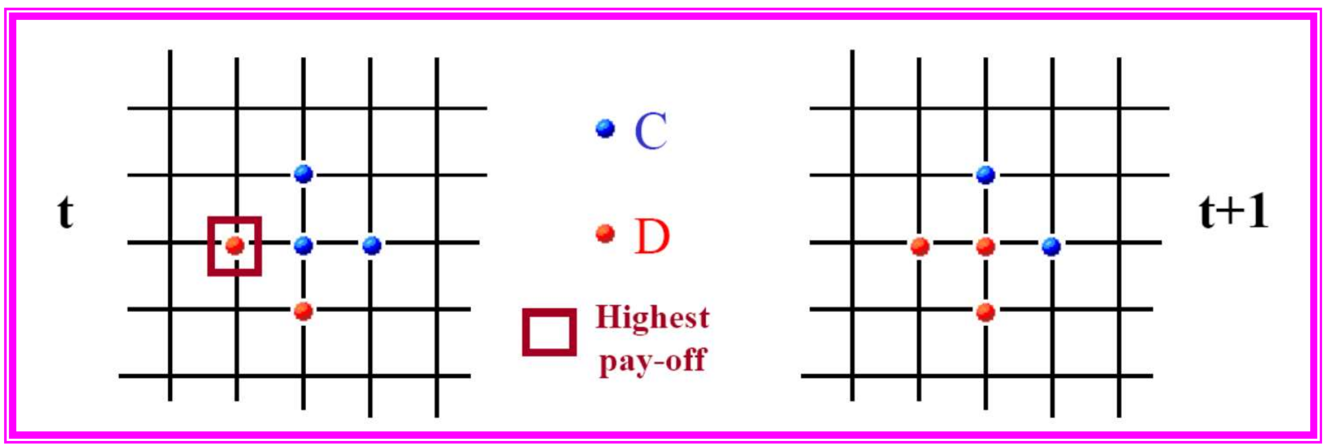
MUCH attention has been given to the Prisoners' Dilemma as a metaphor for the problems surrounding the evolution of cooperative behaviour[1–6]. This work has dealt with the relative merits of various strategies (such as tit-for-tat) when players who recognize each other meet repeatedly, and more recently with ensembles of strategies and with the effects of occasional errors. Here we neglect

M. A. Nowak, and R. M. May, *Nature*, V359, 826-829, 1992







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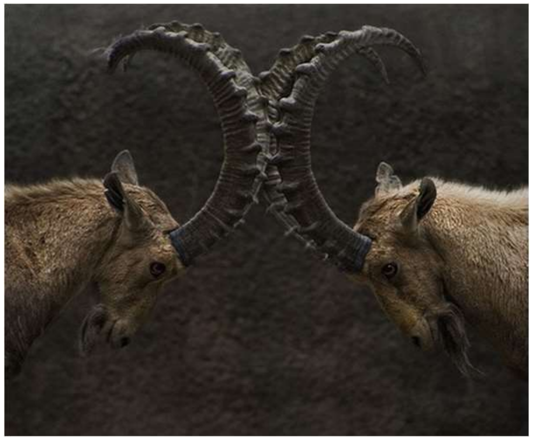
Evolutionary game theory: History and motivation



John Maynard Smith was interested in why so many animals engage in ritualized fighting ("The logic of animal conflict", *Nature*, 1973)

Evolutionary game theory (EGT) refers to the study of large populations of interacting agents, and how various behaviors and traits might evolve.

Differences from classical game theory

- Players = sub-populations, employing a common strategy
- Strategies = behaviors that update or traits encoded in genes
- Payoffs = fitness, which determines update or reproductive rates

Key concept: The fitness of an individual must be evaluated in the context of the population in which it lives and interacts

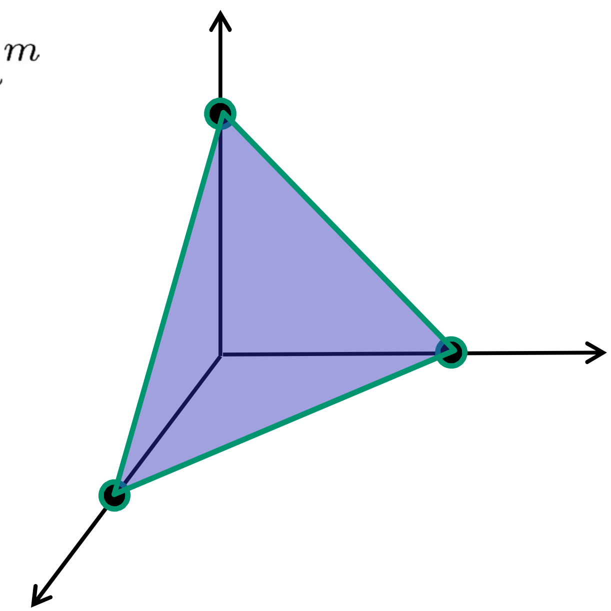
Dynamical system description for evolutionary games

Matrix game (symmetric two-player normal form with finitely many strategies)

Pure strategies: $e_1, e_2, \ldots, e_m$ unit vectors in $\mathbb{R}^m$

Mixed strategies: $p = (p_1, \ldots, p_m) \in \Delta^m$

where $\Delta^m = \{\sum_{i=1}^m p_i\}$
$p_i \geq 0$



- Payoff: Individuals interact in a two-player game
- Let $\pi(e_i, e_j)$ be the payoff of e_i against e_j, then the *m*-by-*m* payoff matrix *A* has entries $A_{ij} = \pi(e_i, e_j)$
- The payoff of *p* against *q* is $\pi(p, q) = \sum_{i,j=1}^{m} p_i \pi(e_i, e_j) q_j = p^T A q$

Dynamical system description for evolutionary games

Matrix game (symmetric two-player normal form with finitely many strategies)

Assumptions:

