The mathematics of actuation, decoupling, robustness properties for generically tilted multirotor platforms

a.k.a.

The Sparkling Mathematics of GTMs

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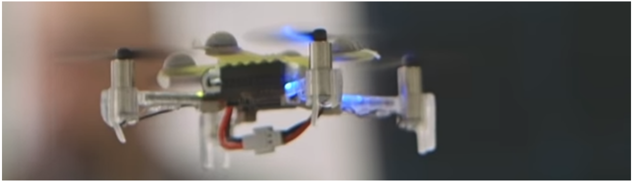
Control Days 2019

SPACE AND AERIAL CONTROL SYSTEMS

Department of Information Engineering, University of Padova



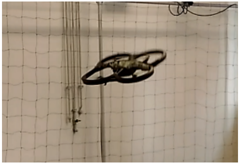
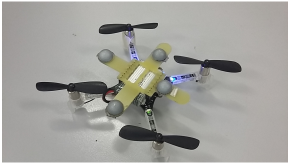
SPARCS





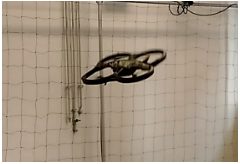

sparcs.dei.unipd.it   SPARCS group   sparcs.dei.unipd@gmail.com

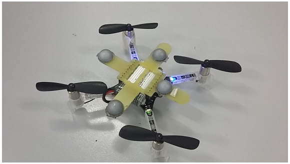
Unmanned
Aerial Vehicles
(**UAVs**)

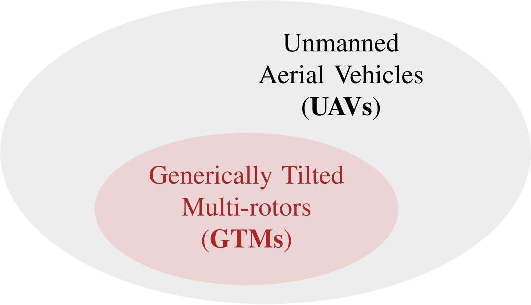




Unmanned
Aerial Vehicles
(UAVs)

- no pilot on-board
  (remotely controlled/autonomous)
- controllable propellers
  (uni/bi-directional spinning)
- navigation sensors
  (GNSS, GPS, IMU)







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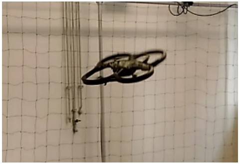

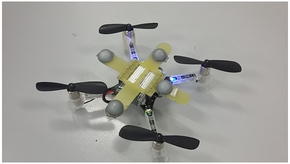

Generically Tilted Multi-rotor (**GTM**)

- rigid body
- $n \geq 4$ propellers
  with arbitrarly oriented spinning axes

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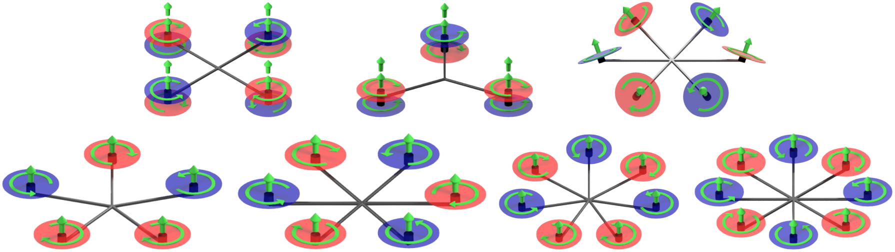
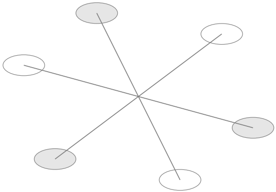


GTM dynamic model



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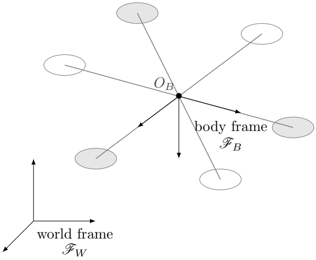
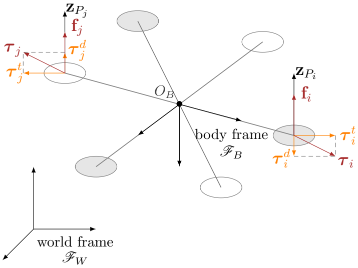