- Well-mixed population (no network structure)
- Random pairwise interaction per unit time
- Payoff translate directly into
  - (biological view) fitness that determines reproductive rate
  - (rational decision-making view) proportional imitation rate
- Individuals use pure strategies

Let n_i denote the number of individuals using strategy e_i at time t, then

$\dot{n}_i = n_i \pi(e_i, x)$ where $N = n_1 + n_2 + \ldots + n_m$ and $x_i = n_i / N$

- The payoff of p against q is $\pi(p,q) = \sum_{i,j=1}^{m} p_i \pi(e_i, e_j) q_j = p^T A q$

Replicator dynamics: $\dot{x}_i = x_i(\pi(e_i, x) - \pi(x, x))$

Key concept: The fitness of an individual must be evaluated in the context of the population in which it lives and interacts

Tempting claims for evolutionary game dynamics

Replicator dynamics: $\dot{x}_i = x_i(\pi(e_i, x) - \pi(x, x))$

Tempting claims for evolutionary game dynamics

Replicator dynamics: $\dot{x}_i = x_i(\pi(e_i, x) - \pi(x, x))$

Tempting claim: For the replicator dynamics of a matrix game, it holds that
(a) A converging trajectory in the interior of Δ^m evolves to a Nash equilibrium

*x** is a Nash equilibrium if for all *x*, $\pi(x, x^*) \leq \pi(x^*, x^*)$

Appealing: Nash equilibria are for individuals as rational decision makers, but replicator dynamics do not assume rationality of the individuals

"Folk theorem" of evolutionary game theory

Replicator dynamics: $\dot{x}_i = x_i(\pi(e_i, x) - \pi(x, x))$

~~**Tempting**~~ **claim:** For the replicator dynamics of a matrix game, it holds that
(a) A converging trajectory in the interior of Δ^m evolves to a Nash equilibrium

*x** is a Nash equilibrium if for all *x*, $\pi(x, x^*) \leq \pi(x^*, x^*)$

Evolutionarily stable strategy (ESS): *x** is ESS if for all other *x*, we have
(i) The Nash equilibrium condition: $\pi(x, x^*) \leq \pi(x^*, x^*)$
(ii) The stability condition: $\pi(x^*, x) > \pi(x, x)$ if $\pi(x, x^*) = \pi(x^*, x^*)$

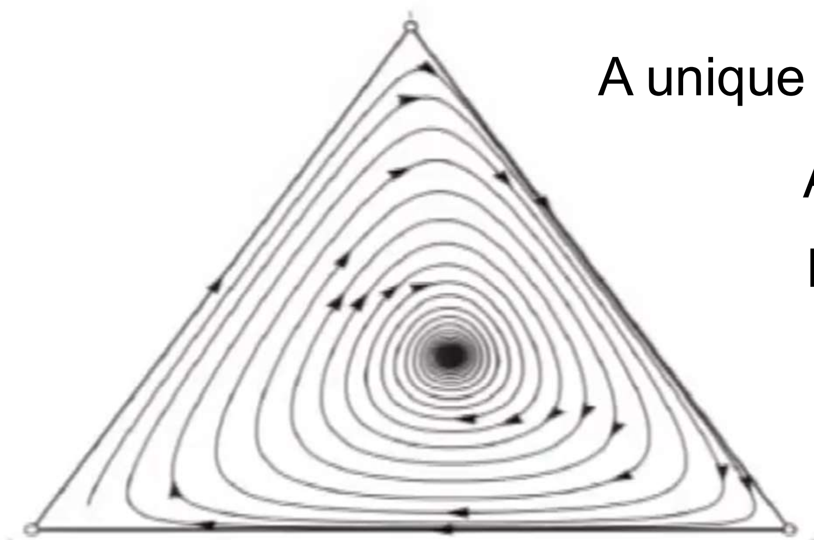
~~**Further**~~ **tempting statement:**
(b) A converging trajectory in the interior evolves to an ESS equilibrium

Generalized Rock-scissors-paper game

Consider the three-strategy matrix game with the payoff matrix

$$A = \begin{array}{c} \\ R \\ S \\ P \end{array} \begin{array}{c} \begin{array}{ccc} R & S & P \end{array} \\ \begin{pmatrix} 0 & 6 & -4 \\ -4 & 0 & 4 \\ 2 & -2 & 0 \end{pmatrix} \end{array}$$

Cyclic dominance: R beats S, S beats P and P beats R.



A unique Nash equilibrium: x^*= (10/29, 8/29, 11/29)

Almost globally asymptotically stable

But not ESS since

$$\pi(e_1, x^*) = \pi(x^*, x^*) = \frac{4}{29}$$
$$\pi(x^*, e_1) = -\frac{10}{29} < 0 = \pi(e_1, e_1)$$

The game with the payoff matrix –*A* reverses the trajectories. The interior trajectories do not converge to a Nash equilibrium, but a heteroclinic cycle

Theorem on ESS for replicator dynamics of a matrix game

(a) *x** is an ESS iff $\pi(x^*, x) > \pi(x, x)$ for all *x* near *x**

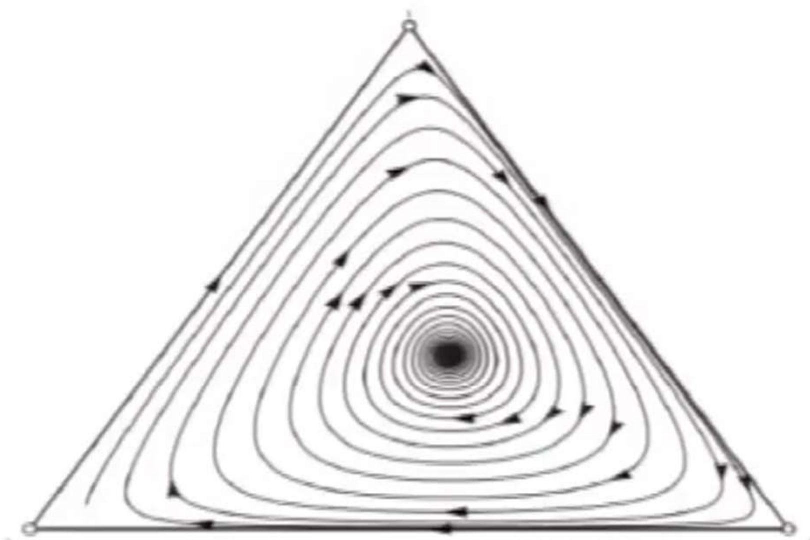
(b) An ESS *x** is a locally asymptotically stable equilibrium

(c) An interior ESS *x** is a almost globally asymptotically stable equilibrium

(a) Check (Weibull 1995, Apaloo 2006, Cressman 2010)

(b)(c) (Hofbauer et al, 1979) $V(x) = \sum x_i^{x_i^*}$

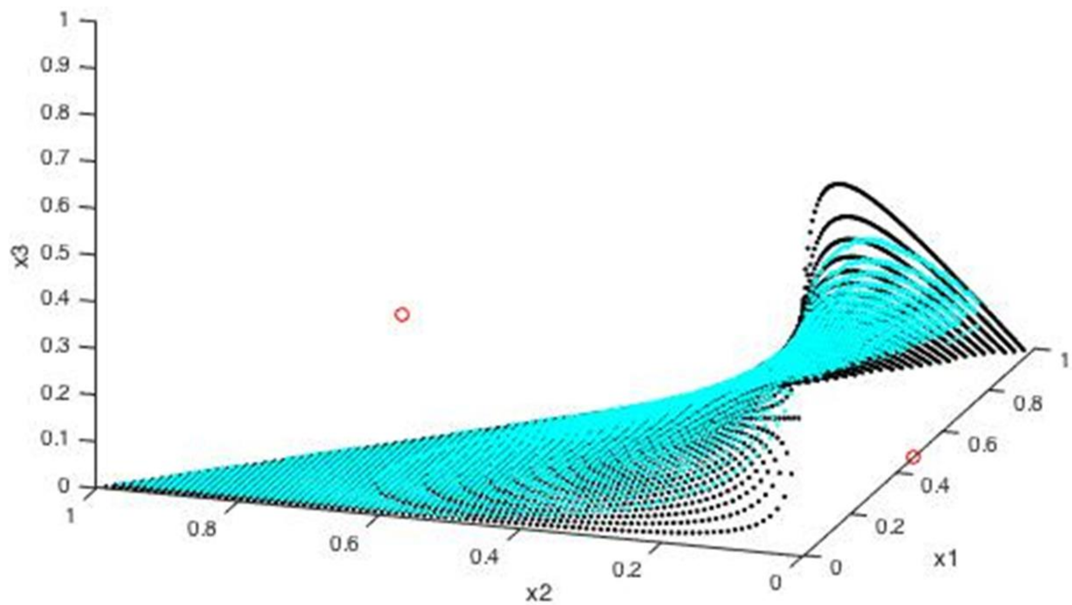
is a strict local Lyapunov function.



The game with the payoff matrix – *A* reverses the trajectories. The interior trajectories do not converge to a Nash equilibrium, but a heteroclinic cycle

Remark on convergence for 4^{th}- or higher-order replicator dynamics

Global analysis in general replies on the construction of Lyapunov function, which is in general difficult



Chaotic behavior can appear.

Two-population replicator dynamics

- Replicator dynamics for the matrix game with strategies $\{e_1, ..., e_m\}$

$$\dot{x}_i = x_i(\pi(e_i, x) - \pi(x, x)),$$

- Two-population replicator dynamics

$$\dot{p}_i^k = p_i^k[U_i^k(\mathbf{p}) - \bar{U}^k(\mathbf{p})], \qquad k = 1, 2$$

where p_i^k is the proportion of individuals in population k using s_i.

- We use x and y to denote the states of the two populations.

Payoff matrices with environmental feedback

Individuals from population 1 interact with individuals from population 2, and vise versa.

Dynamic payoff matrices

$$A(r)_{12} = \begin{bmatrix} a_{11}r + b_{11} & \dots & a_{1m}r + b_{1m} \\ \vdots & \ddots & \vdots \\ a_{m1}r + b_{m1} & \dots & a_{mm}r + b_{mm} \end{bmatrix}$$

$$A(r)_{21} = \begin{bmatrix} c_{11}r + d_{11} & \dots & c_{1m}r + d_{1m} \\ \vdots & \ddots & \vdots \\ c_{m1}r + d_{m1} & \dots & c_{mm}r + d_{mm} \end{bmatrix}$$

Dynamics of the environment

$$\dot{r} = r(1-r)h(x,y)$$

where $h(x,y)$ denotes the impact of population states on the environment, which may enhance or reduce resources.

Co-evolutionary game dynamics model

Consider

$$h(x,y) = \sum_{i\in\mathcal{S}} \mu_i x_i - \sum x_{-i} + \sum_{j\in\mathcal{S}} \rho_j y_j - \sum y_{-j}$$

with $\mu_i > 0,\ \ \rho_j > 0$ representing the ratios of enhancement to degradation.

Replicator dynamics with environmental feedback:

$$\Sigma : \begin{cases} \dot{x}_i = x_i[(A(r)_{12}\mathbf{y})_i - \mathbf{x}^T A(r)_{12}\mathbf{y}] \\ \dot{y}_j = y_j[(A(r)_{21}\mathbf{x})_j - \mathbf{y}^T A(r)_{21}\mathbf{x}] \\ \dot{r} = r(1-r)[\sum_{i,j\in\mathcal{S}}(\mu_i x_i + \rho_j y_j) - \sum(x_{-i} + y_{-j})], \end{cases}$$

where $(x_i, y_j, r) \in \Omega:\ \Delta^{m-1} \times \Delta^{m-1} \times I_{[0,1]}$.