GTM dynamic model

each i-th propeller

- thrust force : $\mathbf{f}_i = c_{f_i}\omega_i^2 \mathbf{z}_{P_i}$
- drag moment : $\boldsymbol{\tau}_i^d = \pm c_{\tau_i}\omega_i^2 \mathbf{z}_{P_i}$
- thrust moment : $\boldsymbol{\tau}_i^t = c_{f_i}\omega_i^2 (\mathbf{p}_i \times \mathbf{z}_{P_i})$



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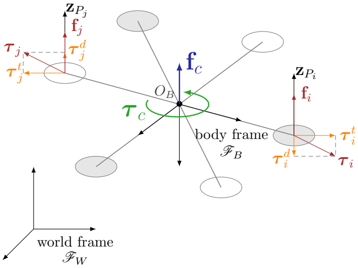
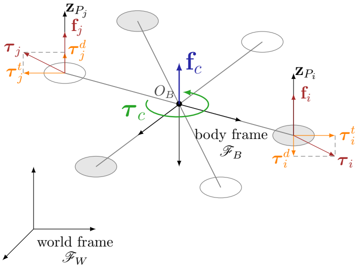


control force & moment

$$\mathbf{f}_c = \sum_{i=1}^n \mathbf{f}_i$$

$$\boldsymbol{\tau}_c = \sum_{i=1}^n (\boldsymbol{\tau}_i^t + \boldsymbol{\tau}_i^d)$$

GTM dynamic model



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kinematics:

$$\dot{\mathbf{p}} = \mathbf{v}$$
$$\dot{\mathbf{R}} = \mathbf{R}[\boldsymbol{\omega}]_\times$$

dynamics:

$$\dot{\mathbf{v}} = -g\mathbf{e}_3 + m^{-1}\mathbf{R}\mathbf{f}_c$$
$$\dot{\boldsymbol{\omega}} = J^{-1}(-\boldsymbol{\omega} \times J\boldsymbol{\omega} + \boldsymbol{\tau}_c)$$

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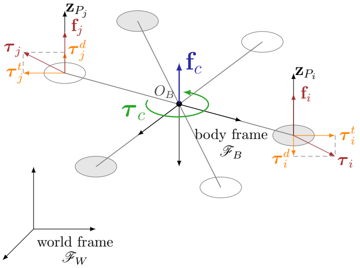
$$\mathbf{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} = \begin{bmatrix} \omega_1^2 \\ \vdots \\ \omega_n^2 \end{bmatrix}$$

control input vector



control force & moment

$$\mathbf{f}_c = \sum_{i=1}^{n} \mathbf{f}_i = f_1(\mathbf{u})$$

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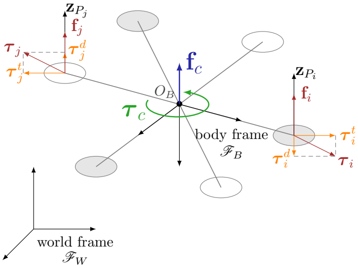

control force & moment

$\mathbf{f}_c = f_1(\mathbf{u}) = \mathbf{F}_1\mathbf{u}$

control force input matrix

$\boldsymbol{\tau}_c = f_2(\mathbf{u}) = \mathbf{F}_2\mathbf{u}$

control moment input matrix

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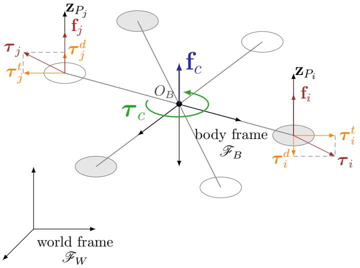

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GMTs structural properties

1. actuation & decoupling
2. rotor-failure robustness

GMTs structural properties

1. actuation & decoupling
2. rotor-failure robustness

both depending on
control input matrices
$\mathbf{F}_1$ and $\mathbf{F}_2$

GMTs structural properties

1. actuation & decoupling
2. rotor-failure robustness

in general
actuation analysis

$$\mathbf{f}_c = \mathbf{F}_1\mathbf{u}$$
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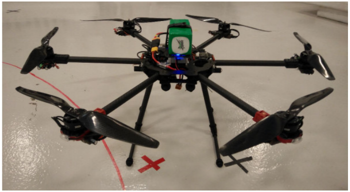
under-actuated systems

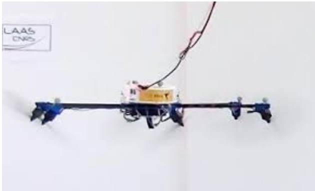
ctrl dofs $<$ 6



Popeye (KTH, Stockholm)

fully-actuated systems

ctrl dofs $=$ 6



Tilt-Hex (LAAS-CNRS, Toulouse)