Specific payoff matrices

Consider the following asymmetric payoff matrices

$$A(r) = (1-r)\begin{bmatrix} T_1 & P_1 \\ R_1 & S_1 \end{bmatrix} + r\begin{bmatrix} R_1 & S_1 \\ T_1 & P_1 \end{bmatrix}$$

$$B(r) = (1-r)\begin{bmatrix} T_2 & P_2 \\ R_2 & S_2 \end{bmatrix} + r\begin{bmatrix} R_2 & S_2 \\ T_2 & P_2 \end{bmatrix}$$

with $P_1 > S_1,\, T_1 > R_1;\, P_2 > S_2,\, T_2 > R_2$.

Each matrix has an embedded symmetry to ensure that mutual cooperation is a Nash equilibrium when $r = 0$ and mutual defection is a Nash equilibrium when $r = 1$.

Then the dynamics become

$$\Sigma_1 : \begin{cases} \dot{x} = x(1-x)[\delta_{PS_1} + (\delta_{TR_1} - \delta_{PS_1})y](1-2r) \\ \dot{y} = y(1-y)[\delta_{PS_2} + (\delta_{TR_2} - \delta_{PS_2})x](1-2r) \\ \dot{r} = r(1-r)[(1+\theta_1)x + (1+\theta_2)y - 2] \end{cases}$$

with $\delta_{PS_i} = P_i - S_i > 0,\, \delta_{TR_i} = T_i - R_i > 0$ and θ_k is the enhancement to degradation ratio in population *k*.

Observation from simple computations

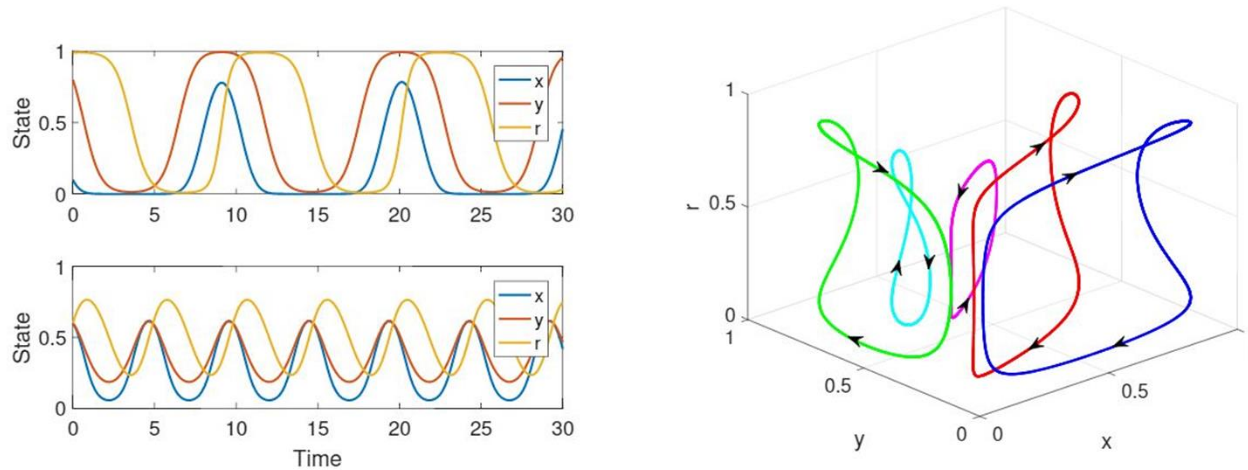
- Invariant cubic domain $I^3_{[0,1]} = [0,1]^3$
- eight isolated fixed points and one interior fixed point

$$\left\{(x, y, r) : (1+\theta_1)x + (1+\theta_2)y = 2, r = \frac{1}{2}\right\}$$

- The eight corner fixed points are unstable;
- The eigenvalues at the interior equilibrium are

$$\lambda_1 = 0,\ \lambda_{2,3} = \pm\sqrt{K}i, K > 0$$

- simulations



(a) Time Evolution (b) Periodic Orbits

Main result to prove

Theorem

The two-population co-evolutionary game dynamics have infinitely many periodic orbits in the interior of $I^3_{[0,1]} = [0,1]^3$.

Reversible system

- Reversible system

A dynamical system is said to be *reversible* if there is an involution *G* in its phase space which reverses the direction of time, i.e. the dynamics are invariant under a change in the sign of time.

- Periodic Orbits

An orbit (not a fixed point) is *periodic and symmetric* with respect to *G* if and only if the orbit intersects $\mathrm{Fix}(G)$ at precisely two points.

For system Σ_1, one can find

$$G : x \to x, y \to y, r \to 1 - r$$

$$Fix(G) : \quad \{r = 1/2\}$$

such that

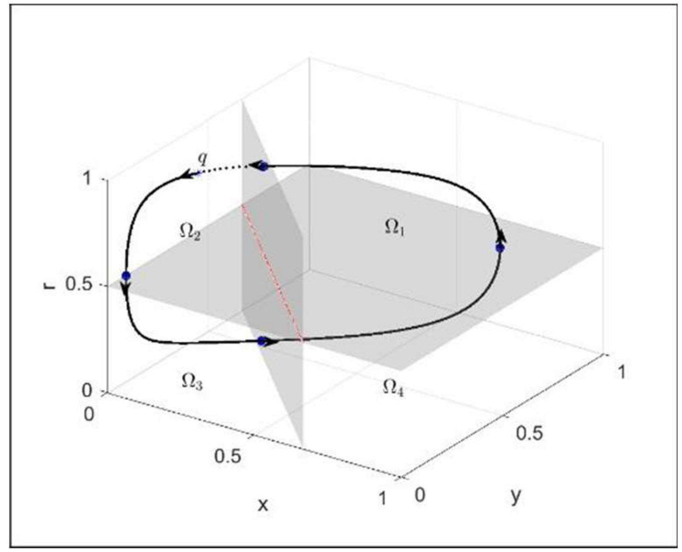
$$\Sigma_1 \overset{G}{\Rightarrow} \tilde{\Sigma}_1 \overset{G}{\Rightarrow} \Sigma_1$$

Viewpoint 1: Reversible system

Divide $I^3_{(0,1)} = (0,1)^3$ into four regions by the two planes

$$\{r = 1/2\}$$

$$\{(1+\theta_1)x + (1+\theta_2)y = 2\}$$



$$\Omega_1 : \{\frac{1}{2} < r < 1, (1+\theta_1)x + (1+\theta_2)y > 2\}$$

$$\Omega_2 : \{\frac{1}{2} < r < 1, (1+\theta_1)x + (1+\theta_2)y < 2\}$$

$$\Omega_3 : \{0 < r < \frac{1}{2}, (1+\theta_1)x + (1+\theta_2)y < 2\}$$

$$\Omega_4 : \{0 < r < \frac{1}{2}, (1+\theta_1)x + (1+\theta_2)y > 2\}$$

A typical trajectory

Consider an arbitrary trajectory starting from point *q*;

it goes across the plane $\{r = 1/2\}$ and into Ω_3;

crosses plane $\{(1+\theta_1)x + (1+\theta_2)y = 2\}$

and enters Ω_4;

then crosses plane $\{r = 1/2\}$ again;

returns to the starting point and forms a periodic orbit.

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A discrete-time stochastic model

Switching probability

$$u_{A\to B}=\frac{1}{1+e^{-\beta(\pi_B-\pi_A)}}$$

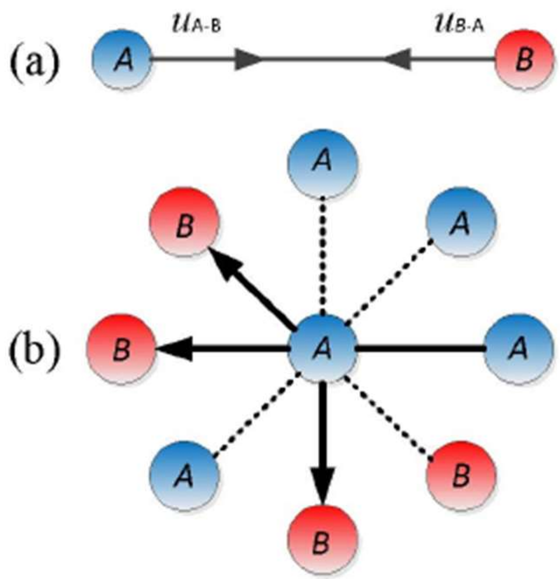
- Individuals' bounded rationality implies their limited cognition and decision-making capabilities
- Computations might be cognitively expensive and thus unfavorable
- False information, error, noise …

$$p_i\left(t+1\right)=p_i\left(t\right)[1-U^i_{A\to B}\left(t\right)]+[1-p_i\left(t\right)]U^i_{B\to A}\left(t\right)$$

$p_i(t)$ is agent i's probability to employ strategy A at time t

A discrete-time stochastic model

Switching probability



$$\begin{cases} U^{i}_{A\to B} = 1 - [1 - u_{A\to B}\left(1 - p_{\Omega_i}\right)]^{m} \\ U^{i}_{B\to A} = 1 - \left(1 - u_{B\to A} p_{\Omega_i}\right)^{m} \end{cases}$$