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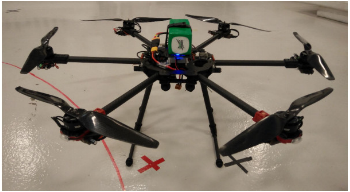

Popeye (KTH, Stockholm)

$$\mathrm{rk}\left(\begin{bmatrix}\mathbf{F}_1 \\ \mathbf{F}_2\end{bmatrix}\right) < 6$$

fully-actuated systems

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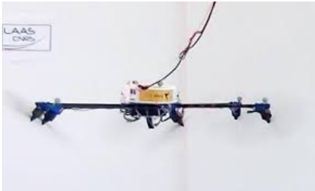

Tilt-Hex (LAAS-CNRS, Toulouse)

$$\mathrm{rk}\left(\begin{bmatrix}\mathbf{F}_1 \\ \mathbf{F}_2\end{bmatrix}\right) = 6$$

in our case
actuation and force&moment decoupling analysis

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actuation and force&moment decoupling analysis

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control force input matrix $\mathbf{F}_1 \in \mathbb{R}^{3\times n}$ $\quad 1 \leq \text{rk}(\mathbf{F}_1) \leq 3$

control moment input matrix $\mathbf{F}_2 \in \mathbb{R}^{3\times n}$ $\quad 1 \leq \text{rk}(\mathbf{F}_2) \leq 3$

- $\text{rk}(\mathbf{F}_2) = 3 \quad \Leftrightarrow$ *full orientation actuation*

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$$\mathbf{u} = \begin{bmatrix}\mathbf{A}_2 & \mathbf{B}_2\end{bmatrix} \begin{bmatrix}\tilde{\mathbf{u}}_A \\ \tilde{\mathbf{u}}_B\end{bmatrix} = \mathbf{A}_2 \tilde{\mathbf{u}}_A + \mathbf{B}_2 \tilde{\mathbf{u}}_B$$

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$$\mathbf{f}_c = \mathbf{F}_1\mathbf{u} = \mathbf{F}_1\mathbf{A}_2\tilde{\mathbf{u}}_A + \mathbf{F}_1\mathbf{B}_2\tilde{\mathbf{u}}_B =: \mathbf{f}_c^A + \mathbf{f}_c^B$$
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$$\mathbf{f}_c \in \mathfrak{F} := \mathrm{Im}(\mathbf{F}_1) \subseteq \mathbb{R}^3$$
$$\mathbf{f}_c^A \in \mathfrak{F}_A := \mathrm{Im}(\mathbf{F}_1\mathbf{A}_2) \subseteq \mathfrak{F}$$
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in our case
actuation and force&moment decoupling analysis

$\mathfrak{F} = \mathrm{Im}(\mathbf{F}_1)$ force space

$\mathfrak{F}_B = \mathrm{Im}(\mathbf{F}_1\mathbf{B}_2)$ zero moment force space

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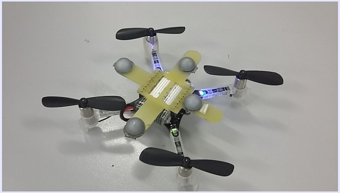
$\mathfrak{F}_B = \mathrm{Im}(\mathbf{F}_1\mathbf{B}_2)$ zero moment force space

		∃ decoupled direction		
			∃ decoupled plane	
	$\dim \mathfrak{F}_B = 0$	$\dim \mathfrak{F}_B = 1$	$\dim \mathfrak{F}_B = 2$	$\dim \mathfrak{F}_B = 3$
$\mathfrak{F}_B \subsetneq \mathfrak{F}$	FC	PC and SD1	PC and SD2	N/A
$\mathfrak{F}_B = \mathfrak{F}$	N/A	UC and SD1	UC and SD2	D3 (UC)
	$(\dim \mathfrak{F} \geq 1)$	$(\dim \mathfrak{F} \geq 1)$	$(\Rightarrow \dim \mathfrak{F} \geq 2)$	$(\Rightarrow \dim \mathfrak{F} = 3)$

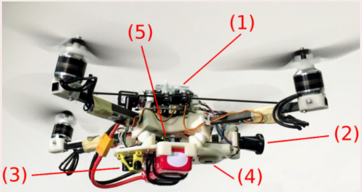
FC - fully coupled
PC - partially coupled
UC - un-coupled

SD1 - single decoupled direction
SD2 - single decoupled plane
D3 - full actuation