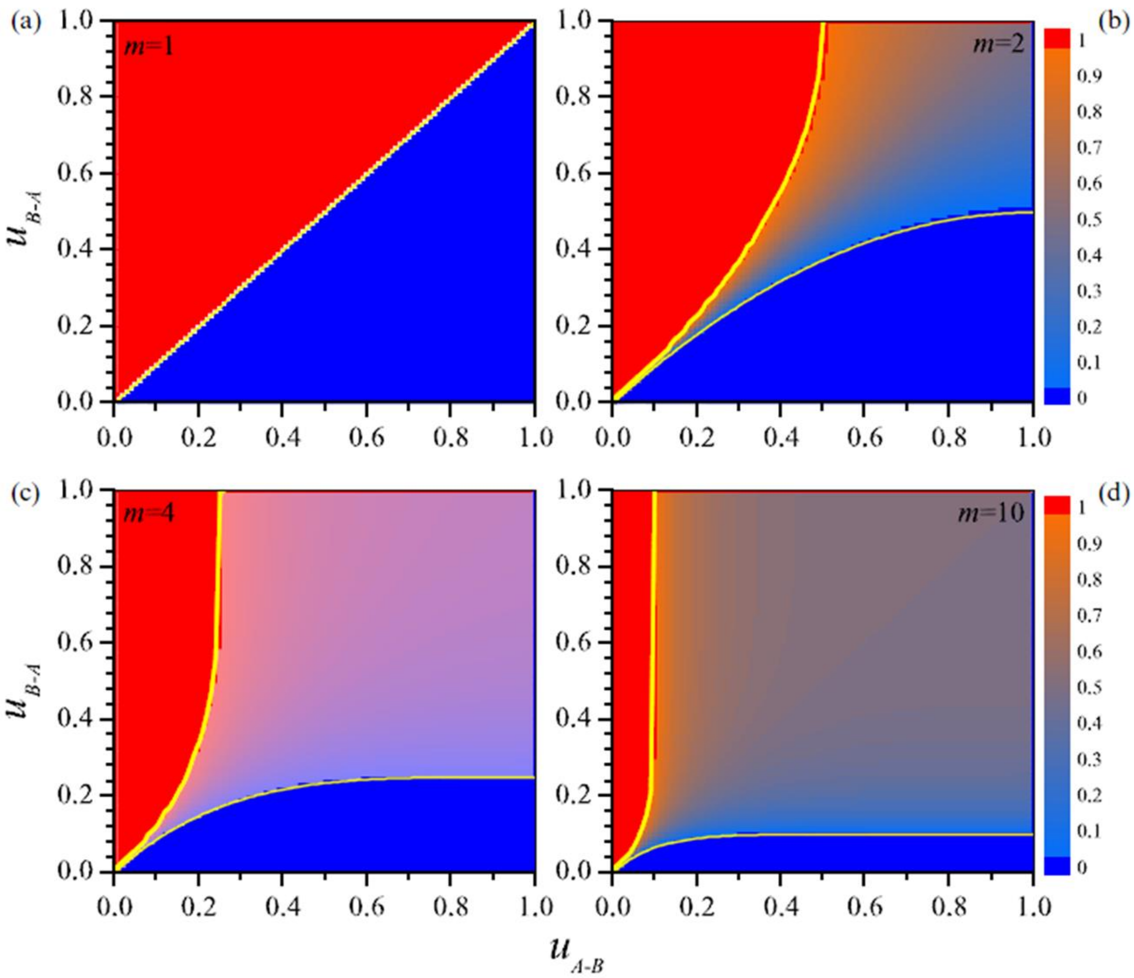
At equilibrium,

$$\hat{p}_i \cdot \hat{U}^{i}_{A\to B} = \left(1 - \hat{p}_i\right) \cdot \hat{U}^{i}_{B\to A}$$

Results

Coexistence of strategies for regular graphs:

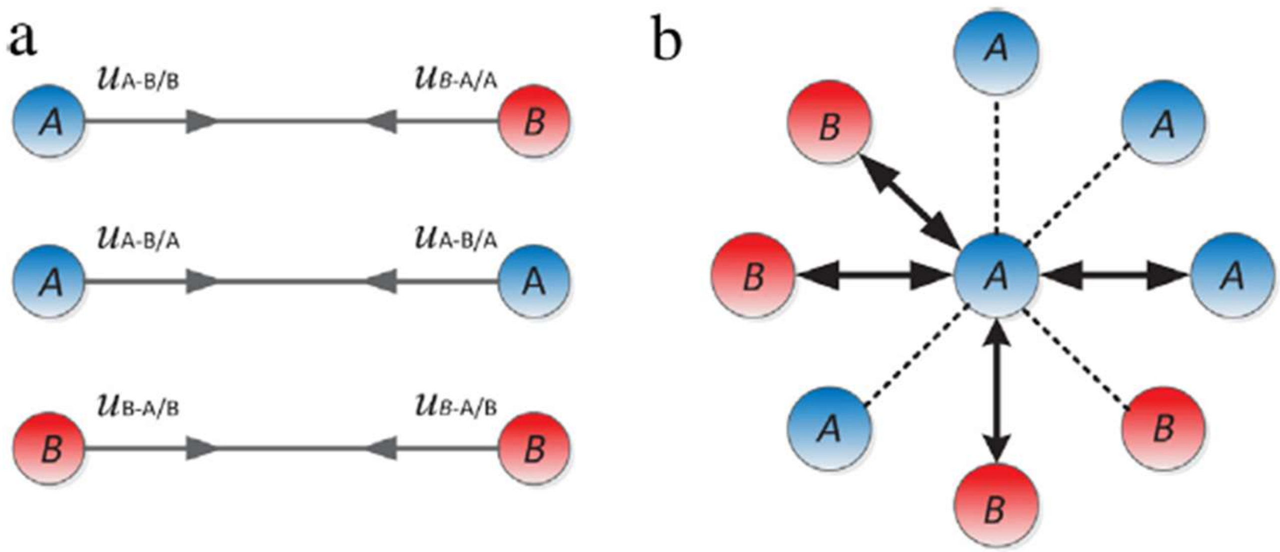
A *persists* if $u_{B\to A} > \frac{1-(1-u_{A\to B})^m}{m}$; B persists if $u_{A\to B} > \frac{1-(1-u_{B\to A})^m}{m}$



More sophisticated model

Strategy revisions:

$$A \rightarrow B \,|\, B \qquad\qquad A \rightarrow B \,|\, A$$



$$P_i(t+1) = P_i(t)\left[1 - U^i_{A\rightarrow B}(t)\right] + \left(1 - P_i(t)\right)U^i_{B\rightarrow A}(t)$$

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Agents' updating rule

Best-response: agents maximize payoff against current neighbor actions.

Imitation: agents copy the action of the highest earning neighbor.

$\mathcal{S}_i^M(t)$ = set of strategies earning the maximum payoff in the neighborhood of agent i:

$$\mathcal{S}_i^M(t) = \left\{ x_j(t) \,\middle|\, y_j(t) = \max_{k \in \mathcal{N}_i \cup \{i\}} y_k(t) \right\}.$$

$$x_i(t+1) = \begin{cases} A & \mathcal{S}_i^M(t) = \{A\} \\ B & \mathcal{S}_i^M(t) = \{B\} \\ x_i(t) & \mathcal{S}_i^M(t) = \{A, B\} \end{cases}$$

Incentive-based control of A-coordinating networks

Suppose we can offer an incentive r for taking a particular action.

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a+r & b+r \\ c & d \end{pmatrix}, \end{array} \qquad a, b, c, d, r \in \mathbb{R}$$

How much would it cost to have all agents converge to A?

Cases:

- Uniform incentives
- Targeted incentives
- Targeted incentives subject to a budget constraint

Uniform incentive-based control

All agents receive the same incentive

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a_i + r_0 & b_i + r_0 \\ c_i & d_i \end{pmatrix}, \end{array} \qquad a_i, b_i, c_i, d_i \in \mathbb{R}$$

Find the minimum value of the uniform incentive such that the entire network converges to *A*?

- ***A*-coordinating**: any agent who updates to Strategy *A* would also do so if some agents currently playing B were instead playing *A*
- ***A*-monotone**: Offering incentives to play *A* will never lead to an agent to switch away from *A*
- **Uniquely-convergent**: Offering incentives leads to a unique equilibrium

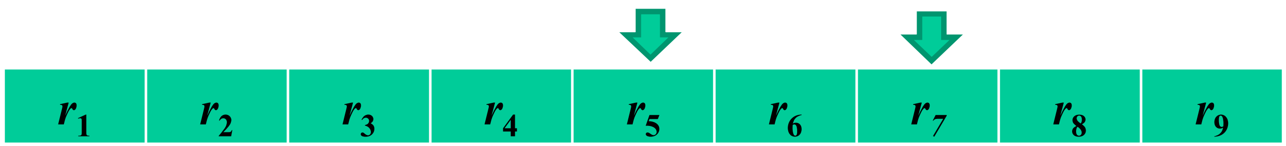
Theorem: Every network of *A*-coordinating agents is *A*-monotone and uniquely convergent.

Uniform incentive-based control

Proposition:
One can construct a finite set ***R*** that contains *r**

Because of the *A*-monotone property, one can carry out the binary search:

***R* :**



Theorem:
Within finite steps, binary search solves the uniform reward problem

Targeted incentive-based control

Suppose it's possible to offer different rewards to individual agents:

$$\begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} a_i + r_i & b_i + r_i \\ c_i & d_i \end{pmatrix} \end{array}, \quad a_i, b_i, c_i, d_i \in \mathbb{R}, \ r_i \in \mathbb{R}_{\geq 0}$$

Problem 1: Find $\mathbf{r} = (r_1, \dots, r_n)$ that minimizes $\sum_i \mathbf{r}$ such that the entire network converges to A.

Problem 2 (budget constraint): Find $\mathbf{r}$ that maximizes the number of agents who converge to A subject to $\sum_i \mathbf{r} \leq \rho$.

Targeted incentive-based control

Computationally complex to solve exactly (conjectured to be NP)

We can compute the incentive $\check{r}_i$ needed such that at least one A-neighbor will switch to B

$$\check{r}_i = \max_{j \in \mathcal{N}_i^B} \max_{k \in \mathcal{N}_j^B} y_k - y_i,$$

where $\mathcal{N}_i^B := \{j \in \mathcal{N}_i \cup \{i\} : x_j = B\}$.

Algorithm: Iteratively choose agents to switch until the desired equilibrium is reached or the budget limit is exceeded.

Targeted incentive-based control

How should we choose these agents?

Several possibilities: max degree, min required incentives, etc.

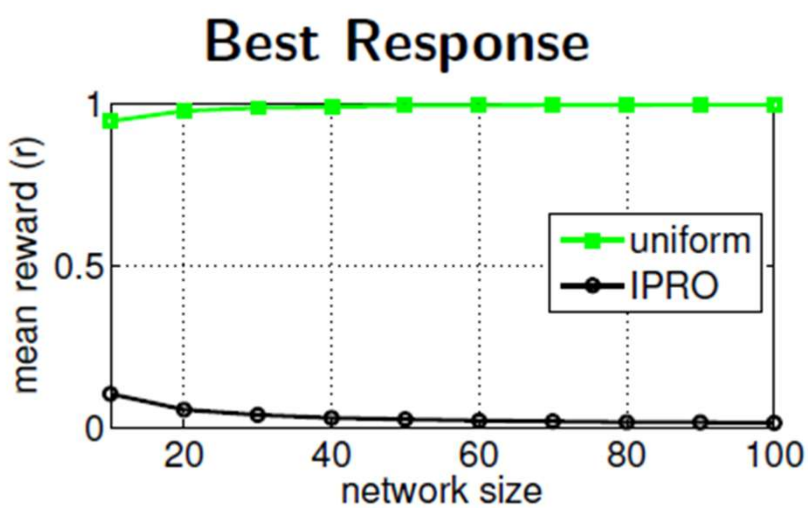
Approach: Iteratively maximize a benefit-to-cost ratio

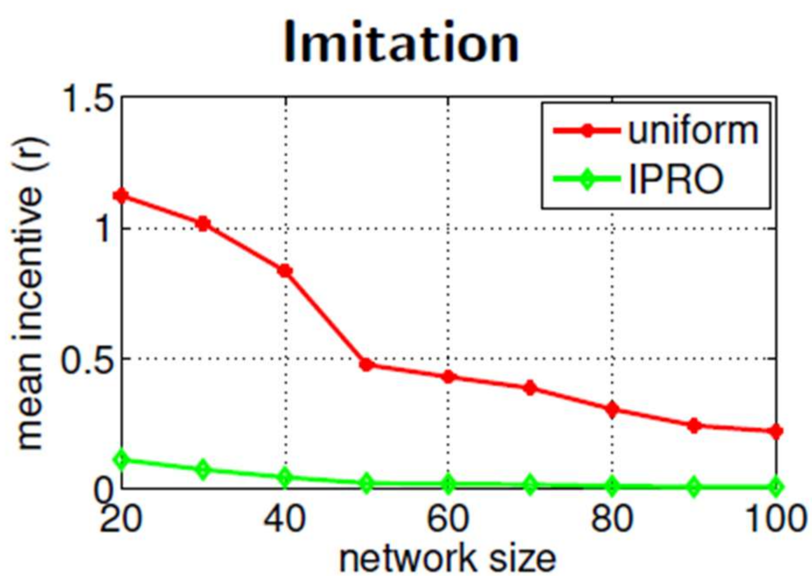
Benefit = # of agents who switch to *A*, cost = incentive

$$\max_i \frac{\Delta\Phi(x)^{\alpha}}{\check{r}_i^{\beta}}, \text{ where } \Delta\Phi(x) = \Phi(x(t_2)) - \Phi(x(t_1)),$$

$$\Phi(x) = \sum_{i=1}^{n} n_i^A(x), \qquad \alpha \text{ and } \beta \text{ are design parameters.}$$