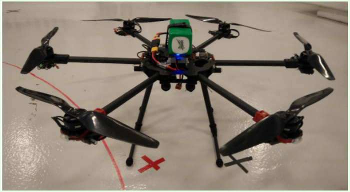
Crazyflie (Bitcraze)

UC & SD1



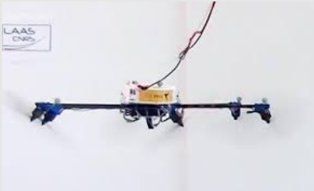
Tilted quadrotor (ETH, Zurich)

PC & SD2



Popeye (KTH, Stockholm)

UC & SD1



Tilt-Hex (LAAS-CNRS, Toulouse)

D3

GMTs structural properties

1. actuation & decoupling
2. rotor-failure robustness

$$\mathbf{f}_c = \mathbf{F}_1\mathbf{u}$$
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hovering realizability with uni-directional propeller spin

fly at a *constant reference position* with any *constant attitude* under the constraint $\mathbf{u} \geq 0$

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hovering realizability with uni-directional propeller spin

fly at a *constant reference position* with any *constant attitude* under the constraint $\mathbf{u} \geq 0$

- $\text{rk}(\mathbf{F}_2) = 3$
- $\exists \mathbf{u} > \mathbf{0}$ s.t. $\mathbf{F}_2\mathbf{u} = \mathbf{0}$
- $\exists \mathbf{u} \geq \mathbf{0}$ s.t. $\mathbf{F}_2\mathbf{u} = \mathbf{0}$ and $\mathbf{F}_1\mathbf{u} \neq \mathbf{0}$

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1) full orientation actuation

2) $\exists$ (at least) a direction in the control input space
   - zero control moment
   - non-zero control force direction

$$\exists \bar{\mathbf{u}} \in \ker(\mathbf{F}_2) \quad \text{s.t.} \quad \mathbf{d}_* := \mathbf{F}_1\bar{\mathbf{u}} \neq \mathbf{0} \text{ and } \|\mathbf{d}_*\| = 1$$

$$\mathbf{f}_c = \mathbf{F}_1\mathbf{u}$$
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hovering realizability with uni-directional propeller spin

fly at a *constant reference position* with any *constant attitude* under the constraint $\mathbf{u} \geq 0$

- $\text{rk}(\mathbf{F}_2) = 3$ → realizability of any control moment
- $\exists \mathbf{u} > \mathbf{0}$ s.t. $\mathbf{F}_2\mathbf{u} = \mathbf{0}$
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hovering realizability ensured for UC GTMs

robustness analysis

full robustness =
capability of realizing static hover after a propeller loss

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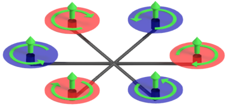
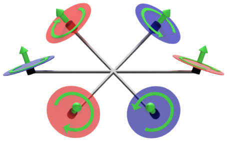


- ✖ collinear star-shaped hexarotor
- ✓ tilted star-shaped hexarotor



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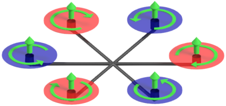

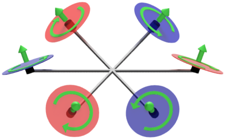

GMTs structural properties

1. actuation & decoupling
2. rotor-failure robustness

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both depending on
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$\mathbf{F}_1 = \mathbf{F}_1\left(\{\mathbf{p}_i, \mathbf{z}_{P_i}\}_{i=1}^n\right)$
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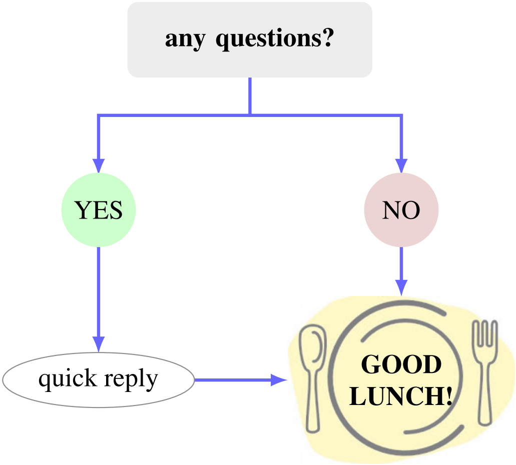
any questions?
YES
NO
quick reply
GOOD
LUNCH!

Thank you for your time!

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DIPARTIMENTO
DI INGEGNERIA
DELL'INFORMAZIONE

SPARCS

> I think that technologies are morally neutral until we apply them. It's only when we use them for good or for evil that they become good or evil.
>
> William Gibson