Simulation results: Uniform vs. Targeted incentives





Outline

- Paradox of cooperation
- Evolutionary matrix games
- Continuous-time replicator dynamics
- Discrete-time dynamics driven by switching probabilities
- Controlling evolutionary games on networks

Outlook

- Extensions to different types of games and more than 2 strategies
- Convergence to equilibria with mixed strategies
- State feedback to render a closed loop system

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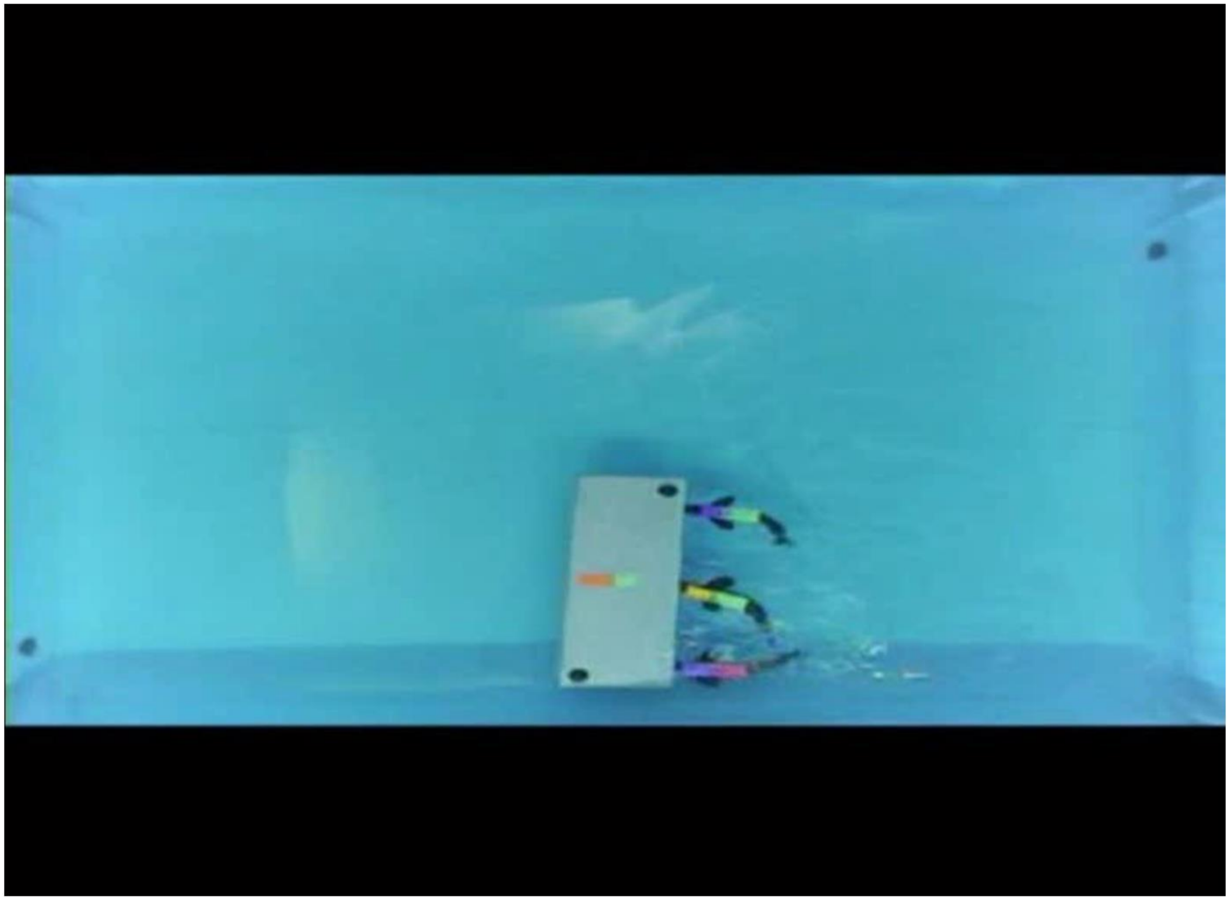

Applications:
Autonomous robots
Traffic flow
Communication networks
Energy grids
Social networks
Pandemic analysis
Marketing

Controlling evolutionary processes is an emerging new topic!

Some selected recent publications from my group on related topics

"Incentive-based control of asynchronous best-response dynamics on binary decision networks," J. R. Riehl, P. Ramazi, and M. Cao. *IEEE Trans. on Control of Network Systems*, 2019

"Evolutionary dynamics of two communities under environmental feedback," Y. Kawano, L. Gong, B. D. O. Anderson, and M. Cao. *IEEE Control Systems Letters*, special issue on Control and Network Theory for Biological Systems, 2019

"A survey on the analysis and control of evolutionary matrix games," J. R. Riehl, P. Ramazi and M. Cao. *Annual Reviews in Control*, 45(6), 87-106, 2018

"Asynchronous decision-making dynamics under best-response update rule in finite heterogeneous populations," P. Ramazi and M. Cao. *IEEE Transactions on Automatic Control*, 63(3), 742-751, 2018.

"Networks of conforming or nonconforming individuals tend to reach satisfactory decisions," P. Ramazi, J. R. Riehl, and M. Cao. *Proceedings of the National Academy of Science of USA (PNAS)*, 113(46), pp12985-12990, 2016

"Crucial role of strategy updating for coexistence of strategies in interaction networks," J. Zhang, C. Zhang, M. Cao and F. J. Weissing. *Phsical Review E*, 042101, 2015

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